DIVISION-BASED METHODS FOR LARGE POINT SETS REGISTRATION

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2016

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by

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Thesis submitted in fulfilment of the requirements
for the degree of
Doctor of Philosophy

March 2016

ACKNOWLEDGEMENT

I would like to extend my deepest gratitude to my supervisor, Associate Professor Dr. Bahari Belaton, from the School of Computer Sciences at Universiti Sains Malaysia, to my co-supervisor, Associate Professor Dr. Munir Zaman, from the Faculty of Engineering at University of Nottingham Malaysia Campus, and to my co-supervisor, Associate Professor Dr. Iman Yi Liao, from the School of Computer Science at University of Nottingham Malaysia Campus for their wise guidance, valuable advice, encouragement, and partial project supporting throughout the duration of my study. Without their cooperation, I could not have completed this study. I would especially like to thank Dr. Iman for her full supervision and generous help during my Ph.D journey. Her teaching will continue to inspire and motivate my throughout all my future research.

I would also like to thank my lab mates Tan Joisan, Pan Zheng and many other friends around me for the discussions, suggestions, and kindhearted assistance.

I would also like to extend my gratitude to the School of Computer Sciences, Universiti Sains Malaysia, for providing the facilities necessary to my study, and to all academic and non-academic members of the School of Computer Sciences for their friendly assistance. My thanks are also to Hebei University, China, for providing this opportunity and their support to me.

My deepest gratitude and appreciation to my beloved daughter He Miaofei, my beloved husband He Qiang, my parents, my parents-in-law, and other family members for their emotional support, encouragement, and patience during my Ph.D study and thesis preparation from January 2011 to today.

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LIST OF ABBREVIATIONS

ANN Artificial Neural Network

BnB Branch-and-Bound

CPD Coherent Point Drift

DA Deterministic Annealing

ECM Expectation Conditional Maximization

EM Expectation Maximization

FGT Fast Gauss Transform

GMMs Gaussian Mixture Models

GPR Gaussian Process Regression

GRBF Gaussian Radial Basis Function

ICP Iterative Closest Point

KC Kernel Correlation

LiDAR Light Detection and Ranging

LLE Local Linear Embedding

LM Levenberg-Marquardt

LRA Low-Rank matrix Approximation

MCT Motion Coherence Theory

ML Maximum Likelihood

MLE Maximum Likelihood Estimation

MSE Mean Squared Error

PCA Principle Component Analysis

PDFs Probability Density Functions

PR Points Registration

QPCCP Quadratic Programming based on Cluster Correspondences Projection

RKHS Reproducing Kernel Hilbert Space

RPM Robust Point Matching

SGCPD Structure-guided Coherent Point Drift

SMM Student's-t Distribution Mixture Models

SVD Singular Value Decomposition

TPS Thin-plate Splines

KAEDAH BERASASKAN PEMBAHAGIAN UNTUK PENDAFTARAN SET TITIK BESAR

ABSTRAK

Pendaftaran set titik adalah satu langkah penting untuk mengukur persamaan antara dua set titik dan digunakan secara meluas dalam penglihatan komputer, grafik komputer, analisis imej perubatan, dan sebagainya. Peralatan semasa mampu menyediakan data dengan butiran terperinci sebagai set titik besar. Walau bagaimanapun, prestasi kaedah pendaftaran konvensional menurun secara mendadak apabila saiz set titik meningkat. Dalam tesis ini, tiga kaedah pendaftaran set titik terkenal dan antara yang mempunyai prestasi terbaik dipertimbangkan untuk mengkaji pengubahan kaedah konvensional kepada kaedah yang menangani pendaftaran set titik besar dengan cekap. Kaedah-kaedah tersebut adalah Lelaran Titik Terdekat (ICP), Peralihan Titik Bersambung (CPD) dan Model Campuran Gaussian berasaskan Plat-nipis Splin. Pertama, dicadangkan kaedah ICP subset berasaskan pembahagian alir-terus untuk pendaftaran tegar set titik besar. Daripada mengenakan pendaftaran kepada set titik penuh seperti ICP asal, kaedah ini mendapatkan semula putaran dan translasi antara set titik penuh dengan hanya menggunakan penghubung di antara sepasang subset. Kaedah ini melelar menerusi semua pasangan subset sehingga penumpuan dicapai. Kedua, pembahagian alir-terus digabungkan dengan kaedah CPD untuk pendaftaran bukan-tegar set titik besar bagi mendapatkan anjakan bukan-linear di antara setiap pasangan titik dalam kalangan dua set. Tidak seperti ICP subset, kaedah ini memanjangkan sedi-

kit setiap subset kepada subset jiran untuk penggelintaran titik penghubung mantap. Hasil pendaftaran bagi semua pasangan subset digabungkan secara terus. Gelintaran heuristik untuk menala parameter lebar kernel Gaussian dalam kaedah CPD asal juga dicadangan, iaitu S-CPD-B. Akhir sekali, bi-GMM-TPS dicadangkan sebagai kaedah berasaskan dua-tahap untuk pendaftaran bukan-tegar set titik besar. Kaedah ini menggunakan skim pengelompokan untuk mendapatkan kelompok dan mendaftarkan pusat kelompok menggunakan GMM-TPS bagi menjajarkan secara kasar kedua-dua set titik penuh. Kaedah ini kemudiannya mendaftarkan secara halus semua pasangan kelompok menggunakan kaedah GMM-TPS sekali lagi. Eksperimen dijalankan secara meluas untuk mengesahkan keberkesanan kaedah yang dicadang menggunakan set data yang tersedia kepada orang ramai termasuk set data titik yang sangat besar dari pangkalan data USF. Keputusan eksperimen menunjukkan bahawa algoritma yang dicadangkan mampu mengurangkan kos pengiraan serta mengekalkan ralat pendaftaran setanding dengan kaedah asal. Secara konsep tiga kaedah khusus pendaftaran berasaskan pembahagian dirumus sebagai satu kerangka pendaftaran berbilang tahap untuk menangani pendaftaran bukan-tegar set titik besar.

