UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2006/2007

Oktober/November 2006

EEE 350 – SISTEM KAWALAN

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat dan **SEPULUH** muka surat LAMPIRAN bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan. TIGA soalan dalam Bahagian A dan TIGA soalan dalam Bahagian B.

Jawab **LIMA** soalan. Jika calon menjawab lebih daripada lima soalan hanya lima soalan pertama mengikut susunan dalam skrip jawapan akan diberi markah.

Gunakan dua buku jawapan yang berasingan supaya jawapan-jawapan bagi soalan Bahagian A adalah dalam satu buku jawapan dan jawapan bagi Bahagian B dalam buku jawapan yang lain. Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah bagi setiap soalan diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam Bahasa Malaysia.

...2/-

Bahagian A
Part A

1 Suatu sistem kawalan mempunyai gambarajah blok seperti ditunjukkan di Rajah 1. R(s) dan C(s) adalah masukan-masukan kepada sistem tersebut dan mempunyai nilai-nilai seperti yang ditunjukkan dalam Jadual 1.

The block diagram of a control system is shown in Figure 1. R(s) and C(s) are the inputs to the system and have the values as shown in Table 1.

(a) Menggunakan permudahan gambarajah blok, dapatkan:-
Using block diagram simplification, obtain:-

(i) fungsi pindah apabila $t = t_a$.
the transfer function when $t = t_a$.

(ii) fungsi pindah apabila $t = t_b$.
the transfer function when $t = t_b$.

(iii) N(s) apabila $t = t_c$.
N(s) when $t = t_c$.

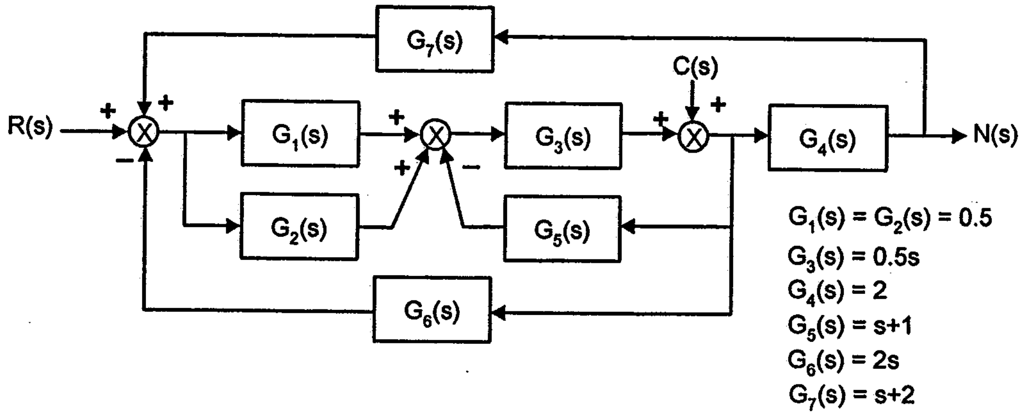
(50%)

(b) Lukiskan graf aliran isyarat yang setara bagi sistem dalam Rajah 1.
Draw the equivalent signal flow graph for the system in Figure 1.

(20%)

(c) Gunakan formula untung Mason kepada graf aliran isyarat dalam (b) dan nyatakan N(s) dalam sebutan R(s) dan C(s).
Apply Mason gain formula to the signal flow graph in (b) and express N(s) in terms of R(s) and C(s).

(30%)



Rajah 1
Figure 1

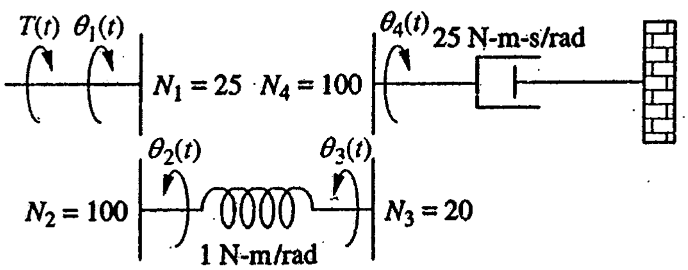
Jadual 1
Table 1

t	R(s)	C(s)
t_a	1	0
t_b	0	1
t_c	1	1

2. (a) Cari fungsi pindah G(s) = $\theta_4(s)/T(s)$ untuk sistem pusingan yang ditunjukkan di dalam Rajah 2(a).

Find the transfer function G(s) = $\theta_4(s)/T(s)$ for the rotational system shown in Figure 2(a).

