

**ENHANCING SECONDARY
STUDENTS' GEOMETRIC THINKING AND
TEACHERS' TPACK THROUGH
LESSON STUDY INCORPORATING
PHASE-BASED INSTRUCTION USING GSP**

LILLA ADULYASAS

UNIVERSITI SAINS MALAYSIA

2016

**ENHANCING SECONDARY
STUDENTS' GEOMETRIC THINKING AND
TEACHERS' TPACK THROUGH
LESSON STUDY INCORPORATING
PHASE-BASED INSTRUCTION USING GSP**

by

LILLA ADULYASAS

**Thesis submitted in fulfillment of the requirements
for the degree of
Doctor of Philosophy**

February 2016

ACKNOWLEDGEMENT

This thesis is a result of compromise, patience, support and encouragement of a number of individuals. I would like to express my sincere gratitude and appreciation to my supervisor Dr. Shafia Binti Abdulrahman, for her patience, commitment, consideration and the strong support during the candidature. The experiences which my supervisor has given me are meaningful memory that will inspire me throughout my life.

I would also like to express my sincere gratitude to my next supervisor Associate Professor Dr. Chew Cheng Meng who has helped and given me suggestions for improving my study. His guidance and supervision has given me the opportunities to learn and grow.

I sincerely express my thanks to my best friends Montawadee and Ekkapon for their endless love, care, support and encouragement in this journey. Your kind assistance through the entire process of this study will always be remembered and appreciated.

I also express my thanks to the teachers and students who participated in this study and all my friends who helped me to successfully complete this study.

Lastly, I would like to express my thanks to my family, my dad, my mom, my husband Ameen and my lovely daughter Aminda who support, love, care, and accompany me all the way along the journey.

Above all, I thank Allah Subhanahu Wa Ta'ala for the countless graces showered upon me to persevere and complete the thesis.

TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENT.....	ii
TABLE OF CONTENTS.....	iii
LIST OF TABLES.....	viii
LIST OF FIGURES.....	xi
LIST OF ABBREVIATIONS.....	xii
ABSTRAK.....	xiii
ABSTRACT.....	xv
 CHAPTER 1 – INTRODUCTION	
1.1 Introduction.....	1
1.2 Background of the Study.....	2
1.3 Problem Statement.....	6
1.4 Objectives of the Study.....	11
1.5 Research Questions.....	12
1.6 Null Hypotheses.....	12
1.7 Significance of the Study.....	13
1.8 Limitations and Delimitations of the Study.....	14
1.9 Definition of Terms.....	15
1.10 Summary	17
 CHAPTER 2 – LITERATURE REVIEW	
2.1 Chapter Overview.....	18
2.2 Teaching and Learning Geometry in the Secondary Level.....	18
2.3 The van Hiele Theory.....	24

2.3.1	The historical background of van Hiele theory and perspective of geometric thinking.....	24
2.3.2	The van Hiele theory of geometric thinking.....	25
2.3.2.1	Van Hiele levels of geometric thinking.....	25
2.3.2.2	Properties of levels.....	26
2.3.2.3	Phases of Learning (Phase-based Instruction).....	27
2.3.3	Previous studies of van Hiele theory of geometric thinking.....	28
2.4	Teaching and Learning by Using Technology.....	30
2.5	Dynamic Geometry Software.....	32
2.5.1	Developing geometric thinking by using dynamic geometry software.....	33
2.5.2	The Geometer's Sketchpad.....	34
2.5.3	Appropriate use of The Geometer's Sketchpad.....	35
2.5.4	The success of the Geometer's Sketchpad.....	37
2.6	Technological Pedagogical and Content Knowledge (TPACK).....	38
2.6.1	Definition and components of TPACK.....	38
2.6.2	TPACK developmental model.....	41
2.6.3	Levels of TPACK.....	42
2.7	Lesson Study.....	44
2.7.1	The lesson study cycle.....	44
2.7.2	Participants in the lesson study.....	47
2.8	Theoretical Framework.....	48
2.8.1	Levels of Geometric Thinking.....	48
2.8.2	Phases-based Instruction.....	49
2.8.3	Levels of TPACK.....	50
2.9	Conceptual Framework of the Study.....	52

CHAPTER 3 – METHODOLOGY

3.1	Chapter Overview.....	55
3.2	Research Design.....	55
3.3	Participants and Site.....	59
3.4	The Research Tools and Instruments.....	61
3.4.1	Lesson plan through phase-based instruction using GSP.....	62
3.4.2	Videotaping.....	64
3.4.3	Pretest and Posttest for assessing van Hiele level of geometric..... thinking of students	65
3.4.4	Interview protocol for assessing teachers' GSP-TPACK.....	66
3.4.5	Classroom observation protocol for observing students' geometric thinking and teacher's GSP-TPACK	66
3.4.6	Focus group discussion.....	67
3.4.7	Validation of the tool and instruments.....	67
3.5	Method of Data Collection.....	71
3.5.1	Data collection for the quasi-experimental research design.....	71
3.5.2	Data collection for the case study research design.....	73
3.6	Procedure of the Research.....	75
3.6.1	First meeting.....	75
3.6.2	Introducing Lesson Study, Phase-based Instruction,..... TPACK and Using GSP in Teaching Geometry	76
3.6.3	Lesson study cycle.....	77
3.6.4	Final meeting.....	80
3.7	Data Analysis.....	81
3.7.1	Data analysis for quantitative approach.....	81
3.7.2	Data analysis for qualitative approach.....	85
3.8	Pilot Study.....	87

3.8.1	Difficulty, Discrimination and Reliability of the instruments.....	87
3.8.2	The effectiveness of the LS-PBI on students' geometric thinking..	88
3.8.3	The effectiveness of the LS-PBI on teachers' GSP-TPACK.....	92
3.9	Chapter Summary.....	96

CHAPTER 4 – RESULTS

4.1	Chapter Overview.....	97
4.2	Results on Research Question 1.....	97
4.2.1	Identifying the level of geometric thinking.....	98
4.2.2	Comparing students' levels of geometric thinking in each group..	100
4.2.3	Comparing students' level of geometric thinking among..... 3 groups after the intervention	103
4.3	Results on Research Question 2.....	106
4.3.1	Whole group interaction during LS-PBI using GSP.....	107
4.3.1.1	The first meeting.....	107
4.3.1.2	Introducing Lesson Study, Phase-based Instruction,..... TPACK and Using GSP in Teaching Geometry	111
4.3.1.3	GSP training.....	114
4.3.1.4	Lesson study cycle.....	116
4.3.2	Individual GSP-TPACK.....	171
4.3.2.1	Somchai's individual GSP-TPACK.....	172
4.3.2.2	Nipa's individual GSP-TPACK.....	177
4.3.2.3	Maliwan's individual GSP-TPACK.....	182
4.3.2.4	Vichuda's individual GSP-TPACK.....	187
4.3.2.5	Achara's individual GSP-TPACK.....	193