DIVISION-BASED METHODS FOR LARGE POINT SETS REGISTRATION

ABSTRACT

Point sets registration is a key step for measuring the similarity between two point sets and widely used in various fields such as computer vision, computer graphics, medical image analysis, to name a few. The current devices can capture data with great details as large point set. However, conventional registration methods slow down dramatically as the size of the point set increased. In this thesis, three well-known and among-best-performance point sets registration methods incorporating division schemes are considered to study transforming conventional methods to efficiently deal with large point sets registration. These methods are Iterative Closest Point (ICP), Coherent Point Drift (CPD), and Gaussian mixture models based on thin-plate splines (GMM-TPS). Firstly, a subset-ICP method is proposed based on streaming division for rigid registration of large point sets. Instead of applying registration on the full point set as the original ICP does, it recovers the rotation and translation using only the correspondence between the pair of subsets. It iterates through all subset pairs until convergent. Secondly, streaming division is incorporated to CPD-B method for non-rigid registration of large point sets to recover the nonlinear displacement between each point pairs among the two sets. Unlike the subset-ICP, it extends each subset marginally to its neighbouring subset for robust point correspondence searching. The registration results of all subset pairs are directly merged. A heuristic search is also pro-

posed to tune the width parameter of the Gaussian kernel in the original CPD method, namely S-CPD-B. Finally, bi-GMM-TPS is proposed as a two stage-based method for large non-rigid point set registration. It employs a clustering scheme to obtain clusters and registers the clusters' centres using GMM-TPS to coarsely align the two full point sets. It then finely registers all cluster pairs using GMM-TPS again. Extensive experiments were conducted to validate the efficiency of the proposed methods on the publicly available datasets including very large point set from USF database. The experimental results demonstrated that the proposed methods are able to reduce the computational cost as well as maintaining registration errors at comparable level with the original methods. Three specific division-based registration methods are conceptually summarised as a division-based multi-stage registration framework for handling large non-rigid point sets registration.

CHAPTER 1

INTRODUCTION

1.1 Background

Over the years, image registration has played a crucial role in a wide range of image applications such as image segmentation, object detection, 3D image reconstruction, and image fusion, to name a few. Image registration involves determining a geometrical transformation that maps each pixel of one image to its appropriate position on another image (Ma, 2014). The many registration approaches for 2D image are dependent mainly on either the intensity (i.e., texture information), or the features, called intensity-based or feature-based methods. The former recovers a transformation between two images by taking full advantage of pixel values taken from the region of interest. Features or point signature (Tam et al., 2013) construct a highly concise and efficient representation of an image. Pin image (Johnson and Hebert, 1999), Gaussian curvature (Gal and Cohen-Or, 2006), shape context (Belongie et al., 2002; Battiato et al., 2012) and integral descriptor (Pottmann et al., 2009), for example, are often used to describe 2D images. Feature-based registration methods are typically insensitive to noise and invariant to illumination and rotation while reducing computational complexity (Szeptycki et al., 2009; Ma, 2014). In general, feature-based image registration methods reliably accommodate registration performance better than intensity-based methods. Among all features, points (or locations in a Cartesian coordinate system) are the most notable feature, and are also quite easily obtained (Hasanbelliu et al., 2011; Ma et al., 2013). Points also serve as the basis of other advanced features, such as curve and surface features, which are yielded by incorporating point coordinates with additional information (Ma, 2014).

A 3D surface is a collection of points or vertices that represents the shape of an object or scene. Surface reconstruction incorporates multiple partial surfaces captured from different directions to form the complete surface of the object (Tam et al., 2013). In this sense, surface registration involves moving multiple data sets into the same coordinate system and aligning the overlapping components (refer to the corresponding points) so as to concatenate them.

Whether 2D image registration or 3D surface registration, the main goal of registration is to find an optimal spatial mapping that transforms one data set onto another data set to make them comparable, or to merge all possible information together. The nature of registration is that a point cloud is moved from a space into another space while maintaining the inherent properties, topological structure, and texture information of the original. Hence, point sets registration that only considers coordinate values is highly challenging, and requires careful and extensive design for application in computer vision, medical image processing, and machine learning fields. To this effect, point sets registration has attracted considerable attention from researchers and developers in recent years.

1.1.1 Point Sets Registration

Point sets, either 2D or 3D, are usually cropped from an object possibly with different viewpoints or with one point set which is a warped (transformed) version of another set, where each point represents a Cartesian coordination in Euclidean space.

An example is displayed in Figure 1.1(a). The registration of two point sets requires determining meaningful correspondences between two point sets or recovering the underlying transformation that can map one set onto another to make them match each other as closely as possible (Chui and Rangarajan, 2003; Myronenko and Song, 2010). Point sets registration makes good use of the spatial locations of an object while neglecting its topological structure and texture information. Naturally, correspondences and transformation are the two core components of registration (Sandhu et al., 2010; Hasanbelliu et al., 2011).

