UNIVERSITI SAINS MALAYSIA

First Semester Examination

Academic Session 2004/2005

October 2004

EEE 501 – ADVANCED ENGINEERING MATHEMATICS

Time : 3 Hours

INSTRUCTION TO CANDIDATE:-

Please ensure that this examination paper contains **FIVE** (5) printed pages and **SIX** (6) question before answering.

Answer **FIVE** (5) questions.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

1. Elastic Deformation :

An elastic membrane in the x_1, x_2 - plane with boundary circle $x_1^2 + x_2^2 = 2$ is stretched so that a point P : (x_1, x_2) goes over into the point Q : (y_1, y_2) given by

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = AX = \begin{bmatrix} 4 & \sqrt{8} \\ \sqrt{8} & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ : in components}$$

$$y_1 = 4x_1 + \sqrt{8}\, x_2$$

$$y_2 = \sqrt{8}\, x_1 + 6\, x_2$$

Find the principle direction, that is, the direction of the position vector x of P for which the direction of the position of rector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation.

(100 marks)

2. (a) Determine the orthogonal trajectories of the given family of curves. Plot or sketch the curves and their trajectories on common axes. Show each step of your calculation.

(50 marks)

$$y = cx^2$$

(b) If in a reactor, uranium $_{92}U^{237}$ loses 10% of its weight within 1 day, what is its half-life? How long would it take for 99% of the original amount to disappear?

(50 marks)

3. Consider the boundary value problem

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

$$\frac{\partial u}{\partial x}(0, y)=\frac{\partial u}{\partial x}(a , y)=u(x , 0)=0$$

$$u(x , b)=f(x)$$

corresponding to a rectangular plate $0<x<a , 0<y<b$ with the edges $x=a$ and $x=b$ insulated. Derive the solution

$$u(x , y)=\frac{a_o y}{2 b}+\sum_{n=1}^{\infty} a_n\left(\cos\frac{n\pi x}{a}\right)\left[\frac{\sinh(n\pi y/a)}{\sinh(n\pi b/a)}\right]$$

where

$$a_n=\frac{2}{a}\int_0^a f(x)\cos\frac{n\pi x}{a}\,dx \quad (n=1,2,3,\cdots)$$

Hint:

Show first $\lambda_0=0$ is an eigen value with $X_0=1$ and $Y_0=1$.

(100 marks)

4. The distribution of an electrical potential ϕ in a cylinder of radius r is given by:

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=0$$

Perform conformal transformation on ϕ using bilinear operator. Hence, plot the complex fields in z and w planes.

(100 marks)

5. (a) Using the L U Factorization matrix, solve this linear equation system bellow :

$$x_1 + 3x_2 = \frac{1}{2} x_3 - 6$$
$$x_1 + \frac{1}{5} x_2 = \frac{2}{5} x_3 + \frac{29}{5}$$
$$x_1 + \frac{4}{3} x_2 = \frac{5}{3} x_3 - 1$$

(50 marks)

(b) From the values in the table below, find the missing value of current (approximation) corresponding to voltage 0.30 volt, use the Lagrange interpolation .

Voltage [volt]	0.00	0.10	0.30	0.50	0.90
Current [ampere]	0.00	0.05	.....	3.11	7.13

Lagrange Formula's :

$$P_n(x) = f(x) = \sum_{k=0}^{n} L_k(x) f(k) = \sum_{k=0}^{n} \frac{l_k(x)}{l_k(x_k)} f(k)$$

where :

$$l_0 = (x - x_1)(x - x_2)\ldots(x - x_n)$$
$$l_k(x) = (x - x_0)\ldots(x - x_{k-1})(x - x_{k+1})\ldots(x - x_n) \qquad 0 < k < n$$
$$l_n(x) = (x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

(50 marks)