CHAPTER 5 – DISCUSSION, RECOMMENDATIONS AND CONCLUSIONS

5.1	Chapter Overview.....	201
5.2	Summary of the Results.....	203
5.3	Contributions of the study.....	208
5.4	Implications of the Study.....	208
5.4.1	LS-PBI using GSP on students’ geometric thinking.....	208
5.4.2	LS-PBI using GSP on teacher’s GSP-TPACK.....	211
5.5	Limitations of the study.....	213
5.5.1	Limitations of LS-PBI using GSP on students’ geometric thinking..	214
5.5.2	Limitations of LS-PBI using GSP on teachers’ GSP-TPACK.....	215
5.6	Recommendations for future research.....	216
5.7	Conclusions.....	217
	REFERENCES.....	219
	APPENDICES.....	232
	APPENDIX A Lesson plans through phase-based instruction using GSP	233
	APPENDIX B Examples of GSP activities.....	261
	APPENDIX C Student worksheets.....	267
	APPENDIX D Pretest and Posttest.....	301
	APPENDIX E Interview protocol.....	313
	APPENDIX F Classroom observation protocol.....	314
	APPENDIX G Focus group discussion questions.....	316
	APPENDIX H Curriculum Vitae.....	317

LIST OF TABLES

	Page
Table 2.1 Standard M 3.1 for grade 1 - 6	20
Table 2.2 Standard M 3.1 for grade 7 - 12	21
Table 2.3 Standard M 3.2 for grade 1 - 6	22
Table 2.4 Standard M 3.2 for grade 7-12	22
Table 3.1 Background information of teacher participants	60
Table 3.2 Background information of student participants	61
Table 3.3 Details of the lesson plans	63
Table 3.4 IOC of the pretest and posttest	69
Table 3.5 IOC of the pretest and posttest after revised unacceptable items	70
Table 3.6 Timeline of the procedure of the research	81
Table 3.7 Usiskin's operational definitions	83
Table 3.8 Difficulty, Discrimination and Reliability of the pretest and posttest	87
Table 3.9 Frequency and percentage of students in each group at each van Hiele level	89
Table 3.10 The comparison of students' pretest and posttest score	90
Table 3.11 The comparison of the posttest scores among 3 groups of students (1)	91
Table 3.12 The comparison of the posttest scores among 3 groups of students (2)	91
Table 4.1 Frequency and percentage of students' pretest and posttest in each group at each van Hiele level	98
Table 4.2 Mean and standard deviation of the pretest and posttest of the students in each group	100
Table 4.3 Kolmogorov-Smirnov Normality Test	101
Table 4.4 Comparing students' achievement before and after the intervention in each group	101
Table 4.5 Levene's test of homogeneity of variance	104

Table 4.6	Comparing students' posttest mean scores among 3 groups after the intervention	104
Table 4.7	Pairwise comparisons of students' achievements among 3 groups after the intervention	105
Table 4.8	The initial level of teachers' GSP-TPACK during the first meeting (1)	109
Table 4.9	The initial level of teachers' GSP-TPACK during the first meeting (2)	112
Table 4.10	The levels of teachers' GSP-TPACK during discussion of the problems in teaching and learning geometry	118
Table 4.11	The levels of teachers' GSP-TPACK during planning lesson plan 1	121
Table 4.12	The levels of teachers' GSP-TPACK during planning lesson plan 2	124
Table 4.13	The levels of teachers' GSP-TPACK during planning lesson plan 3	126
Table 4.14	The levels of teachers' GSP-TPACK during lesson study cycle of Lesson plan 1	138
Table 4.15	The levels of teachers' GSP-TPACK during lesson study cycle of Lesson plan 2	145
Table 4.16	The levels of teachers' GSP-TPACK during lesson study cycle of Lesson plan 3	154
Table 4.17	The levels of teachers' GSP-TPACK during lesson study cycle of Lesson plan 4	162
Table 4.18	Summarizing levels of teachers' GSP-TPACK in a whole group interaction during LS-PBI using GSP at each stage of lesson study cycle	170
Table 4.19	The level of Somchai's GSP-TPACK before lesson study cycle	173
Table 4.20	The level of Somchai's GSP-TPACK after lesson study cycle	176
Table 4.21	The level of Nipa's GSP-TPACK before lesson study cycle	178
Table 4.22	The level of Nipa's GSP-TPACK after lesson study cycle	181
Table 4.23	The level of Maliwan's GSP-TPACK before lesson study cycle	184
Table 4.24	The level of Maliwan's GSP-TPACK after lesson study cycle	186
Table 4.25	The level of Vichuda's GSP-TPACK before lesson study cycle	189
Table 4.26	The level of Vichuda's GSP-TPACK after lesson study cycle	192

Table 4.27	The level of Atchara's GSP-TPACK before lesson study cycle	195
Table 4.28	The level of Atchara's GSP-TPACK after lesson study cycle	198
Table 4.29	Summarizing the individual level of teachers' GSP-TPACK before and after LS-PBI using GSP	199

LIST OF FIGURES

	Page
Figure 2.1 TPACK Model	40
Figure 2.2 Visual Description of the TPACK Developmental Model	42
Figure 2.3 Steps of Typical Lesson Study Cycle	46
Figure 2.4 The Conceptual Framework of the Study	54
Figure 3.1 Embedded Design: Embedded Experimental Model	56
Figure 3.2 One Group Pretest-Posttest Design for Each Group of Students	72

LIST OF ABBREVIATIONS

LS	Lesson Study
PBI	Phase-Based Instruction
LS-PHI	Lesson study incorporating Phase-Based Instruction
GSP	The Geometer's Sketchpad Software
TPACK	Technological Pedagogical and Content Knowledge
GSP-TPACK	Technological Pedagogical and Content Knowledge which specify to GSP