(50%)

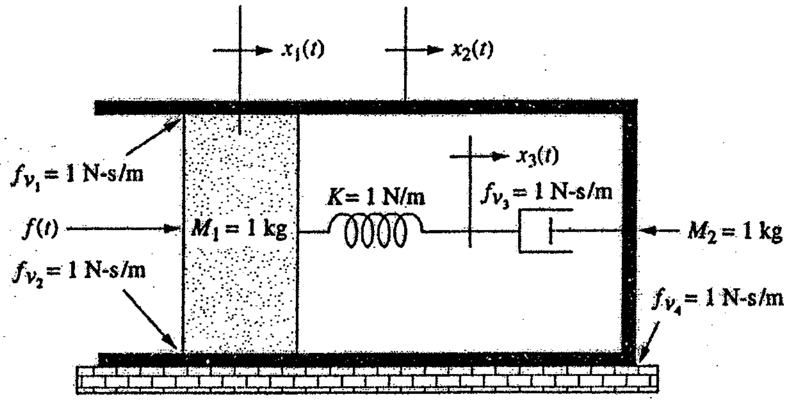


Rajah 2(a)
Figure 2(a)

(b) Cari fungsi pindah M(s) = $X_2(s)/F(s)$ untuk sistem yang ditunjukkan di dalam Rajah 2(b).

Find the transfer function M(s) = $X_2(s)/F(s)$ for the system shown in Figure 2(b).

(50%)



Rajah 2(b)
Figure 2(b)

3. (a) Sistem suapbalik tunggal yang ditunjukkan dalam Rajah 3 mempunyai:-
A unity feedback system shown in Figure 3 has:-

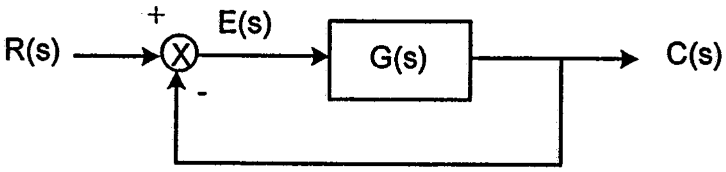
$$G(s) = \frac{K}{s(s+1)(s+2)(s+5)}$$

(i) Cari julat K untuk kestabilan.
Find the range of K for stability.

(ii) Cari nilai K, frekuensi ayunan dan lokasi sebenar bagi kutub gelung-tertutup apabila sistem berada pada kestabilan jidar.
Find the value of K, frequency of oscillation and actual location of the closed-loop poles when system is marginally stable.

(iii) Lakarkan lokasi kutub-kutub tersebut pada satah s.
Sketch the location of the poles on the s-plane.

(40%)



Rajah 3
Figure 3

(b) Selesaikan persamaan kebezaan berikut menggunakan kaedah Penjelmaan Laplace. Andaikan fungsi-fungsi memaksa adalah sifar sebelum t = 0.

Solve the following differential equations using Laplace Transform method. Assume the forcing functions are zero just before t = 0.

(i) $\frac{dx}{dt} + 7x = 5\cos 2t$

(ii) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = \delta(t)$

(iii) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 10u(t)$

(60%)

Bahagian B
Part B

4. Salah satu daripada sistem kawalan gelung-tertutup otomotif yang dikaji mempunyai rangkap pindah seperti berikut:

One of the automotive closed-loop control system studied has the following transfer function:

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)} \qquad H(s) = 1$$

(a) Lakarkan plot londar punca bagi sistem gelung-tertutup di atas. Nyatakan dengan jelas langkah-langkah lakaran tersebut.

Sketch the root locus for the closed-loop system stated above. State your subsequent steps in sketching it.

(60%)

(b) Tentukan kutub gelung-tertutup pada londar punca tersebut supaya kutub gelung-tertutup terserlah mempunyai nilai redaman bersamaan 0.5.

Determine the closed-loop poles on the root locus such that the dominant closed-loop poles have damping ratio of 0.5.

