UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2007/2008

Oktober/November 2007

EEE 453 – REKABENTUK SISTEM KAWALAN

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA BELAS muka surat dan ENAM muka surat LAMPIRAN bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan.

Jawab **LIMA** soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah bagi setiap soalan diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam bahasa Malaysia.

1. (a) Merujuk kepada sistem kawalan, terangkan konsep bagi
With reference to control systems, explain the concept of

(i) Vektor keadaan
State vector

(ii) Pembolehubah keadaan
State variable

(iii) Ruang keadaan
State space

(15%)

(b) Lakarkan satu gambarajah blok bagi mewakili satu sistem kawalan lelurus, masa-berterusan dalam keadaan ruang.
Tuliskan persamaan keadaan dan persamaan keluaran bagi sistem tersebut. Daripada persamaan keadaan dan persamaan keluaran, dapatkan rangkap pindah bagi sistem tersebut dalam sebutan Laplace.

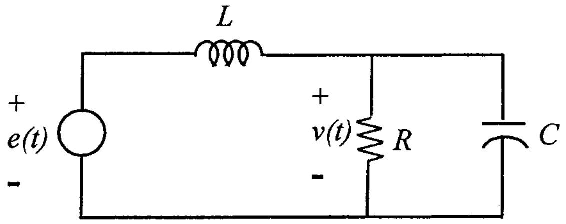
Draw the block diagram representation of the linear, continuous-time control system represented in state space.
Write the state equation and output equation for the system.
From the state and output equations, obtain the transfer function of the system in the Laplace domain.

(35%)

(c) Rajah 1 menunjukkan suatu rangkaian RLC. Input kepada sistem adalah sumber voltan, $e(t)$, dan output bagi sistem adalah voltan merintangi R. Diberi pembolehubah keadaan $x_1(t) = i_L(t)$, $x_2(t) = V_c(t)$, cari perwakilan ruang keadaan bagi sistem tersebut.

Figure 1 shows an RLC network. The input to the system is the voltage source, $e(t)$, and the output of the system is voltage across R. Given the state variables $x_1(t) = i_L(t)$, $x_2(t) = V_c(t)$, find the state-space representation of the system.

(50%)



Rajah 1
Figure 1

2\. (a) Apa itu taraf bagi suatu matrik?
What is the rank of a matrix?

Tentukan taraf bagi matrik berikut
Determine the rank of the following matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 4 \end{bmatrix}$$

(20%)

(b) (i) Merujuk kepada sistem kawalan, terangkan konsep bagi
With reference to control systems, explain the concept of

(1) Kebolehkawalan
Controllability

(2) Kebolehperhatian
Observability

(10%)

(ii) Pertimbangkan sistem berikut:
Consider the following system:

$$\frac{Y(s)}{U(s)} = \frac{s+6}{s^2+5s+6}$$

Dapatkan perwakilan keadaan ruang bagi sistem tersebut dalam bentuk
Obtain a state-space representation of the system in the form of

(1) kanonikal bolehkawal
controllable canonical

(2) kanonikal bolehperhati
observable canonical

(20%)

(c) (i) Dapatkan matrik transisi keadaan $\Phi(t)$ bagi sistem berikut:
Obtain the state-transition matrix $\Phi(t)$ for the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

...5/-

dengan keadaan awal
with the initial condition

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ (30%)

(ii) Pertimbangkan sistem berikut
Consider the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

Adakah sistem tersebut bolehkawal keadaan sepenuhnya?
Is the system completely state controllable?

(20%)

3. (a) (i) Terangkan perbezaan antara sistem regulator dengan sistem kawalan.
Explain the difference between regulator systems and control systems.

(15%)

(ii) Apakah keadaan yang diperlukan dan mencukupi untuk perletakan kutub yang sebarangan?
What is the necessary and sufficient condition for arbitrary pole placement?

(15%)

4. (a) Dengan berpandukan kepada Teori Pengenalan Sistem, jawab soalan berikut.

With reference to system identification theory, answer the following questions.

(i) Nyata sebab fundamental kepada Pengenalan Sistem dengan berpandukan kepada perkaitan antara masukan dan keluaran.

State the fundamental reasoning behind system identification with reference to input and output relationship.