MENINGKATKAN PEMIKIRAN GEOMETRI PELAJAR SEKOLAH MENENGAH DAN TPACK GURU MENERUSI “LESSON STUDY” DAN PENGAJARAN BERASASKAN FASA MENGGUNAKAN GSP

ABSTRAK

Geometri telah terbukti sebagai salah satu topik matematik yang sukar bagi pelajar di kebanyakan negara. Hal ini menunjukkan bahawa pengajaran dan pembelajaran geometri di Thailand adalah kurang berkesan kerana pelajar yang berkesan adalah hasil daripada pendekatan pengajaran yang berkesan dan penggunaan bahan bantu mengajar yang sesuai. Kajian ini bertujuan menentukan sejauh manakah penggunaan “Lesson Study” dan pengajaran berasaskan fasa (LS-PBI) menggunakan GSP meningkatkan tahap pemikiran geometri pelajar sekolah menengah serta mengkaji perubahan dalam pengetahuan teknologi, konten dan pedagogi menggunakan GSP (GSP-TPACK) guru sebelum, semasa dan selepas LS-PBI menggunakan GSP. Pendekatan kuantitatif menggunakan reka bentuk kajian kuasi-eksperimen telah digunakan untuk menjawab persoalan kajian mengenai tahap pemikiran geometri pelajar manakala pendekatan kualitatif telah digunakan untuk menjawab persoalan kajian mengenai GSP-TPACK guru. Dalam kajian ini, lima orang guru dan tiga kumpulan pelajar pelbagai keupayaan di sebuah sekolah bandar di wilayah Yala, Thailand telah dipilih sebagai peserta kajian. Intervensi dalam kajian ini ialah lima pengajaran berasaskan fasa menggunakan GSP dalam topik "Hubungan antara bentuk geometri 2D dan 3D" yang dijalankan dalam kitaran “Lesson Study”. Tiga kumpulan pelajar diajar oleh tiga orang guru yang berbeza mengikut giliran. Ujian pra dan pasca telah digunakan untuk menilai tahap pemikiran geometri van Hiele pelajar manakala pemerhatian, perbincangan kumpulan fokus dan temuduga individu digunakan untuk menilai GSP-TPACK guru. Dapatan kajian

menunjukkan dua kesimpulan utama mengenai pemikiran geometri pelajar dan GSP-TPACK guru. Pertama, LS-PBI menggunakan GSP meningkatkan tahap pemikiran geometri pelajar sekolah menengah seperti yang dijangkakan dalam pengujian hipotesis bahawa terdapat perbezaan yang signifikan pada tahap pemikiran geometri pelajar Kumpulan 1, 2 dan 3 sebelum dan selepas LS-PBI menggunakan GSP serta perbezaan yang signifikan pada tahap pemikiran geometri pelajar ketiga-tiga kumpulan selepas LS-PBI menggunakan GSP. Tahap awal pemikiran geometri van Hiele pelajar dalam Kumpulan 1, 2 dan 3 adalah kebanyakannya pada tahap 1 sebelum intervensi dan kebanyakannya pada tahap 3 selepas intervensi. Skor ujian pasca adalah lebih tinggi daripada skor ujian pra bagi setiap kumpulan. Selain itu, pelajar dalam Kumpulan 3 yang diajar menggunakan rancangan pengajaran terakhir yang dimurnikan mendapat skor min ujian pasca yang tertinggi. Kedua, tahap GSP-TPACK guru meningkat daripada tahap 0 sebelum LS-PBI menggunakan GSP ke tahap 5 selepas LS-PBI menggunakan GSP. Sebelum intervensi, tahap awal GSP-TPACK guru berada pada tahap 0 dan 1. Guru-guru menunjukkan peningkatan tahap GSP-TPACK semasa menggunakan LS-PBI. Selepas LS-PBI menggunakan GSP, tahap GSP-TPACK guru meningkat ke tahap 3, 4 dan 5. Dapatan kajian menunjukkan keberkesanan LS-PBI menggunakan GSP dalam meningkatkan tahap pemikiran geometri pelajar dan tahap GSP-TPACK guru.

ENHANCING SECONDARY STUDENTS' GEOMETRIC THINKING AND TEACHERS' TPACK THROUGH LESSON STUDY INCORPORATING PHASE-BASED INSTRUCTION USING GSP

ABSTRACT

Geometry has in many countries, proved to be one of the topics in mathematics that is problematic for students to understand. This shows that teaching and learning geometry in Thailand has not been very effective because the effective student must come from effective teaching approaches and appropriate use of teaching tools. This study aims to determine the extent to which lesson study incorporating phase-based instruction (LS-PBI) using GSP enhances secondary students' levels of geometric thinking and to examine the changes in secondary teachers' GSP-TPACK before, during and after LS-PBI using GSP. The quantitative approach utilizing quasi-experimental research design was employed to answer the research question on students' levels of geometric thinking while qualitative approach was employed to answer the research question on teachers' GSP-TPACK. In this study, five teachers and three groups of mix-ability students in an urban school in Yala province, Thailand were chosen as the research participants. The intervention in this study is phase-based instruction using GSP of five lesson plans in the topic of "Relationship between 2D and 3D geometric shapes" which were carried out in the lesson study cycle. Three groups of students were taught this topic in turn by three different teachers. Pretest and Posttest were employed for assessing students' van Hiele level of geometric thinking while observation, focus group discussion and individual interview were employed for assessing teachers' GSP-TPACK. The results show two major conclusions on students' geometric thinking and teachers' GSP-TPACK. First, LS-PBI using GSP enhanced secondary students' levels of geometric thinking as