In the context of registration, correspondences refer to the points with the maximum similarity (Domokos et al., 2012; Kim et al., 2013; Battiato et al., 2012) between two point sets. The similarity is measured by either the closest distance (Besl and McKay, 1992; Ezra et al., 2008; Yang et al., 2013) or features of point such as correlation coefficient (Tsin and Kanade, 2004; Nandakumar and Jain, 2004), entropy (Warfield et al., 2001), curvature (Szeptycki et al., 2009; Zeng and Gu, 2011), moment (James, 2007), to name a few. Three established correspondences among two images of fish are illustrated in Figure 1.1(b).

Geometrical transformations (motions, mappings) map a point from one space to another space. They are typically classified as either rigid or non-rigid transformations. Rigid transformations do no alter the size and shape of an object, on the contrary, non-rigid transformations do change the object's size or shape (Galarza et al., 2007; Bottema, 2008). Further details regarding transformations are presented in Appendix A. Similarly, registration can be classified as rigid or non-rigid that are also denoted as linear (global) or non-linear (deformable) registration. The term "deformable" refers

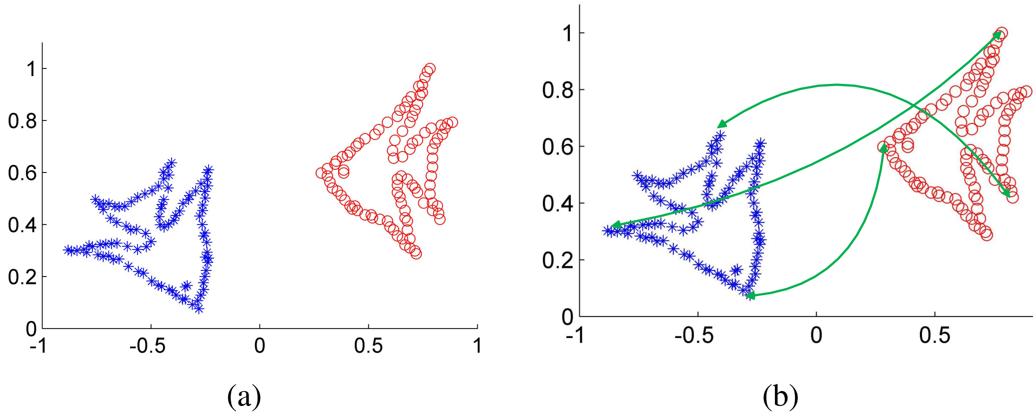

Figure 1.1: Point sets registration, (a) two fish sets to be registered; (b) three of the correspondences between the two fish sets.

to the observed objects are associated through non-linear dense transformation or a spatially varying deformation model (Sotiras et al., 2013). Rigid registration is relatively simple owing to only three degrees of freedom (DOF) for 2D data and six DOF for 3D data to be determined (Bellekens et al., 2014). Non-rigid registration is more challenging due to more complicated registration model, more unknown parameters of non-rigid motion than rigid motion, and the topological ambiguity of the data (such as misaligned data, noisy data or outliers), among other issues.

Many previous researchers have designed and tested point sets registration algorithms in an effort to obtain satisfying solutions for non-rigid registration problems, which are a common requirement in computer vision and medical image analysis fields. Section 2.4 of Chapter 2 provides a comprehensive literature review about them. Without loss of generality, most registration methods consist of the following steps: 1) modeling two point sets; 2) establishing the correspondences; 3) estimating the spatial transformation; 4) optimization techniques; and 5) refining iteratively. A general registration procedure is shown in Figure 1.2.

