UNIVERSITI SAINS MALAYSIA

Semester I Examination
Academic Session 2004/2005

October 2004

EEE 503 – STOCHASTIC PROCESSES

Time : 3 hours

INSTRUCTION TO CANDIDATE:

Please ensure that this examination paper contains **FIVE** **(5)** printed pages and **SIX** **(6)** questions before answering.

Answer **FIVE** **(5)** questions.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

1. (a) The count of students dropping the course "Stochastic Processes" is known to be a Poisson process of rate 0.1 drops per day. Starting with day 0, the first day of the semester, let $D(t)$ denote the number of students that have dropped after t days. What is $P_{D(t)}(d)$?

(50%)

(b) Let Y_k denote the number of failures between successes k-1 and k of a Bernoulli (p) random process. Also, let Y_1 denote the number of failures before the first success. What is PMF $P_{Y_k}(y)$? Is Y_k an independent identically distributed *(iid)* random sequence?

(50%)

2. In a production line for $1000\ \Omega$ resistors, the actual resistance in ohms of each resistor is a uniform *(950, 1050)* random variable R. The resistances of different resistors are independent. The resistor company has an order for 1% resistors with a resistance between $990\ \Omega$ and $1010\ \Omega$. An automatic tester takes one resistor per second and measures its exact resistance. (This test takes one second). The random process $N(t)$ denotes the number of *1%* resistors found in t seconds. The random variables T_r seconds is the elapsed time at which r *1%* resistors are found.

(a) What is p, the probability that any single resistor is a *1%* resistor ?

(15%)

(b) What is the *PMF of* $N(t)$?

(15%)

(c) What is $E[T_1]$ seconds, the expected time to find the first *1%* resistors ?

(20%)

(d) What is the probability that the first *1%* resistor is found in exactly 5 seconds?

(20%)

(e) If the automatic tester finds the first *1%* resistor in 10 seconds, what is $E[T_2|T_1 = 10]$, the conditional expected value of the time of finding the second *1%* resistor ?

(30%)

3. $X(t)$ is a wide sense stationary stochastic process with autocorrelation function $R_X(\tau) = 10\ \sin(2\pi 1000t) / (2\pi 1000t)$. The process $Y(t)$ is a version of $X(t)$ delayed by 50 microseconds: $Y(t) = X(t - t_o)$ where $t_o = 5 \times 10^{-5}$ s.

(a) Derive the autocorrelation function of $Y(t)$.

(b) Derive the cross-correlation function of $X(t)$ and $Y(t)$.

(c) Is $Y(t)$ wide sense stationary ?

(d) Are $X(t)$ and $Y(t)$ jointly wide sense stationary ?

(100%)

4. (a) With reference to Figure 1 the transfer function G(s) of the causal Wiener filter is given by

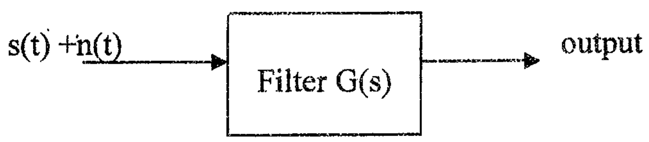

Figure 1

$$G(s) = \frac{1}{S_{s+n}^{+}}\left[positive - time \quad part \quad of \quad \frac{S_{s+n,s}\, e^{\alpha s}}{S_{s+n}^{-}} \right]$$

Where S_{s+n}^{+} is that part of spectral function S_{s+n} that has all its poles and zeros in the left half of the s-plane and the poles and zeros of S_{s+n}^{-} are mirror images of those of S_{s+n}^{+}. $R_{s+n,n}$ is the cross-correlation between s(t) and n(t). Assuming the prediction time to be zero, calculate G(s) if the signal and noise are uncorrelated and their respective spectral density functions are

$$S_s = \frac{2}{1-s^2} \quad , \quad S_n = 1$$

(70%)

(b) A random process is modeled as

$$x_{k+1} = \Phi_k\, x_k + w_k$$

where w_k is a n x 1 vector consisting of white noise sequence with power spectral density equal to unity. Write the covariance matrix of w_k.

(30%)

5. (a) Consider a random process with a spectral density function given by

$$S_y(j\omega) = \frac{16}{\omega^4 + (-4\omega)^2 + 16}$$

Determine the state space model of the process.

(40%)

(b) Draw the Kalman filter loop and explain different stages of the filter iterative process.

(60%)

6. Figure 2 shows combination of two independent noisy measurements of the same signal s(t). Using Weiner Filtering theory determine $G_1(s)$ and $G_2(s)$ that minimize the mean square error in the estimation of s(t) subject to the condition that the resulting structure does not introduce any delay or distortion in s(t).

(100%)

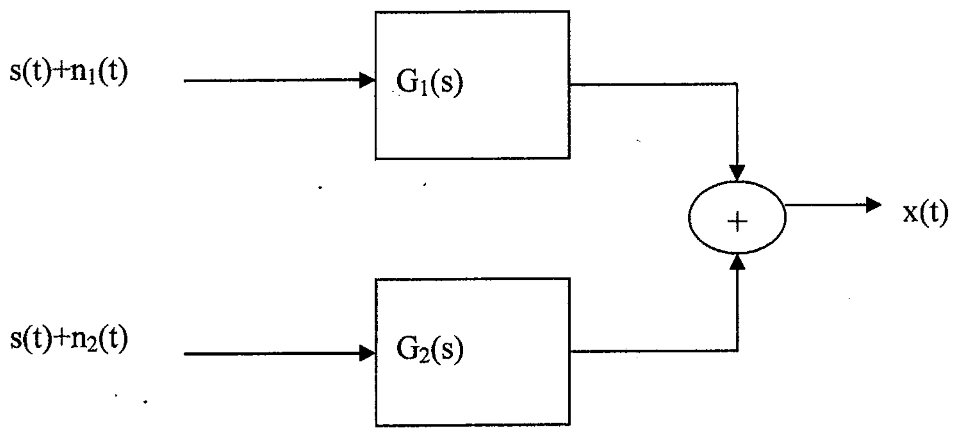

Figure 2

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