(ii) Dengan bantuan gambarajah, terangkan asas gambaran dalam permasalahan sistem pengenalan kepada sebuah sistem yang perlu dikenal pasti.

With an aid of a diagram, explain the basic representation of system identification problem when given a system to be identified.

(iii) Prosedur dalam Pengenalan sistem dapat dinyatakan untuk menyelesaikan masalah mengenali sesebuah sistem. Nyatakan prosedur-prosedur tersebut.

The system identification can be explained in procedures that can be used to solve a system identification problem. State the procedures.

(60%)

(b) Terdapat TIGA (3) jenis definasi ralat yang dapat digunakan dalam meminimakan ralat. Nyatakan semua TIGA (3).

There are THREE (3) types of error definition that can be used to minimize error. State all THREE (3).

(6%)

(c) Nyatakan DUA (2) gambaran Model Sistem Dinamik.
State TWO (2) representation of Dynamic System Models.

(4%)

(d) Sebuah sistem dinamik yang digambarkan dengan satu-pembolehubah masa-tak berubah sistem diskret lelurus yang diberikan.

Given that a dynamic system is represented by the single-variable time-invariant linear discrete system shown below.

$$y(k)+a_1(y-1)+\ldots+a_n y(k-n)=b_0 u(k)+b_1 u(k-1)+\ldots+b_n u(k-n)$$

(i) Buktikan bahawa persamaan di atas dapat tulis sebagai
Proof that, the equation above can be written as

$$A(q^{-1})y(k))=B(q^{-1})u(k)$$ (20%)

(ii) Dengan mengunakan persamaan di atas, dapatkan persamaan rangkap pindah.
Using the equation above, obtain the transfer function of the equation.

(10%)

5. (a) Berpandukan Kaedah Kuasa Dua Terkecil, jawab soalan berikut.

With reference to least square theory, answer the following questions.

(i) Dapatkan pemerolehan matematik untuk persamaan garis lurus serta lakaran yang mengambarkan perkaitan ralat dan data keluaran.

Derive the mathematical solution for straight line least square error and provide a sketch to demonstrate the error relation to data output.

(30%)

(ii) Dengan menggunakan kaedah kuasa dua terkecil, padankan satu persamaan lurus ke dalam titik (x,y). Lakarkan hasilnya melalui titik dan garisan yang sesuai di kertas geraf..

Using the method of least square fit a straight line to the given points (x,y). Check the result by sketching the points and the line on a graph paper.

i	*x*	*y*
1	2	0
2	3	4
3	4	10
4	5	16

(40%)

(b) Di dalam kaedah Prinsipal kuasa dua terkecil yang melibatkan persamaan polynomial kita perlu memenuhi kehendak.
In the least square principle involving polynomial we seek to satisfy.

$$b_0 + b_i x_j + b_2 x_j^2 + \ldots + b_m x_j^m = y_j$$
$$(j = 1, \ldots, n)$$

dengan terbaik. Matrik C diperkenalkan dalam perumpamaan Cb=y. Tunjukkan bahawa Persamaan Normal dapat ditulis sebagai $C^{\tau}Cb=C^{\tau}y$.

as best as possible. Introduce a matrix C such that this can be written as Cb=y. Show that the normal equation can be written as $C^{\tau}Cb=C^{\tau}y$.

(20%)

(c) Satu persamaan sistem lelurus parameter-n dengan bilangan permerhatian m diberikan sebagai
Given that, a n-parameter linear system with m observations is given by

$$y(i) = \theta_1 x_1(i) + \theta_2 x_2(i) + \ldots + \theta_n x_n(i)$$
$$i = 1,2,\ldots m$$

Tuliskan persamaan untuk
Write the equation for

(i) Fungsi Kemerosotan
Regression function

(ii) Pekali Kemerosotan
Regression coefficient

(10%)

6. (a) Bincangkan objektif pengoptimuman parameter dengan merujuk kepada keluaran sebenar [c(t)] dan masukan rujukan r(t). Tuliskan persamaan yang mewakili indek ketentuan J.