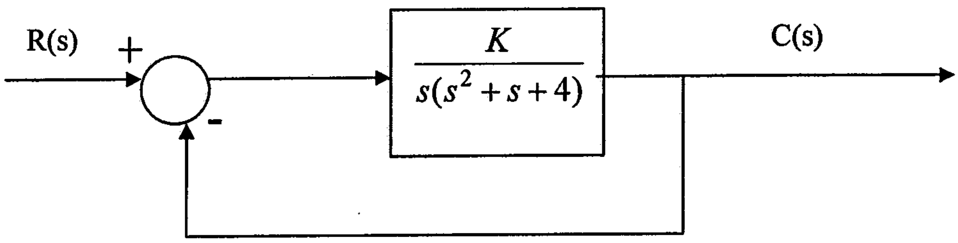
(25%)

(c) Tentukan nilai K pada kutub gelung-tertutup tersebut.

Determine the corresponding value of K at the dominant closed-loop poles.

(15%)

5.



Rajah 4
Figure 4

Pertimbangkan sistem kawalan suap-balik tunggal seperti yang ditunjukkan dalam Rajah 4.

Consider the unity-feedback control system shown in Figure 4.

(a) Dengan menggunakan teknik sambutan frekuensi iaitu lakaran bode, tentukan nilai bagi gandaan K yang menjurus kepada jidar fasa bersamaan 50°.

By using frequency response technique i.e. bode plot, determine the value of gain K such that the phase margin is 50°.

(30%)

(b) Apakah nilai jidar gandaan bagi nilai gandaan K yang diperolehi?

What is the value of the gain margin for the gain K obtained previously?

(20%)

(c) Apakah nilai lebar jalur, ω_{BW} bagi sistem tersebut dengan nilai K yang diperolehi dalam soalan 5(a). Kirakan masa penetapan bagi sistem tersebut dengan menggunakan nilai lebar jalur dan jidar fasa yang diperolehi sebelumnya.

What is the bandwidth, ω_{BW} of the system with K obtained in question 5(a). Calculate the settling time of the system by using the value of the bandwidth and the phase margin obtained.

(50%)

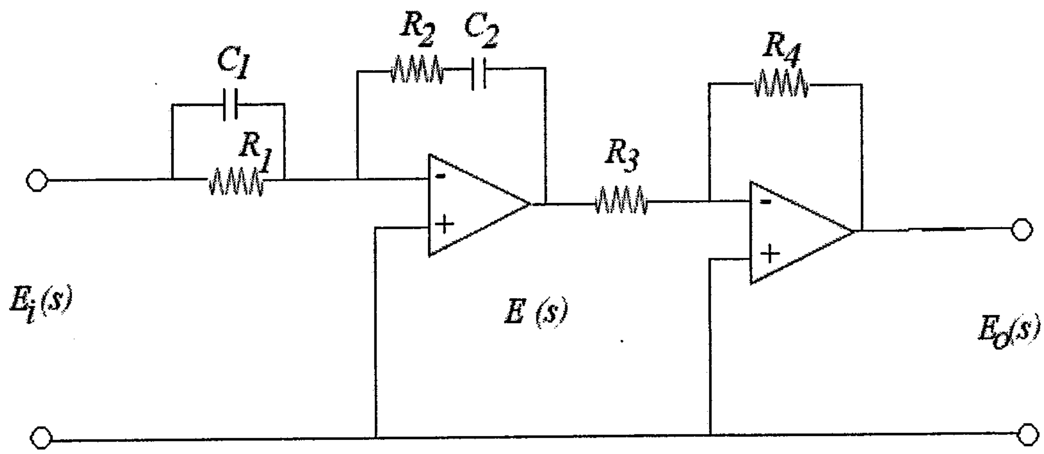
6. Pertimbangkan pengawal PID elektronik seperti yang ditunjukkan dalam Rajah 5.

Consider the electronic PID controller shown in Figure 5.

(a) Tentukan nilai bagi R_1, R_2, $R_{3,}$ $R_{4,}C_1$ dan C_2 dalam pengawal tersebut yang mempunyai rangkap pindah $G_c(s) = E_0(s)/E_i(s)$ iaitu

Determine the values of R_1, R_2, $R_{3,}$ $R_{4,}C_1$ and C_2 *of the controller such that the transfer function* $G_c(s) = E_0(s)/E_i(s)$ *is*

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right)$$
$$= 30.3215\frac{(s+0.65)^2}{s}$$



Rajah 5
Figure 5

(30%)

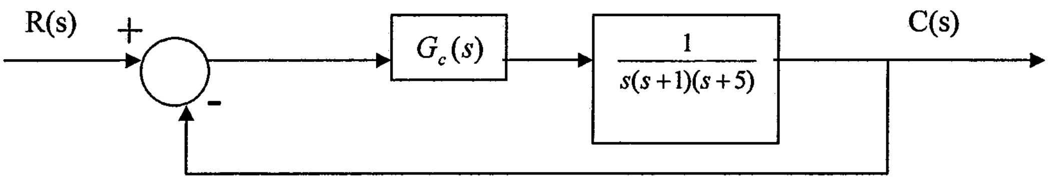
(b) Sebuah sistem kawalan aras cecair akan dikawal menggunakan pengawal PID. Gambarajah blok bagi sistem tersebut ditunjukkan dalam Rajah 6.