expected in the hypothesis testing that there are statistically significant differences in Groups 1, 2 and 3 students' levels of geometric thinking before and after LS-PBI using GSP and also the difference in the levels of geometric thinking among the three groups of students after LS-PBI using GSP. It shows that the initial van Hiele levels of students' geometric thinking in Group 1, Group 2 and Group 3 were predominantly at level 1 before the intervention and were predominantly at level 3 after the intervention. Besides, the posttest score was greater than the pretest score in every group and students in Group 3 who have learned with the last revised lesson plans got the highest mean score of posttest. Second, the secondary teachers' level of GSP-TPACK changes from level 0 before LS-PBI using GSP to level 5 after LS-PBI using GSP. Before the intervention, the teachers' initial levels of GSP-TPACK were at level 0 and level 1. During LS-PBI using GSP, the teachers' levels of GSP-TPACK progressed continuously. After LS-PBI using GSP, the teachers' levels of GSP-TPACK progressed to level 3, 4 and 5. The findings suggested the effectiveness of LS-PBI using GSP in enhancing both students' level of geometric thinking and teachers' level of GSP-TPACK.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Geometry is an important content, which plays a useful role in representing and solving the problems in both areas of mathematics and real-world situations [National Council of Teachers of Mathematics (NCTM), 2000]. According to Pattanatrakulsuk (2002), if there is no geometry, there will be no mechanical science; if there is no mechanical science, which is the foundation of science theory, there will be no science and if there is no science there will be no technology. Therefore, since geometry is claimed to be the essential foundation used in mathematics, it helps students to improve in terms of real-life problem solving along with the development of spatial perception. Besides, it prepares students for studying in higher mathematics courses and sciences and for any careers which require mathematical skills, it facilitates general thinking and problem solving abilities and it can develop cultural and aesthetic values (Sherard, 1981). Moreover, it is also said by NCTM that in order to increase the development of the students' justification and reasoning abilities, capping in work with the proof in the secondary grades, geometry can be counted as the natural place (NCTM, 2000). This is the reason for the placement of geometry in the mathematics curriculum from pre-kindergarten to high school in many countries.

Regarding the learning of geometry, students should be able to analyze characteristics and properties of geometric shapes, develop mathematical arguments about geometric relationships, use visualization, spatial reasoning and geometric modeling to solve problems (NCTM, 2000). Particularly in the secondary school level, geometric content in the curriculum emphasizes on practicing, drawing,

creating, investigating, observing, conjecturing about geometric properties, proving, analyzing, and explaining to make conclusion and reasoning (Serkoak, 1996). According to Serkoak (1996), when students have an understanding of geometric concepts, they will be able to learn geometry at the higher level without difficulties and have good attitude towards learning geometry, which is an essential precedence to having the ability to solve the problems in their real lives and to understand and to appreciate mathematics.

Therefore, the importance of geometric thinking, which is the ability to think reasonably in geometric context (Van de Walle, 2004), is absolutely imperative. The ability to think geometrically will lead students to having spatial visualization – a vital aspect of geometric thinking, geometric modeling and spatial reasoning that will provide ways for students to understand and explain physical environments and can be an important tool in problem solving (NCTM, 2000).

1.2 Background of the Study

The Basic Education Core Curriculum (Ministry of Education of Thailand, 2008) states that geometry is claimed to be a considerable content standard in the curriculum for Thai students from primary school to secondary school. It comprises two standards:

- Standard M 3.1: Ability to explain and analyze the two-dimensional and three dimensional geometric figures.
- Standard M 3.2: Ability to visualise, spatially reasoning, and applying the models of geometry to solve the problems

Ministry of education of Thailand (2008)

In particular, at secondary school level, the students are required to understand and elaborate the compass and straight edge two-dimensional geometric figures. It is also to describe the characteristics and properties of three-dimensional geometric figures, such as prisms, pyramids, cylinders, cones and spheres.

Phase-based instruction (PBI) defined by van Hiele (1986) was one of the teaching instructions, which Thai scholars have studied in order to enhance Thai students' geometric achievement and develop their geometric thinking. The van Hiele theory describes the level of development in learning geometry. The five levels of geometric thinking (Burger & Shaughnessy, 1986); van Hiele, 1986) are:

Level 1: Recognition

Level 2: Analysis

Level 3: Order

Level 4: Deduction

Level 5: Rigor

van Hiele (1986) also proposed the *phased-base instruction* (PBI) as a teaching strategy to move up the levels of geometric thinking. The five phases of instruction are:

Phase 1: Information

Phase 2: Guided Orientation

Phase 3: Explication

Phase 4: Free Orientation

Phase 5: Integration

The studies in Thailand have found that there was an increase in the level of geometric thinking of students who were taught geometry through phase-based instruction (Chatbunyong, 2005; Chutkaew, 2006; Heamwatsadugit, 2002) and an

increase in the attitude towards geometry (Chutkaew, 2006). Besides, students also had geometric achievements more than the criterion (Namchitrong, 2003).

In addition, the Office of the Basic Education Commission of Thailand (OBEC) organizes projects for promoting effective teaching and learning mathematics according to the method proposed by The Institute for the Promotion of Teaching Science and Technology (IPST) using the Geometer's Sketchpad in every school which is a member of the OBEC in order to enhance students' geometry achievement. The Geometer's Sketchpad (GSP) is a dynamic geometry software developed by Nicholas Jackiw, which can create, explore and analyze many concepts of mathematics such as algebra, geometry, calculus, and many other areas (Jackiw, 2001). This software has the ability to draw, measure, calculate and script geometric shapes and figure (Liu & Cummings, 2001). Students can construct and explore an object by dragging it because it relies on very simple commands to create, edit, and manipulate accurate geometrical constructions. The procedure of discovery which is aided by the GSP is what can be firstly visualized and analyzed by the students in order to solve the problems and to make assumptions before endeavoring the proof (Bakar, Tarmizi, Ayub, & Yunus, 2009). Integrating technology such as GSP in the teaching and learning of geometry is an integration among technology, pedagogy and mathematical content. Many researchers are interested in the integration of technology, pedagogy and content. Therefore, the researchers describe the terms of technological, pedagogical and content knowledge as the interconnection and intersection among three construct knowledge of technology, pedagogy and content (Mishra & Koehler, 2006; Niess et al., 2009). Technological, pedagogical and content knowledge (TPACK) framework is designed under Shulman's descriptions of pedagogical content knowledge (PCK) – the knowledge of teaching which is

relevant to the teaching of specific content– to describe the interaction of PCK with technology to produce effective teaching (Koehler & Mishra, 2009). This is consistent with the International Society for Technology and Education (ISTE), which challenges teachers in an increasingly technology-savvy society to think about the skill and knowledge of technology that students would need (ISTE, 2000). Therefore, teachers need to be competent in integrating technology in their teaching as the NCTM stated that in order to make teachers attempt to design an optimistic learning environment which stimulates the cooperative problem-solving, features technology in a more consequential way, activates intellectual exploration, and helps student thinking, their experiences into that learning must be taken into accounts for the teachers (NCTM, 2000).