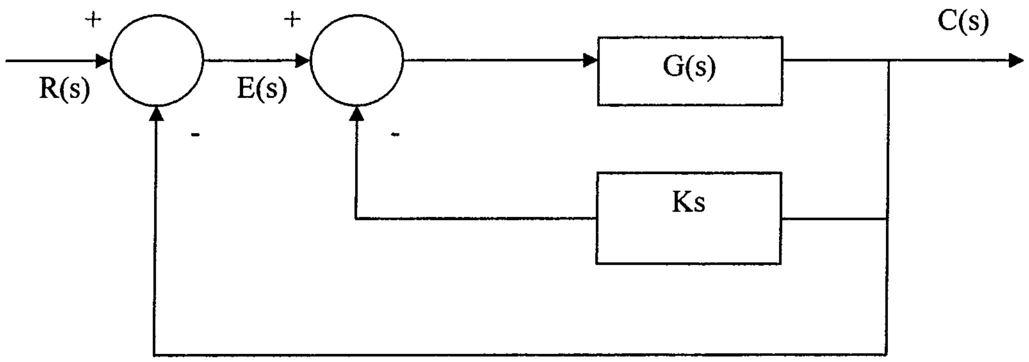
Discuss the objectives in parameter optimization with reference to actual output (c(t)) and reference input r(t). Write the equation representing the performance index, J.

(30%)

(b) Dengan berpandukan kepada gambarajah blok di Rajah 2, $G(s)=100/s^2$ dan $R(s) = 1/s$. Dapatkan persamaan untuk E(s) dan Tentukan nilai optimal parameter K yang mana Indek Ketentuan J adalah minimum.

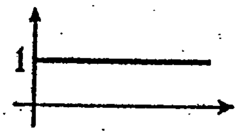
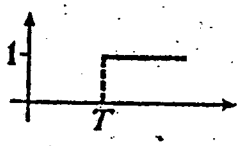
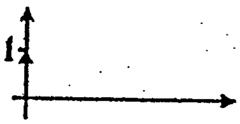
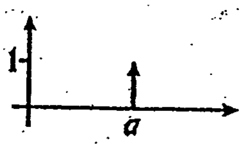
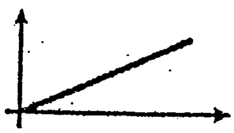
By referring to the block diagram shown in Figure 2, given that $G(s) = 100/s^2$ and $R(s) = 1/s$. Obtain the expression for E(s) and Determine the optimal value of parameter K such that the performance index, J is minimum.

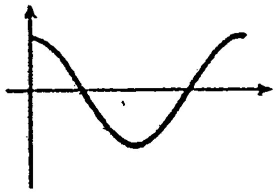
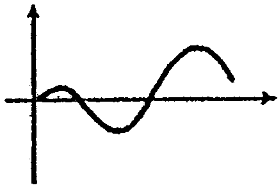
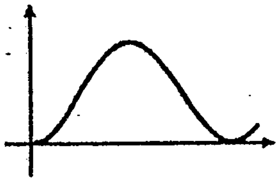
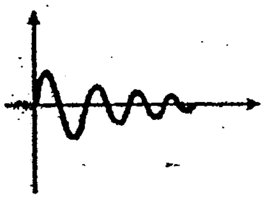
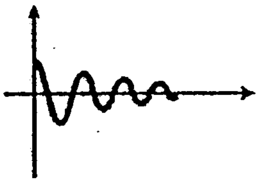
(40%)

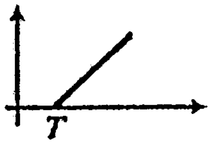
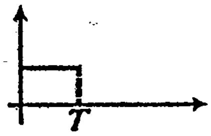
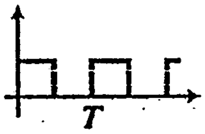
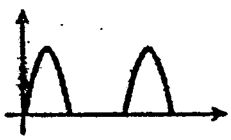
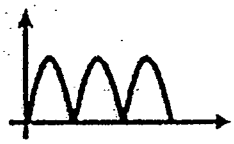