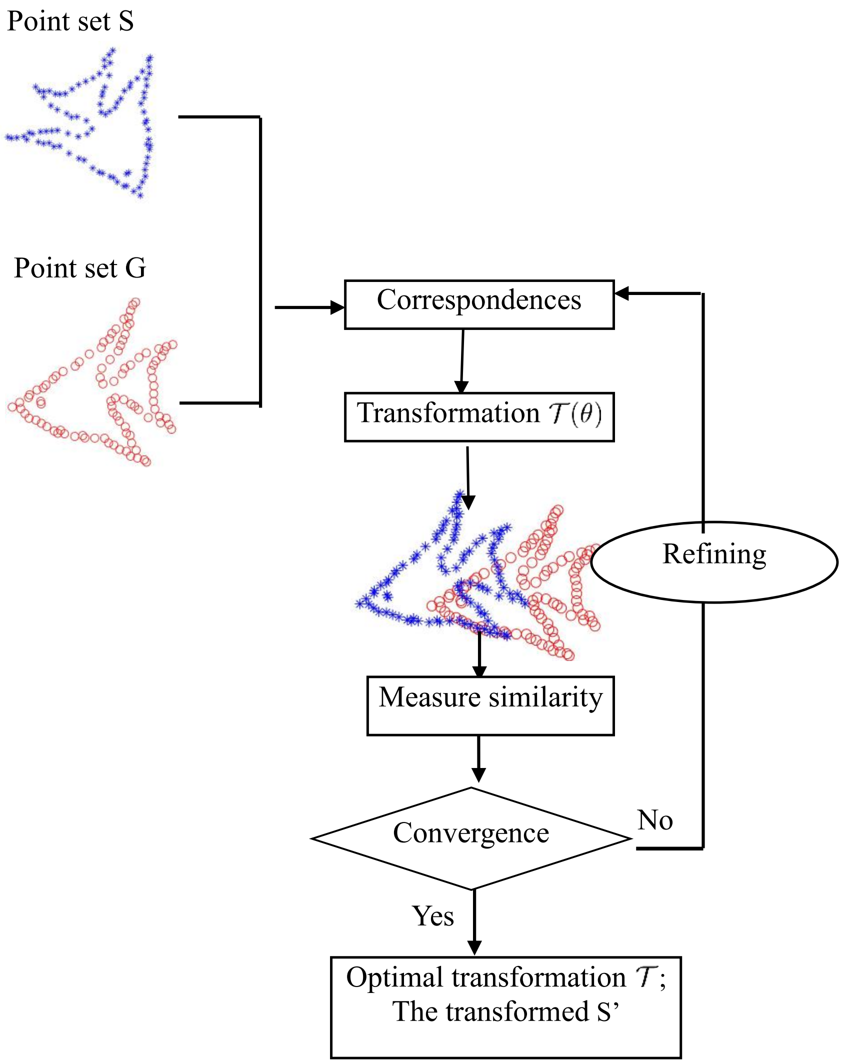

Figure 1.2: Steps involved in point sets registration.

Point matching, point set registration, and point alignment frequently appear in the related literatures. On the one hand, one considers alignment as the initially coarse operation that moves two point sets into a standard reference system to make them possible overlapping, while registration is the fine operation to obtain more accurate results (Rouhani and Sappa, 2013). For instance, a doctor may want to know whether a tumor has changed or not according to two images obtained at different time or modalities. It is much easier for the doctor to find any change when the two images are aligned well. On the other hand, there is no distinct difference between point matching and point set registration as stated in (Sandhu et al., 2010) and (Hasanbelliu et al., 2011). Hence, registration is also called matching and the two terms are fully interchangeable (Fischer and Modersitzki, 2008; Chui and Rangarajan, 2000b; Lian and Zhang, 2014).

1.2 Motivations and Problem Statement

Large point set processing has become popular in many real-world applications. There are two primary reasons for this: one, that the size of point sets has become extremely large because many specific applications require high-quality image reconstruction or high-fidelity 3D modeling. Two, it is possible to collect large amounts of points with the advent of 3D laser scanners. For instance, the *Cyberware*^{TM} 3030PS laser scanner scans human faces in cylindrical coordinates to yield 3D USF face database, where each mesh face is composed of 75972 vertices (Blanz and Vetter, 2003). The Microsoft Kinect camera is a structured light laser scanner that can obtain coloured 3D point sets with more than 300000 points at a frame rate of 30HZ (Houshiar et al., 2013). A large set of points provides more information about an object, but also poses significant challenges because conventional registration algorithms may become impractical.

The three existing registration algorithms Iterative Closest Point (ICP), coherent point drift (CPD) and Gaussian mixture models based on thin-plate splines (GMM-TPS) are not reliable for large point sets registration, because they are either too slow or show high registration error with large numbers of registered points. More specifically, Iterative Closest Point (ICP) (Besl and McKay, 1992) is a widely used rigid point sets registration method performed within Expectation Maximization (EM) optimization scheme, where Euclidean transformation is updated iteratively and nearest neighbours are searched as the correspondences. Establishing precise correspondence can facilitate high registration accuracy, but it is an $\mathscr{O}(N^2)$ time-consuming process (Jost and Hügli, 2003) (where N is the number of points) that occupies the majority of the computational cost of ICP. Coherent point drift (CPD) (Myronenko et al., 2007; Myronenko and Song, 2010) is a probability-based, robust, rigid and non-rigid registration method, where point sets registration is formulated as the maximum likelihood of a Gaussian Mixture Model (GMM). This registration method can afford large point sets registration, but its registration performance degrades considerably as the number of points increases. An alternative probability-based, robust, rigid and non-rigid point matching registration method based on the Gaussian mixture model, GMM-TPS (Jian and Vemuri, 2005, 2011), becomes impractical when applied to large point sets. In addition, GMM-TPS is not reliable when the data exhibits topological ambiguity.

Overall, the size of the data inevitably becomes large, which leads to the registration of large point sets using existing methods being impractical. Thus, the main research problem of the thesis is the efficient registration of two large point sets, especially large non-rigid point sets by means of the chosen registration algorithms.

1.3 Research Objectives

This thesis attempts to produce efficient registration methods by means of division schemes for large point sets registration, especially, large and non-rigid point sets registration. The division scheme incorporates into each existing registration methods ICP, CPD, and GMM-TPS in an effort to reduce computing time while maintaining comparable registration error when registering two large point sets. Therefore, the main research objectives are as follows:

1. To present an algorithm based on the ICP method which can reduce computational cost and registration error for large rigid point sets registration;
2. To propose an algorithm based on the CPD which can reduce computational cost and registration error for registering large, non-rigid point sets;
3. To explore a flexible clustering-based GMM-TPS registration approach to register large, non-rigid point sets.

1.4 Research Contributions

According to the above objectives, three specific division-based methods are proposed to tackle the large point sets registration problem. The main research contributions of this research are as follows:

1. A subset-ICP algorithm is proposed in which ICP is combed with streaming division to reduce the computational cost of large rigid point sets registration. In addi-

tion, the proposed subset-ICP implicitly employs structure information to improve the convergence range and flexibility to deformation;

2. The CPD-B algorithm determines the optimal width parameter of the Gaussian kernel in CPD to refine the registration accuracy of CPD. Streaming division is associated with CPD-B to develop the proposed S-CPD-B algorithm to reduce the computational cost of registration between large, non-rigid point sets;

3. A two-stage GMM-TPS based on K-means clustering, bi-GMM-TPS, is proposed to efficiently handle large, non-rigid point sets registration and to remedy topological ambiguity in the globally misaligned data.