A liquid level control system is to be controlled by using PID controller. The block diagram of the system is shown in Figure 6.

Gunakan pengawal PID dalam soalan 6(a) untuk **mencari jidar fasa** bagi sistem kawalan aras cecair yang dikawal oleh pengawal PID. Kamu boleh menggunakan salah satu kaedah analisis sambutan frekuensi yang telah kamu pelajari seperti londar punca, plot bode, Nyquist atau Carta Nichols untuk mencari jidar fasa tersebut.

Use the PID controller ($G_c(s)$) in question 6(a) to ***find the phase margin*** *of the PID-controlled liquid level control system. You may use any of the frequency response analysis you had learnt like root locus, bode plot, Nyquist or Nichols Chart to find the phase margin.*

(70%)



Rajah 6
Figure 6

- oooOooo -

Lampiran A
Appendix A

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Lampiran A
Appendix A

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form 0/0 or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}.$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2+b^2}, \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1e^{j\theta_1})(r_2e^{j\theta_2}) = r_1r_2e^{j(\theta_1+\theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \tfrac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \tfrac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x-y) + \sin(x+y)]$$

$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

Lampiran B
Appendix B

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$	$\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{d}{dx}\sin ax = a\cos ax$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}\cos ax = -a\sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx}\ln(ax) = \frac{1}{x}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}\log(ax) = \frac{\log e}{x}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}e^{bx} = be^{bx}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Lampiran C
Appendix C

TABLE 2.1: Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\frac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=0}^{n}\frac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\frac{m!n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\frac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\frac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \text{Re}\,\lambda_2 > \text{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\dfrac{1-\gamma^{k+1}}{1-\gamma}\right]u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\dfrac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right]u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\dfrac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \dfrac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\lvert\gamma_2\rvert > \lvert\gamma_1\rvert$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\dfrac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \dfrac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right]u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\dfrac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\dfrac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right]u[k]$	
10	$\lvert\gamma_1\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\dfrac{1}{R}\left[\lvert\gamma_1\rvert^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right]u[k]$	γ_2 real
			$R = \left[\lvert\gamma_1\rvert^2 + \gamma_2^2 - 2\lvert\gamma_1\rvert\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\dfrac{(\lvert\gamma_1\rvert\sin\beta)}{(\lvert\gamma_1\rvert\cos\beta-\gamma_2)}\right]$	

Table 11.1: (Unilateral) z-Transform Pairs

	$f[k]$		$F[z]$
1	$\delta[k-j]$		z^{-j}
2	$u[k]$		$\dfrac{z}{z-1}$
3	$ku[k]$		$\dfrac{z}{(z-1)^2}$
4	$k^2u[k]$		$\dfrac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$		$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$		$\dfrac{1}{z-\gamma}$
7	$\gamma^k u[k]$		$\dfrac{z}{z-\gamma}$
8	$k\gamma^k u[k]$		$\dfrac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$		$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\dfrac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$		$\dfrac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos \beta k\, u[k]$		$\dfrac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin \beta k\, u[k]$		$\dfrac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$		$\dfrac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma=\lvert\gamma\rvert e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ $r=\sqrt{\dfrac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\dfrac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\dfrac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$		$\dfrac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

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Appendix D

Table 11.2

Z- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k]+f_2[k]$	$F_1[z]+F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z]+\frac{1}{z^m}\sum_{k=1}^{m}f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z]+f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z]+\frac{1}{z}f[-1]+f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z]+\frac{1}{z^2}f[-1]+\frac{1}{z}f[-2]+f[-3]$
Left-shift	$f[k+m]u[k]$	$z^mF[z]-z^m\sum_{k=0}^{m-1}f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z]-zf[0]$
	$f[k+2]u[k]$	$z^2F[z]-z^2f[0]-zf[1]$
	$f[k+3]u[k]$	$z^3F[z]-z^3f[0]-z^2f[1]-zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k]*f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty}F[z]$
Final value	$\lim_{N\to\infty}f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.