Lesson study (LS) is one of the professional teacher development processes which Japanese teachers use to improve the performance of teacher and the quality of teaching and learning in their classroom in order to provide an environment for teacher which impact student understanding (Stigler & Hiebert, 1997). Becker, Ghenciu, Horak, and Schroeder (2008, p.491) described the process of lesson study as follows:

The fundamental concept of lesson study is that a group of the teachers join in to work collaboratively on one particular lesson taken from the course. The group of teachers pinpoints the goals for the lesson, then makes a plan of the lesson, observes the lesson which has been taught in class (based on the student learning), revises the lesson to improve, and observe the lesson which has been taught in a second time and repeat the process if necessary. The result is hopefully a highly successful lesson plan that could be used by anyone teaching the topic.

Although many scholars in Thailand are trying to find their ways to make students develop their geometric thinking, the teaching and learning of mathematics and in particular geometry, in Thailand has not been very effective. The examination

results evaluated by the National Institute of Education Testing Service (NIETS, 2012) in the Ordinary National Educational Test of middle school students in Thailand show that the average mathematics scores of secondary school students from 2008-2012 are 32.66%, 26.05%, 24.18%, 32.08% and 26.95% respectively. It is astounding that all of these results are less than 50%. Additionally, the results from The Programme for International Student Assessment (PISA) 2009 found that the average score of Thai students is 419 which is statistically significant below the average of The Organization for Economic Co-operation and Development and Thailand was ranked in the period of 48-52 in average score out of 65 participating countries (OECD, 2010). Besides this, the trend of Thai students' scores decrease continuously from PISA 2000 to PISA 2009 (OECD, 2010). Moreover, if we focus specifically in geometry achievement from the Trends in International Mathematics and Science Study (TIMSS) 2007 and 2011, the average geometry achievement of Thai students are 442 and 415 respectively which were significantly lower than the international average (500) and Thailand was ranked twenty-eighth in average geometry achievement out of 49 participating countries (Mullis et al., 2008; Mullis, Martin, Foy, & Arora, 2012). These findings suggest that the teaching and learning of mathematics and in particular geometry, in Thailand can benefit from further innovations and improvement.

1.3 Problem Statement

Despite the fact that geometry is very important and many studies in Thailand have attempted to develop students' geometric thinking, the statistical data shows that Thai students still lack behind in mathematics and geometry in comparison to national and international averages. Many studies have found that Thai students have

difficulties in learning geometry. Chatbunyong (2005) investigated the problem in learning geometry of Thai students and found that the main problem about learning geometry is students do not know how to start proving, and other problems are the misunderstanding of students in the properties of geometric shape, cannot give their reasoning in proving, cannot find the way in proving, cannot connect the information given in question with what the questions ask and cannot use the properties of geometric shape to help in their proving. This is consistent with the study of Maneewong (1999). Moreover, there are a number of students, which cannot apply the concepts of solving a problem to some other similar problems in the same topic (Fongjangvang, 2008). Sawangsri (2002) investigated Thai Students' geometric thinking in Suphanburi province of Thailand by using the geometric test developed by Usiskin (1982) and found that 75.28% of the 90 students have geometric thinking in level 0, 24.72% are at level 1 and no-one is above level 2 and above. This shows that Thai students' level of geometric thinking is relatively low. According to Usiskin (1982), if students have their geometric thinking lower than level 2, then they will not be successful in learning geometry in high school or at other higher levels.

If we consider the failure in learning geometry of Thai students we will find that the teaching of geometry in Thailand does not follow a step-by-step procedure – a number of teachers skip the beginning step of teaching (Suthtakeit, 1999). Moreover, the geometric content in Thai curriculum does not systematically prepare students for learning geometry in high school or at the higher level because geometry does not feature in the Thai curriculum for high school. Therefore, Thai students do not have the opportunity to learn geometry in high school and this will be a problem for students who are required to study geometry in the university (Chamnankit,

2007). It is evidenced that the teaching and learning of geometry in Thailand is discontinuous and unsustainable.

The van Hiele theory which describes five levels of geometric thinking and the five phases of instruction has been applied in many studies, related to teaching and learning of geometry and this instruction shows it has been successful in developing students' geometric thinking (Cannizzaro & Menghini, 2006; Chang, Sung, & Lin, 2007; Chew, 2009; Duatepe, 2005; Erdogan & Durmus, 2009; Hanlon, 2011; Liu & Cummings, 2001; Patsiomitou & Koleza, 2008). In the "free orientation" stage of the phase-based instruction, students will have the opportunity to learn by general tasks to find their own way in the network of relation of solving problem (van Hiele, 1986). Thus, teachers can give the opportunities and environment which encourages students to think independently as much as possible by emphasizing phase-based instruction in order to enhance students' geometric thinking. However, teachers in Thailand tend not to use the van Hiele theory of geometric thinking in their classroom settings (Chamnankit, 2001).

Besides the van Hiele theory, a much more important concern is to find ways to make students understand the concepts in geometry. Throughout the last decade, researchers have studied ways to teach geometry by considering students' difficulties. These studies showed that using the technology, such as GSP, was useful in developing students' understandings of geometric concepts (Chew, 2009; Connor, Moss, & Grover, 2007; Liu & Cummings, 2001; McClintock, Jiang, & July, 2002; Myers, 2009; Patsiomitou & Koleza, 2008). These studies indicate that GSP is a useful tool for enhancing children's thinking through van Hiele's hierarchy because it allows students to discover relationships among geometric concepts through

investigation (Key Curriculum, 1999; Liu & Cummings, 2001; Pokay & Tayeh, 1997).

Hence, the integration of technology, pedagogy with the teaching content is important in developing students' understanding of a particular mathematical content. In this context, it is essential that teachers develop their TPACK. TPACK is defined as a notion which emerges from the interaction among content, pedagogy and technology knowledge. TPACK is a term that has been described to be the basis of successful teaching in relation with the use of technology which provides the teachers with the understanding as the representation of the concepts of using technologies and pedagogical techniques which technology is used in such constructive ways to deliver the teaching content, knowledge of what leads the concepts to be difficult or easy to learn together with how technology can do to redress some of the problems that students deal with; knowledge of the students before knowledge and theories of epistemology; and knowledge of how technology can be applied to extend the existing knowledge to develop new epistemologies or strengthen old ones (Koehler & Mishra, 2009).

However, despite the availability of hardware and software in the technology-rich secondary school, a study by Norton et al. (2000) found that teachers rarely use computers in their teaching because they believe in their existing pedagogy; they are concerned about time constraint and their preference towards some particular text resources. Moreover, some teachers had restricted images of the potential of computer in mathematics teaching and learning because they have absorbed images of teacher-centered and content-focus pedagogy (Norton et al., 2000).