1.5 Overview of Methodology

The three proposed registration methods are the significant contributions to the thesis and the abundant research will be further accommodated in the respective chapter of each method. This section briefly describes the research methodology to fit the research objectives.

1.5.1 Designing of Division-based Registration Scheme

Divide-and-conquer strategy has been widely applied in computer science which works recursively by dividing a task into two or more subtasks of the same type, until those are easily to be solved (Leiserson et al., 1990; Brassard and Bratley, 1996). The prominent advantage of the methods based on divide-and-conquer is the reducing of computational complexity. Within the scope of point sets registration, only making good use of the spatial locations information without considering possibly other structural information, large point set could be divided into several relatively smaller subsets, and those subsets are usually considered to be independent to each other. It is relatively easy to perform registration on smaller point cloud than larger one because the number of points involved is greatly decreased.

The division-based registration methods are associated with the division scheme, and the general procedure of registering large point sets is displayed in Figure 1.3. Division-based registration methods come with reduced computational cost, to this effect, and with comparable registration error.

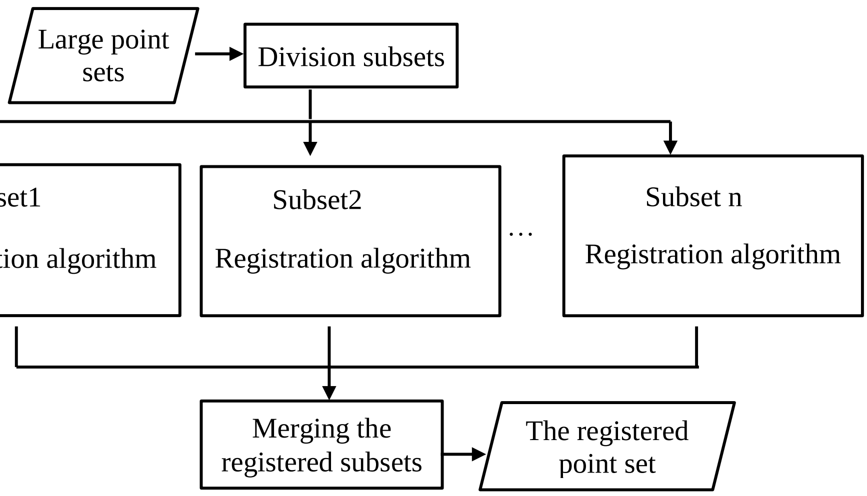

Figure 1.3: Typical procedure of three division-based registration methods.

1.5.2 Three Specific Division-based Registration Methods

Three registration methods, subset-ICP, S-CPD-B, and bi-GMM-TPS, are proposed sequentially to efficiently handle the large point sets registration problem. This type of methods can also benefit the articulated registration, such as motion tracking of human poses (Horaud et al., 2011; Ge et al., 2014; Ge and Fan, 2015), and Chinese character registration (Sun et al., 2014).

The core idea of subset-ICP is that streaming is first used to divide the two point sets into multiple subsets respectively. These subsets and their partners are then registered to recover the Euclidean transformation before it is used to update the point set and its subsets. These steps are repeated until terminal conditions are satisfied. In essence, subset-ICP is a piecewise rigid registration method. Streaming is employed to obtain subsets, which benefits subset-ICP with implicit structure information.

CPD-B, which is a data-driven method, tunes the width parameter of the Gaussian kernel in CPD to refine the registration accuracy of CPD. The refined CPD (CPD-B) once combined with the streaming division strategy forms a S-CPD-B method. Similar to subset-ICP, streaming is employed to divide point set. In order to establish accurate correspondences, the subsets of the target set are further extended by margin set to make the extended subsets overlapping, and the subsets of the source set are kept independent to each other. The subset registered by the CPD-B algorithm are merged together to achieve the final result.

The bi-stage GMM-TPS method reduces computational cost when bi-GMM-TPS encounters large point sets, and also manages topological ambiguity in the data. Coarse alignment is first performed on two centre sets provided by K-means clustering, which obtains the global deformation of the data by means of a TPS transformation. The recovered TPS transformation is then coupled to the source point set and its clusters. A tuned cluster and its corresponding nearest cluster from another set are registered by

GMM-TPS method again to release the local deformation. The registered clusters are then collected and integrated together to obtain the holistic result.

Subset-ICP is a rigid registration approach, although it is able to manage slight deformation. Thus, it can provide stable registration results for large, rigid registration problems. Large, non-rigid point sets registration is the focus of this study due to the fact that it shows more general applications than rigid registration. S-CPD-B is developed to meet the requirements of solving large, non-rigid registration problem at low computational cost and registration error. S-CPD-B is similar to subset-ICP, where the correspondences and transformation are considered two unknown variables to be determined. The subsets are obtained by streaming division in the two proposed methods discussed above. To obtain the transformation without dependence on the correspondences, meanwhile combining existing registration methods with more general division methods, a more flexible bi-stage registration scheme, bi-GMM-TPS, is proposed to handle large, non-rigid, point sets. In general, each of the proposed methods is expected to yield better registration performance and wider convergence than traditional methods. The evolution of the three specific division-based registration methods can be summarized as follows: 1) from scattered points to probability modeling (GMM), 2) from rigid to non-rigid, and 3) from with correspondences to without explicit correspondences.

1.6 Operational Definition of Certain Terms

The scope of this research is the registration of two large point sets where only spatial locations are employed ($p = [x,y]^T$ or $p = [x,y,z]^T$) without considering structure or texture information.