Rajah 2
Figure 2

A Table of Laplace Transforms

		$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$
1. Sum		$af(t) + bg(t)$	$aF(s) + bG(s)$ or $a\bar{f}(s) + b\bar{g}(s)$
2. First derivative		$\frac{d}{dt}f(t)$ or $f'(t)$	$sF(s) - f(0)$ or $s\bar{f}(s) - f_0$
3. Second derivative		$\frac{d^2}{dt^2}f(t)$ or $f''(t)$	$s^2F(s) - sf(0) - f'(0)$ or $s_2\bar{f}(s) - sf_0 - f_1$
4. Third derivative		$\frac{d^3}{dt^3}f(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$
5. Fourth derivative		$\frac{d^4}{dt^4}f(t)$	$s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0) - f'''(0)$
6. Definite integral		$\int_0^t f(t)\,dt$	$\frac{1}{s}F(s)$ or $\frac{1}{s}\bar{f}(s)$
		$\int_{-\infty}^t f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^0 f(t)\,dt$
7. Exponential multiplier		$e^{-\alpha t}f(t)$	$F(s+\alpha)$ or $\bar{f}(s+\alpha)$
8. Time shift		$f(t-T)u(t-T)$	$e^{-sT}F(s)$ or $e^{-sT}\bar{f}(s)$
9. Periodic function		$f(t) = f(t+T)$	$\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$
10. Convolution		$f(t) \times g(t) = \int_0^t f(t-u)g(u)\,du$	$F(s)G(s)$ or $\bar{f}(s)\bar{g}(s)$
11. Unit step		$u(t)$ or $H(t)$	$\frac{1}{s}$
12. Delayed step		$u(t-T)$	$\frac{1}{s}e^{-sT}$
13. Unit impulse		$\delta(t)$	1
14. Delayed unit impulse		$\delta(t-a)$	e^{-as}
15. Linear ramp		t	$\frac{1}{s^2}$

		$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$
28. Cosine wave		$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
29.		$\cos(\omega t \pm \phi)$	$\frac{s\cos(\phi) \pm \omega \sin(\phi)}{s^2+\omega^2}$
30.		$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
31.		$1-\cos(\omega t)$	$\frac{\omega^2}{s(s^2+\omega^2)}$
32.		$\sin(\omega t) - t\cos(\omega t)$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
33. Exponentially damped		$e^{-\alpha t}\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
34.		$e^{-\alpha t}\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
35.		$e^{-\alpha t}[\sin(\omega t) - \omega t\cos(\omega t)]$	$\frac{2\omega^3}{[(s+\alpha)^2+\omega^2]^2}$
36. Hyperbolic function		$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$
37.		$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$
38. Damped Hyperbolic		$e^{-\alpha t}\sinh(\omega t)$	$\frac{\omega}{(s+\alpha)^2-\omega^2}$
39.		$e^{-\alpha t}\cosh(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2-\omega^2}$
40.		$e^{-\alpha t}[\sinh(\omega t) - \omega t\cosh(\omega t)]$	$\frac{-2\omega^3}{[(s+\alpha)^2-\omega^2]^2}$

		$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$
41. (a)	$\zeta < 1$ and $\omega_d = \omega_0(1-\zeta^2)^{1/2}$ where ω_d is the frequency of free damped oscillation.	$u(t) - e^{-\zeta\omega_0 t} \times \left[\cos(\omega_d t) + \frac{\zeta\omega_0}{\omega_d}\sin(\omega_d t)\right]$	$\left[s\left(\frac{s^2}{\omega_0^2} + \frac{2\zeta s}{\omega_0} + 1\right)\right]^{-1}$ where ω_0 is the frequency of undamped oscillations, i.e. if $\zeta = 0$
(b)	$\zeta = 1$	$u(t) - e^{-\omega_0 t}[1 + \omega_0 t]$	
(c)	$\zeta = 1$ and $\beta = \omega_0(\zeta^2 - 1)^{1/2}$	$u(t) - e^{-\zeta\omega_0 t} \times \left[\cosh(\beta t) + \frac{\zeta\omega_0}{\beta}\sinh(\beta t)\right]$	
42.	Delayed ramp 	$(t-T)u(t-T)$	$\frac{1}{s^2}e^{-sT}$
43.	Rectangular pulse 	$u(t) - u(t-T)$	$\frac{1}{s}(1 - e^{-sT})$
44.	Rectangular periodic 	$f(t) = \begin{cases} 1 & 0 < t < T/2 \\ 0 & T/2 < t < T \end{cases}$	$\frac{1}{s(1 + e^{-sT/2})}$
45.	Half-wave-rectified sine, period $T = 2\pi/\omega$ 	$f(t) = \begin{cases} \sin(\omega t) & 0 < t < t/2 \\ 0 & T/2 < t < T \end{cases}$	$\frac{\omega}{(s^2+\omega^2)(1 - e^{-\pi s/\omega})}$
46.	Full-wave-rectified sine, period $T = 2\pi/\omega$ 	$f(t) = \lvert\sin(\omega t)\rvert$	$\frac{\omega}{s^2+\omega^2}\frac{(1 + e^{-\pi s/\omega})}{(1 - e^{-\pi s/\omega})}$