There are numerous barriers that can block the implementation of technology in teaching. Peggy (1999) stated “these barriers range from personal fears (What will I do if the technology fails and my lesson can’t proceed? How will I gain the confidence I need?) to technical and logistical issues (How does this software package work? Where or when should I use computers?) to organizational and pedagogical concerns (How can I ensure that students obtain adequate computer time without missing other important content? How do I weave computers into current curricular demands?)” (p.48). Kastberg and Leatham (2005) said that, it will not encourage teachers to integrate technology in their teaching if accessing to technology does not have knowledge of related curriculum material. This indicated that teacher lack knowledge of integration technology in their pedagogical and teaching content.

Although it may appear that ICT is the important factor that makes students succeed in learning mathematics, OECD (2010) mentioned in the PISA 2009 Results that the use of ICT in the teaching and learning of mathematics does not have an effect on teaching and learning, moreover, the details showed that students who use the most ICT have the minimum score. Therefore, it seems that only technology is not enough to improve student learning. Teachers need to consider and improve their teaching as well because in today’s world, the needs and interests of children are very different from the children in the past decades and the traditional approach may not respond to the potential of children (Battista & Clement 1999; Garrity, 1998; Schoenfeld, 1983).

Hence, it follows that teacher aspect is a factor which is the key to successful learning because effective students must come from effective teachers. This suggests that professional teacher development is also one element which is important and

brings out the professionals in the teacher, which will lead to student success. Lesson study is one of the professional teacher development programs which many scholars have studied for developing teaching process and it obviously shows success in teaching and learning because it provides opportunities for teacher to work collaboratively, have a deep understanding of the pedagogy and cultivate the skill of observation, analysis and reflection of the teacher (Becker, Ghenciu, Horak, & Schroeder, 2008; Chassels & Melville, 2009; Fernandez, 2005; Isoda, 2010; Knapp, Bomer, & Moore, 2008; Lewis, Perry, & Hurd, 2009; Roback, Chance, Legler, & Moore, 2006). In addition, Stigler and Hiebert (1997) also stated in their book that the improvement of teaching and learning comes from how our education system is able to find the way to use the lesson study to build the professional knowledge of teaching. For these reasons, this study aims to enhance the secondary students' geometric thinking and teachers' GSP-TPACK through lesson study incorporating phase-based instruction (LS-PBI) using GSP in order to support the effective teaching and learning in Thailand.

1.4 Objectives of the Study

Specifically the objectives of this study are as follows:

1. To determine the extent to which LS-PBI using GSP enhances secondary students' levels of geometric thinking.
2. To examine the changes in secondary teachers' GSP-TPACK before, during and after LS-PBI using GSP.

1.5 Research Questions

The study is grounded in the following research questions:

- 1) To what extent does LS-PBI using GSP enhance secondary students' levels of geometric thinking?
 - 1a) Is there a statistically significant difference in Group 1 students' levels of geometric thinking before and after LS-PBI using GSP?
 - 1b) Is there a statistically significant difference in Group 2 students' levels of geometric thinking before and after LS-PBI using GSP?
 - 1c) Is there a statistically significant difference in Group 3 students' levels of geometric thinking before and after LS-PBI using GSP?
 - 1d) Is there a statistically significant difference in the levels of geometric thinking among Group 1, Group 2 and Group 3 students after LS-PBI using GSP?
- 2) What are the changes in secondary teachers' GSP-TPACK before during and after LS-PBI using GSP?

1.6 Null Hypotheses

The null hypotheses for the study are as follows:

Ho 1a: There is no statistically significant difference in Group 1 students' levels of geometric thinking before and after LS-PBI using GSP.

Ho 1b: There is no statistically significant difference in Group 2 students' levels of geometric thinking before and after LS-PBI using GSP.

Ho 1c: There is no statistically significant difference in Group 3 students' levels of geometric thinking before and after LS-PBI using GSP.

Ho 1d: There is no statistically significant difference in the levels of geometric thinking among Group 1, Group 2 and Group 3 students after LS-PBI using GSP.

1.7 Significance of the Study

Nowadays, traditional instruction does not seem effective in developing students' geometric thinking. Although, there is a wealth of publications explaining the advantages of using GSP into mathematics classroom in an effort to make students familiar with using computer software to study geometry in order to shift students' geometric thinking to the higher van Hiele levels, the use of phase-based instruction LS-PBI using GSP as an instructional tool in the teaching and learning geometry is yet to be explored. The stages in the phase-based instruction, particularly the fourth stage which is "Free Orientation" will encourage students to think independently and foster geometric thinking of students in solving problem. Moreover, technology integration with the pedagogy and teaching content is very important because TPACK will provide a dynamic framework for viewing the essential teachers' knowledge for designing the curriculum and instruction with digital technology in order to support the changes in students in classroom contexts nowadays (Niess et al., 2009). Besides, the lesson in this particular content will be developed because of lesson study process, a group of teachers have to discuss collaboratively about how to improve the instruction and design lesson by focusing their attention on their students' understanding.

Considering this fact, there is a need to design an experimental study about the use of LS-PBI using GSP on students' geometric thinking and teachers' GSP-TPACK. In addition, this study aims to report the benefits of this teaching and

learning process by quantitative and qualitative methods in order to provide a more complete picture of the issue. From this perspective, the insights obtained from this study will be very helpful for researchers and educators who have struggled to determine how to enhance students' geometric thinking and teachers' GSP-TPACK with better design instruction. The findings will be significant in validating the use of this teaching and learning process in learning geometry content in the secondary school level. The curriculum developers might modify the curriculum according to the outcomes of this study or suggest this instructional process in the secondary school level.

1.8 Limitations and Delimitations of the Study

The following limitations and delimitations are applicable in this study:

1. This study is conducted within an urban school in Yala Province of Thailand.
2. This study is limited to the number of participants available from 7th grade students of this school in semester 1 of 2013 academic year.
3. This study is limited to the topic "Relationship between 2D and 3D geometric shapes" as indicated in The Basic Education Core Curriculum (Ministry of Education of Thailand, 2008) for student in grade 7.
4. The facilities, such as PC computers, GSP software and all teaching and learning materials are contained in this school.
5. The participants have experience in using GSP software before participating in this experiment.
6. The instruments accurately reflect the abilities of the participants.
7. The results are limited to the sample participated in this study.