(a) Large point set. The cardinality of the point sets used in existing registration methods ranges from a few tens (2D fish data with 92 points (Chui and Rangarajan, 2003)) to no more than 5000 (3D bunny, dragon data with 4000$\sim$5000 points (Jian and Vemuri, 2011)). Jian and Vemuri had written in a notable study that "They (the proposed methods GMM-TPS) are efficient for most **moderate** non-rigid point sets registration problems" (Jian and Vemuri, 2011). In addition, CPD associated with FGT can register **large** point sets where data contains 92, 398, 453, 1889, 8171 or 35947 points (Myronenko and Song, 2010).

According to the aforementioned overview, the term "large point set" in this thesis refers to a collection of more than 5000 points in Euclidean space. Note that 5000 is a relative concept that reflects real-world registration applications, state-of-the-art registration methods, and computer hardware. A stricter description will be included in a future work.

(b) Computational complexity. Computational complexity is used to analyse an algorithm for solving a problem or the problem itself. The computational complexity (efficiency) of an algorithm refers to a measure of how many steps the algorithm will require for an instance or input of a given size, and the number of steps is measured as a function of that size (e.g., big Oh notation); while the computational complexity of a problem refers to the inherent tractability/intractability of the problem (Arora and Barak, 2009; Hall, cited 2015).

In this thesis, the usage of computational complexity is for the specific proposed algorithms (refer to Section 3.1, Section 4.4.4, and Section 5.3.4) as several other literatures did (Besl and McKay, 1992; Myronenko and Song, 2010; Jian and Vemuri, 2011; Ma et al., 2013; Gao et al., 2014; Ge et al., 2014; Ge and Fan, 2015). Additionally, the quantitative evaluation of registration includes computing time (seconds) and the registration error (MSE) information for the proposed algorithms. Computing time of CPU sometimes is called computational cost. Generally, registration error is defined as the mean of the distances among the points from two point sets (Salvi et al., 2007). The term "registration accuracy" is also used as an alternative measure of "registration error" in the thesis and the relation between them satisfies that the summation is equal to one.

1.7 Structure of Thesis

The remaining six chapters of this thesis are organised as follows.

Chapter 2 comprehensively reviews several state-of-the-art point sets registration methods. The literatures are organised according to two typical categories, rigid versus non-rigid. Within each category, descriptions of the related articles include two core components of registration (i.e., correspondences and transformations). Division-based registration approaches including approximate and exact registration are also summarised.

The subset-ICP method is proposed for rigid registration in Chapter 3. The development of the proposed method, the derivation process of similarity transformation parameters are described exactly. Experimental evaluations of the proposed subset-ICP are discussed in detail and compared extensively to other rigid registration methods.

Chapter 4 covers the S-CPD-B approach for large non-rigid registration. First, the algorithm is proposed that tunes the width parameter of the Gaussian kernel in CPD. The development of the S-CPD-B algorithm is then discussed, as well as theoretical analysis of its computational complexity. Comprehensive experimentations validates the efficiency of the proposed method even though on the very large USF face dataset.

Chapter 5 presents the bi-GMM-TPS approach, established based on clustering, for large, non-rigid registration. A detailed description of the bi-GMM-TPS algorithm, including its computational complexity and performance during experiments are included. Comparative experiments are also presented to further detail the advantages of the proposed method.

Chapter 6 summarises and discusses the proposed subset-ICP, S-CPD-B, and bi-GMM-TPS algorithms. Experimental comparisons among the three methods on large, public datasets are first performed. A summary of the division-based scheme and its core components are then addressed. Similarity measurements used to evaluate the registration performance of registration methods are also summarised.

Chapter 7 provides a conclusion and description of future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter summarises and discusses prior research regarding registration methods to support the research in this thesis; more specifically, it provides a comprehensive overview of point sets registration. Registration of point sets involves finding meaningful correspondences among points, or recovering the underlying spatial transformation. There already exist several different registration methods for real-world applications, which can be grouped into two basic categories according to which spatial transformations are required: rigid registration, and non-rigid registration. For rigid registration, the underlying transformations, naturally, are rigid. These include rotation or translation transformations. For non-rigid registration, the underlying transformations are non-rigid, such as affine or curved transformations.

This thesis adopts a hierarchical classification scheme (see Figure 2.1) to review currently existing point sets registration methods. The methods are firstly grouped into one of two primary categories (rigid vs. non-rigid), then specific registration algorithms are divided again according to their two core components (correspondences and transformations). Under the designed classification criteria of registration approaches, extensive summarisation is extended based on several state-of-the-art registration methods such as Iterative Closest Point (ICP), Principle Component Analysis (PCA) and Singular Value Decomposition (SVD), Robust Point Matching based on thin-plate spline (TPS-RPM), Coherent Point Drift (CPD), and Gaussian mixture models based on thin-plate spline (GMM-TPS).

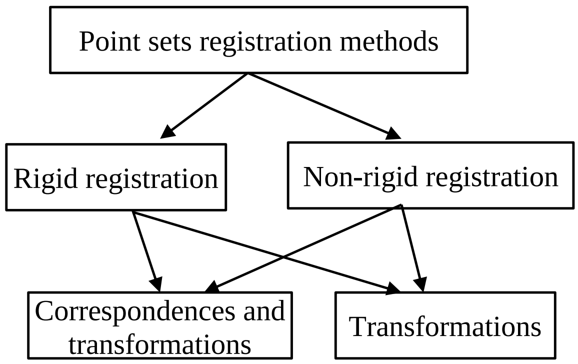

Figure 2.1: Classification strategies of point sets registration methods in the thesis.