Initial value theorem $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$

Final value theorem $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Z-Transform Table

	$x(t)$ or $x(k)$	$\mathcal{Z}[x(t)]$ or $\mathcal{Z}[x(k)]$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\cdots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\cdots-zx(k-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}[(1-z^{-1})X(z)]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n} x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^mx(k)$	$\left(-z\frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^{n} x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$

Table 1 Tabulation of Definite Integral for Continuous-Time Systems

$$J_n = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)}ds$$

$$B(s) = \sum_{k=0}^{n-1} b_k s^k$$

$A(s) = \sum_{k=0}^{n} a_k s^k$; $A(s)$ has zeros in left half plane only.

$$J_1 = \frac{b_0^2}{2a_0a_1}$$

$$J_2 = \frac{b_1^2 a_0 + b_0^2 a_2}{2a_0a_1a_2}$$

$$J_3 = \frac{b_2^2 a_0 a_1 + (b_1^2 - 2b_0b_2)a_0a_3 + b_0^2 a_2 a_3}{2a_0a_3(-a_0a_3 + a_1a_2)}$$

$$J_4 = \frac{b_3^2(-a_0^2a_3 + a_0a_1a_2) + (b_2^2 - 2b_1b_3)a_0a_1a_4 + (b_1^2 - 2b_0b_2)a_0a_3a_4 + b_0^2(-a_1a_4^2 + a_2a_3a_4)}{2a_0a_4(-a_0a_3^2 - a_1^2a_4 + a_1a_2a_3)}$$

Table 2 Tabulation of Definite Integral for Sampled-Data Systems

$$J_n = \frac{1}{2\pi j}\oint_{\substack{unit\\circle}} X(z)X(z^{-1})\frac{dz}{z}$$

$$X(z) = \frac{b_0z^n + b_1z^{n-1} + \cdots + b_n}{a_0z^n + a_1z^{n-1} + \cdots + a_n}$$

$$J_1 = \frac{(b_0^2 + b_1^2)a_0 - 2b_0b_1a_1}{a_0(a_0^2 - a_1^2)}$$

$$J_2 = \frac{B_0a_0e_1 - B_1a_0a_1 + B_2(a_1^2 - a_2e_1)}{a_0[(a_0^2 - a_2^2)e_1 - (a_0a_1 - a_1a_2)a_1]}$$

where

$$B_0 = b_0^2 + b_1^2 + b_2^2$$
$$B_1 = 2(b_0b_1 + b_1b_2)$$
$$B_2 = 2b_0b_2$$
$$e_1 = a_0 + a_2$$

$$J_3 = \frac{a_0B_0Q_0 - a_0B_1Q_1 + a_0B_2Q_2 - B_3Q_3}{[(a_0^2 - a_2^2)Q_0 - (a_0a_1 - a_2a_3)Q_1 + (a_0a_2 - a_1a_3)Q_2]a_0}$$

$$B_0^2 = b_0^2 + b_1^2 + b_2^2 + b_3^2$$
$$B_1 = 2(b_0b_1 + b_1b_2 + b_2b_3)$$
$$B_2 = 2(b_0b_2 + b_1b_3)$$
$$B_3 = 2b_0b_3$$
$$Q_0 = (a_0e_1 - a_3a_2)$$
$$Q_1 = (a_0a_1 - a_1a_3)$$
$$Q_2 = (a_1e_2 - a_2e_1)$$
$$Q_3 = (a_1 - a_3)(e_2^2 - e_1^2) + a_0(a_0e_2 - a_3e_1)$$
$$e_1 = a_0 + a_2$$
$$e_2 = a_1 + a_3$$