8. The results are limited to this teaching and learning process as defined in this study.

1.9 Definition of Terms

Definitions of important terms in this study are presented in this section.

Van Hiele levels : the levels of geometric thinking range from level 1 to level 5 which are Visualisation (level 1), Analysis (level 2), Abstraction (level 3), Informal deduction (level 4) and Rigour (level 5) (Burger & Shaughnessy, 1986; van Hiele, 1986).

Phase-based instruction (PBI) : the teaching instruction proposed by van Hiele in order to make the students' geometric thinking level's way up. It has five phases which are Information (phase 1), Guided Orientation (phase 2), Explication (phase 3), Free Orientation (phase 4), and Integration (phase 5) (van Hiele, 1986).

Geometric thinking : the ability to think reasonably in geometric context (Van de Walle, 2004) which have five levels of thinking as seen through the van Hiele levels of geometric development defined by van Hiele's theory.

Lesson study (LS) : the process which instructors work collaboratively to study one particular lesson selected from a course whereby they identify the goals of the lesson, plan the lesson, observe the lesson that is being taught by one instructor and its success in terms of student learning, revise the lesson, observe the lesson that is being taught a second time by the other instructors and repeats the process if necessary. The results will be a very successful lesson plan which can be used by any instructor who teaches in the same content (Becker, Ghenciu, Horak, & Schroeder, 2008).

The Geometer's Sketchpad (GSP) : the dynamic geometry software named "The Geometer's Sketchpad" which was created by Nicholas Jackiw in 1991, This

software has the ability to draw, measure, calculate, and script geometric shapes and figures (Liu & Cummings, 2001). Students can construct and explore the object by dragging the object with the mouse. The user can easily to create, edit, and manipulate accurate geometrical constructions on the computer screen.

Technological Pedagogical and Content Knowledge (TPACK) : the interconnection and intersection among three construct knowledge of technology, pedagogy and content (Niess et al., 2009; Mishra & Koehler 2006). TPACK's framework builds on Shulman's descriptions of pedagogical content knowledge (PCK) -the knowledge of teaching which is applicable to the teaching of specific content- to describe the interaction of PCK with technology to produce effective teaching (Koehler & Mishra, 2009).

GSP-TPACK : technological pedagogical and content knowledge of the teachers in using GSP.

LS-PBI using GSP : the process of teaching and learning in this study by focusing on the use of lesson study incorporating phase-based instruction using GSP as a tool to facilitate teaching and learning geometry in the classroom.

Students: Thai secondary school students in grade 7 of an urban school in Yala Province of Thailand who is in the experimental groups of this study.

Teachers : Teachers in an urban school in Yala Province of Thailand who is in the experimental groups of this study and also in a group of teachers in the lesson study cycle.

Geometry : the geometry content in mathematics subject in Thai's curriculum for Thai secondary school students in grade 7. This study focuses particularly on the content of "Relationship between 2D and 3D geometry"

1.10 Summary

The understanding of geometric concepts can be enhanced through well-designed teaching and learning processes and appropriate tools. This study is an effort to enhance secondary students' geometric thinking and teachers' GSP-TPACK through lesson study incorporating phase-based instruction (LS-PBI) using GSP. GSP will be provided to facilitate teachers and students throughout this teaching and learning process of learning geometry.

The study posed quantitative method to answer the research question about the effectiveness of the use of LS-PBI using GSP on student's geometric thinking and teachers' GSP-TPACK in teaching and learning this geometric content by using pretest and posttest scores. Additionally, there is qualitative method to answer the research question about the role that LS-PBI using GSP plays on students' geometric thinking and teachers' GSP-TPACK.

The next chapters are Chapter 2 to Chapter 5. Chapter 2 will present the related literature which pertains to all variables of this study -lesson study, phase-based instruction, GSP, geometric thinking and TPACK-, the literature which related to the methodology to be used in this study and the theoretical framework. Next, Chapter 3 will describe the research methodology of this study. Chapter 4 will present the results of this study. Finally, Chapter 5 will conclude and discuss the results to give the recommendations for future study.

CHAPTER 2

LITERATURE REVIEW

2.1 Chapter Overview

This research focuses on identifying the impacts of lesson study incorporating phase-based instruction using GSP on students' geometric thinking and teachers' technological pedagogical and content knowledge. The literature review for this study will present the related literature which pertains to all the variables of this study, the literature which related to the methodology to be used in this study and the theoretical framework of this study.

2.2 Teaching and Learning Geometry in the Secondary Level

Geometry is the important subject which is related to our real life and has fascinated people for a long time and it plays an important role in any area of sciences and arts. In mathematics, geometry is unifying subject for an entire mathematics curriculum and is a rich source in visualizing the concept of algebraic, arithmetic, statistical and calculus (Napitupulu, 2002).

The main reason why geometry should be combined with other mathematical areas is that it is claimed to be useful as it represents to solve the problems in both mathematical areas and real-world situations (NCTM, 2000). Moreover, geometric representation is valuable, it helps students make sense in learning the topic about area and fraction, it can give insight about data through histogram and scatter plots and it can serve to connect geometry and algebra through coordinate graphs. Students can engage with geometric ideas by using correct model, drawing and also using dynamic geometry software. Besides, to make the students explore conjecture and

able to acquire reasons based on geometric concepts from the school at their early stage, the activities and the tools which the instructors use should be well-designed, well-appropriated, and well-supported (NCTM, 2000). That is why geometry is contained in the curriculum from the early year of school.

Regarding geometric content, NCTM (2000, p.41) pointed out that for all the students from pre-kindergarten to grade 12, the instructional program should allow student to:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships ;
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- Apply transformations and use symmetry to analyze mathematical situations;
- Use visualization, spatial reasoning, and geometric modeling to solve problems

In Thailand, Ministry of Education Thailand has adapted the guideline for teaching geometry from many sources and has developed The Basic Education Core Curriculum 2008 for Thai students in mathematics and geometry is an important content standard in this curriculum for Thai students from primary school to secondary school.

Geometric figures and properties of one-dimensional geometric figure, visualization of geometric models, geometric theories, and geometric transformation

through translation, reflection, and rotation are included as the geometric content arranged for all students (Ministry of Education Thailand, 2008).