The remainder of this chapter is organised as follows: Section 2.2 summarises the classifications of existing image registration methods. Next section discusses rigid point sets registration methods including ICP and its numerous extensions (conforming to the first research objective). Following that, a series of state-of-the-art, non-rigid point registration methods such as CPD, and GMM-TPS (conforming to the second and third research objectives) are discussed in Section2.4. Division-based registration approaches, including approximate registration and exact registration, are then presented in Section 2.5. Section 2.6 provides a brief summary to end the chapter.

2.2 Image Registration Methods

Research and development of imaging techniques and their numerous applications have fostered growing diversity of image registration methods. The earliest survey on image registration focused primarily on geometrical correlation and information compensation, which are realized by identifying corresponding points in images (Kashef and Sawetauk, 1984). Another study surveyed the latest medical image registration methods according to nine factors (Maintz and Viergever, 1998; Mani and rivazhagan, 2013). The nine primary factors formulated by Van den Elsen and Viergever (1993) are introduced as follows:

(a) **Dimensionality**: This factor includes spatial dimensionality and temporal dimensionality.

(b) **Nature of registration basis**: This intrinsic factor consists of landmark-based, segmentation-based, and voxel property-based methods. Extrinsically, this factor comprises the frame, markers or calibration of equipment. The non-image part of the system refers to changes in the coordinate system (such as calibrated coordinate systems).

(c) **Nature of transformation**: This factor involves the types of transformations used, for example, rigid, scaling, affine, projective, or curved transformations. Further information regarding this factor can be found in Appendix A.

(d) **Domain of transformation**: This factor involves local transformation vs. global transformation.

(e) **Interaction**: This factor defines interactive as with or without initialization, and semi-interactive as initialization and automatic registration.

(f) **Optimization procedure**: Optimization methods solve the transformation.

(g) **Modalities**: The images involved stem from one or more modalities.

(h) **Subjects**: Images originate from inter-subject, intra-subject or atlas.

(i) **Objects**: This factor defines the imaging area of the subjects.

The factors of registration methods addressed in the study (Maintz and Viergever, 1998) were originally highly elaborative, but ultimately formed the basic criteria used by subsequent researchers. For example, a simplified, five-criterion classification for image registration methods was proposed by Goshtasby (2005) which involve: 1) whether or not mapping is rigid, 2) the dimensionality of data, 3) data representation structure, 4) whether subjects are the same or different, and 5) automation level. In an effort to formulate the registration problem as an optimization problem, researchers divided the optimization-based registration methods used for medical images, especially variational-based approaches which tackle deformable shapes, into two groups according to a fixed or iterative Tichonov regularization (Fischer and Modersitzki, 2008).

Although the set of criteria discussed above were established based on a medical image registration study, similar criteria have been utilized for human face registration, remote sensing image registration, and among other registration methods. The interested readers can refer to the relevant surveys (Dawn et al., 2010; Sotiras et al., 2013; Sariyanidi et al., 2014).

2.3 Rigid Point Sets Registration

Rigid registration of two point sets identifies either a rigid transformation or establishes the correspondences between two sets. Rigid transformation includes isotropic transformations that preserves the distances in Euclidean space. So, it is also called

Euclidean transformation (Galarza et al., 2007; Bottema, 2008). Rigid transformation typically comprises a linear combination of rotation and translation motions that can be modeled by six Degree Of Freedom (DOF) for 3D data or three DOF for 2D data. Without loss of generality, the transformation is denoted as T, and then the aligned data is $T \circ X$. For this reason, rigid transformation is also categorized as linear transformation. Regardless of the particular or generic registration methods employed, correspondences and transformations are the two key components of rigid point sets registration algorithms. Existing rigid registration methods can thus be described according to the combination of these two components.

2.3.1 Rigid Registration Methods Determining Correspondences and Transformations

In this registration procedure, the correspondences and transformation parameters are unknown. The Expectation Maximization (EM) algorithm, which updates the correspondences and transformation parameters iteratively, is appropriate for deriving the closed-form solution.

2.3.1(a) Iterative Closest Point (ICP) Registration Algorithm

Iterative Closest Point (ICP) is a popular rigid point sets registration method that has been applied successfully to a wide variety of registration problems (Besl and McKay, 1992; Chen and Medioni, 1992). ICP has become the generic framework of rigid point sets registration for determining unknown correspondences and unknown Euclidean transformation. Given two point sets $X = \{x_1, x_2, \ldots, x_M\}$, and $Y = \{y_1, y_2, \ldots, y_N\}$, where $x_j, y_i \in \mathbb{R}^d$, the main idea of ICP is that for each point in set Y, the closest point in set X is searched to form a correspondence set. Based on set Y and the correspondence set, an orthogonal rotation matrix R and translation vector t are then updated iteratively until the terminal conditions are satisfied. The three basic steps of ICP are as follows:

Step 1: Search a closest point x_j for $y_i \in Y$, then define a correspondences set as $N_Y(X) = \left\{x_j | d(y_i, x_j) = \text{argmin}_{x \in X} d(y_i, x)\right\}$;

Step 2: Compute rotation matrix R^k and translation vector t^k using the Singular Value Decomposition (SVD) technique based on sets $N_Y(X)$ and Y; and

Step 3: Updates Y using transformation (R^k, t^k) and accumulates R and t as:

$$Y = R^k Y + t^k. \tag{2.1}$$

$$R = R^k R. \tag{2.2}$$

$$t = R^k t + t^k. \tag{2.3}$$

In each iteration, correspondences can be computed using the nearest-neighbour scheme and transformation parameters can be determined by either SVD or quaternion technique (Horn, 1987). Though effective, ICP is an expensive computational algorithm with $\mathcal{O}(MN)$ because correspondences must be computed for each point of set X in each iteration. Furthermore, its convergence relies heavily on better initialization and tends toward the local minimum.