The Ministry of Education of Thailand (2008) has described about geometric standard (Standard 3) which include Standard M 3.1 and Standard M 3.2 as follows;

Table 2.1

Standard M 3.1 for grade 1-6

Grade level indicators					
Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
1. Distinguish triangles, quadrilaterals, circles and ellipses.	1. Identify 2D geometric figures whether in the form of triangles, quadrilaterals, circles or ellipses. 2. Identify 3D geometric figures whether in the form of cuboids, spheres or cylinders. 3. Distinguish between rectangles and cuboids, and between circles and spheres.	1. Identify 2D geometric figures that are components of an object in the form of a 3D geometric figure. 2. Identify 2D geometric figures with axis of symmetry from a given figure. 3. Write linear points, straight lines, rays, parts of straight lines, angles and symbols.	1. Identify kind, name and components of angles and write symbols. 2. Can identify which pair of straight lines or parts of straight lines form a parallel, as well as use symbols to indicate kind of parallel. 3. Identify components of a circle. 4. Can identify which figure or which part of an object has the form of a rectangle, and can identify whether it is a square or a rectangle.	1. Identify characteristics and differentiate between various kinds of 3D geometric figures. 2. Identify characteristics, relationship and differentiate between various kinds of quadrilaterals. 3. Identify characteristics, components, relationships and differentiate between various kinds of triangles.	1. Identify kinds of 2D geometric figures that are components of 3D geometric figures. 2. Identify characteristics of diagonals in various kinds of quadrilaterals. 3. Identify which pair of straight lines is parallel.

Table 2.1- continued

5. Can identify which 2D geometric figures have axes of symmetry, and identify the number of axes.

Table 2.2

Standard M 3.1 for grade 7-12

	Grade level indicators			Key stage indicators
	Grade 7	Grade 8	Grade 9	Grade 10-12
1. Construct and explain steps of basic geometric construction.		-	1. Explain characteristics and properties of prisms, pyramids, cylinders, cones and spheres.	-
2. Construct 2D geometric figures by using basic geometric construction, and explain steps of construction without emphasizing proof.				
3. Search for, observe and project about geometric properties.				
4. Explain characteristics of 3D geometric figures from a given image.				
5. Identify 2D images from front view and side view of a given 3D geometric figure.				
6. Draw or create a 3D figure from a cube, when given 2D image from front view, side view and top view.				

Table 2.3

Standard M 3.2 for grade 1-6

Grade level indicators					
Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
-	1. Draw 2D geometric figures by using geometric models.	1. Draw 2D geometric figures given in various models. 2. Identify various geometric figures in the surroundings.	1. Use geometric figures to create various designs.	1. Construct angles by using a protractor. 2. Create rectangles, triangles, and circles. 3. Create parallels by using a set square.	1. Create cuboids, cylinders, cones, prisms and pyramids from nets of 3D geometric figures or 2D geometric figures given. 2. Construct various kinds of quadrilaterals.

Table 2.4

Standard M 3.2 for grade 7-12

Grade level indicators			Key stage indicators
Grade 7	Grade 8	Grade 9	Grade 10-12
-	1. Use properties of congruence of triangles and those of parallels for reasoning and problem-solving. 2. Use Pythagoras' Theorem and converse for reasoning and problem-solving. 3. Understand and apply geometric transformation through translation, reflection and rotation. 4. Identify images from translation, reflection and rotation of models, and explain the method of obtaining the images when given such models and images.	1. Use properties of similar triangles for reasoning and problem-solving.	-

Moreover, this curriculum has mentioned about the learners' quality in learning geometry that Grade 3 students must have knowledge and understanding of 2D and some 3D geometric figures, i.e., triangle, quadrilateral, circle, ellipse, cuboid, sphere, and cylinder as well as point, line segment and angle, Grade 6 students must have knowledge and understanding of characteristics and properties of 2D and 3D geometric figures, i.e., triangles, squares, circles, cylinders, cones, prisms, pyramids, angles and parallel lines, Grade 9 students must utilize the compass and straight edge to construct and explain stages of constructing 2D geometric figures; can explain characteristics and properties of 3D geometric figures, i.e., prisms, pyramids, cylinders, cones, and spheres.

In Thailand, it was found that Thai students have difficulties in learning 2D and 3D geometric contents as indicated in the Basic Education Core Curriculum 2008 such as students have misunderstanding in the properties of 2D and 3D geometric shaped, cannot give their reasoning in proving, cannot use the properties of 2D and 3D geometric shapes to help in their proving (Chatbunyong, 2005; Maneewong, 1999). Moreover, some students cannot apply the concepts of solving a problem of 2D and 3D geometric shapes to some other similar problems in the same topic (Fongjangvang, 2008). These indicate that students do not understand the relationship between 2D and 3D geometric shapes which lead them to have the misconception in properties of 2D and 3D geometric shapes. If students understand the properties of 2D and 3D geometric shape well, they will be able to learn geometry effectively.

From The Basic Education Core Curriculum 2008 for Thai students, we can see that at grade 7, the first grade of the secondary school level in Thailand, is the starting point of Thai students for learning the characteristics and properties of 2D

and 3D geometric shapes which is a critical infrastructure of learning geometry. From these standards we can see that geometric content is no longer present in the high school level. Therefore, we should pay more attention in teaching and learning geometry in the first year of secondary school level. This will give students a strong foundation in learning geometry in the higher level.

2.3 The van Hiele Theory

2.3.1 The historical background of van Hiele theory and perspective of geometric thinking

Since 1800s, there have been many attempts to develop geometric understanding by many educators, curriculum developers and also those who were concerned with teaching and learning geometry. In 1950s, Pierre van Hiele and his wife Dina van Hiele-geldof develop a theory about teaching and learning of geometric concepts in their doctoral dissertations at the University of Utrecht (Usiskin, 1982). Then they presented in a short paper entitled “The Child’s Thought and Geometry” which describes their theory regarding the knowledge of geometric concepts acquired by the learners will progress through five developmental levels (Fuys et al., 1988). Since Diana died shortly after she finished her dissertation, her husband was the one who continued improving and advancing the theory which was the results of their dissertations. In the late 1950s, he wrote three papers which were applied in the curriculum development of the soviet academician Pyshkalo in 1968 and got published in the well-known book “Mathematics as an Educational Task” of his mentor Freudenthal in 1973. But, it received a little attention from the educators until this theory caught the attention of Wirszup who is the first one to talk about the van Hiele’s theory in 1974 before publishing his talk (Usiskin, 1982). Moreover,