2.3.1(b) Variant ICP Registration Algorithms

In an effort to remedy disadvantages in the original ICP, later researchers attempted to reduce the computational cost of establishing correspondences. Obviously, the fewer the number of points to be registered, the faster the ICP registration works. A coarse-to-fine selection point scheme was used in Picky ICP to decrease the number of points to be registered (Zinsser et al., 2003). Other researchers have adopted local search strategies, such as the neighbourhood search algorithm (see Figure 2.2) (Jost and Hügli, 2002a, 2003) and *k-d* tree nearest-neighbour search algorithm (Zhang, 1994; Zinsser et al., 2003), in an effort to shrink the search space of the closest points. Utilizing a local instead of global search to obtain correspondence pairs may neglect some coherent information, however; basically forming a tradeoff between computational complexity and registration accuracy. Existing correspondences establishment methods are compared briefly in Table 2.1.

Table 2.1: Techniques used to establish correspondences in ICP and its extensions.

Algorithm	**Techniques**	**Feature**
ICP and accelerated ICP	Nearest neighbours	global
Multi-resolution ICP	Neighbourhood	local
Picky ICP	*k-d* tree	local

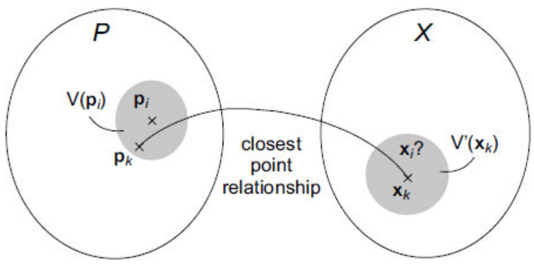

Figure 2.2: Neighbourhood search scheme used by Jost and Hügli (2003).

Another relevant study employed a coarse-to-fine, multi-resolution scheme to improve registration accuracy, in which the coarser solution is improved successively by the next finer representation (Jost and Hügli, 2002b, 2003). The number of hierarchical iterations essentially affects the computational complexity of the coarse-to-fine strategy. A comprehensive look-up matrix used was introduced to ICP to denote the CICP algorithm, which establishes correspondences (Almhdie et al., 2007). In CICP, a line-by-line followed by a column-by-column method (instead of only line-by-line employed in Picky ICP) was used to search corresponding pairs. This method guarantees that the correspondences form a bijection, however, it may mistake good correspondence pairs for noisy data and mistakenly reject them.

In an effort to improve the convergence rate of ICP, a linear approximation and parabolic interpolant were used to accelerate the updating of registration parameters by Besl and McKay (1992). Similarly, an extrapolation method was applied to transformation parameters to speed up the convergence of Picky ICP by Zinsser et al. (2003). Researchers highlighted the fact that the number of iterations of ICP has a polynomial relationship to the number of input points under the root mean squared (RMS) distance and one-sided Hausdorff distance (Ezra et al., 2008). Another study introduced a uniform sampling scheme to ICP for the normals on nearly-flat meshes to improve the convergence of range image registration (Rusinkiewicz and Levoy, 2001). To speed up the convergence of standard ICP registration, gradient-based optimization strategies such as nonlinear Levenberg-Marquardt have been utilized (called LM-ICP algorithm) to update transformation parameters by Fitzgibbon (2001). In addition, LM-ICP is tuned by a robust Huber kernel to improve robustness to noise and outliers. Any enhanced initialization of ICP variants implies that two required point sets are closer at the time they begin to be registered. Evolutionary computation has been used to optimize the initial Euclidean transformation parameters by Santamaría et al. (2011).

Branch-and-bound (BnB) searching, which minimizes the risk of trapping into local minima, is a method commonly used to search global solutions (Olsson et al., 2009; Pfeuffer et al., 2012; Yang et al., 2013). A notable example, the BnB global search algorithm, was proposed to determine rotation transformation and correspondences simultaneously by Li and Hartley (2007). When correspondences are given, Euclidean or similarity transformation is computed via branch-and-bound algorithm to solve a global, non-convex optimization problem (Olsson et al., 2009). An innovative study proposed a globally optimal ICP, Go-ICP, with branch-and-bound (BnB) for 3D Euclidean registration where in subcubes of solution space, the rotation and translation provided by a nested BnB search algorithm provide the initialization of local ICP registration (Yang et al., 2013). This method assists ICP to escape local minima (Figure 2.3). The BnB search algorithm can afford accurate solutions with high computational cost and without convergence dependent on initialization. However, the domain of unknown transformation parameters are given in closed-form, which limits the transformations to Euclidean, similarity, and affine cases. It is worth mentioning that similarity transformations with isotropic scaling ($s \neq 1$) are an extension of Euclidean transformation and that similarity transformations with anisotropic scaling (Chen et al., 2015) are affine transformations.

It is necessary to consider the robustness of ICP-based algorithms to noise and outliers, because both are unavoidable during data collection. Researchers have used predefined distance thresholds to remove outlier candidates when points find their closest neighbours (Besl and McKay, 1992; Zinsser et al., 2003; Jost and Hügli, 2003; Almhdie et al., 2007). However, most of the related algorithms can not guarantee convergence. When the ratio of outliers is known, trimmed ICP can determine the optimal alignment; when the alignment is given, RANSAC-based methods or nonlinear esti-