UNIVERSITI SAINS MALAYSIA

First Semester Examination
2014/2015 Academic Session

December 2014/January 2015

EEE 232 – COMPLEX ANALYSIS
[ANALISIS KOMPLEKS]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SIX (6)** pages of printed material and **ONE (1)** page of Appendix before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM (6)** mukasurat bercetak berserta Lampiran **SATU (1)** muka surat bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of FIVE (5) questions. Answer **ALL** questions. All questions carry the same marks.

[***Arahan:*** *Kertas soalan ini mengandungi LIMA (5) soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Prove that $tan5\theta = \frac{5tan\theta - 10tan^3\theta + tan^5\theta}{1 - 10tan^2\theta + 5tan^4\theta}$

Buktikan bahawa $tan5\theta = \frac{5tan\theta - 10tan^3\theta + tan^5\theta}{1 - 10tan^2\theta + 5tan^4\theta}$

(25 marks/*markah*)

(b) Identify the locus of z on a complex plane when equation $\frac{z-j}{z-2}$ is purely imaginary.

Kenal pasti lokus z *pada satah kompleks apabila persamaan* $\frac{z-j}{z-2}$ *adalah semata-mata khayalan.*

(30 marks/*markah*)

(c) Show that the stream function given by $u = tan^{-}\left(\frac{y}{x}\right)$ is harmonic. Then, find the velocity potential $v(x, y)$ where $w = u(x, y) + jv(x, y)$

Tunjukkan bahawa fungsi aliran yang diberikan oleh $u = tan^{-}\left(\frac{y}{x}\right)$ *adalah harmonik. Seterusnya, cari potensi halaju* $v(x, y)$ *di mana* $w = u(x, y) + jv(x, y)$

(45 marks/*markah*)

2. (a) Given that the function $f(x) = (2x - y) + j(ax + by)$ is differentiable. Hence find the constants a and b. Determine $f^{'}(z)$

Diberi bahawa fungsi $f(x) = (2x - y) + j(ax + by)$ *adalah boleh beza. Seterusnya, dapatkan pemalar* a *dan* b. *Tentukan* $f^{'}(z)$

(20 marks/*markah*)

(b) The voltage in a cable is given by the expression
Voltan dalam kabel diberikan oleh ungkapan

$$V = coshpx + \frac{Z_0}{Z_r} sinhpx$$

Calculate V in the form $a + jb$, giving a and b correct to 2 decimal places, when $px = 0.40 + j0.93$, $Z_0 = 15 - j20$ and $Z_r = 3 + j4$

Kira V dalam bentuk $a + jb$, dengan memberikan a dan b betul kepada 2 tempat perpuluhan apabila $px = 0.40 + j0.93$, $Z_0 = 15 - j20$ and $Z_r = 3 + j4$

(25 marks/*markah*)

(c) Using Cauchy's Integral Theorem, evaluate the following:
Dengan menggunakan Teorem Kamiran Cauchy, nilaikan yang berikut:

(i) $\oint_C \frac{(z+4)}{(z^2+2z+5)} dz$; $C: |z + 1 - j| = 2$

(ii) $\oint_C \frac{e^{2z}}{(z+1)^4} dz$; $C: |z| = 2$

(iii) $\oint_C \frac{tanz}{(z^2-1)} dz$; $C: |z| = \frac{3}{2}$

(55 marks/*markah*)

3. (a) Find all the roots of $\left(\frac{1+j\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and display them on an Argand diagram.

Dapatkan semua punca bagi $\left(\frac{1+j\sqrt{3}}{2}\right)^{\frac{3}{4}}$ dan paparkan punca tersebut pada gambarajah Argand.

(20 marks/*markah*)

(b) Find all the solutions of $cosz = j\frac{3}{4}$ with $z = x + jy$

Dapatkan semua penyelesaian bagi $cosz = j\frac{3}{4}$ dengan $z = x + jy$

(20 marks/*markah*)

(c) Evaluate $\int_C (z^2+3z)dz$ along the contour of a

Nilaikan $\int_C (z^2+3z)dz$ *di sepanjang kontur suatu*

i) circle $|z|=2$ from the point $(2,0)$ to $(0,2)$

bulatan $|z|=2$ *dari titik* $(2,0)$ *ke titik* $(0,2)$

ii) straight line from the point $(2,0)$ to $(0,2)$

garis lurus dari titik $(2,0)$ *ke titik* $(0,2)$

iii) straight line from the point $(2,0)$ to $(2,2)$ and then from $(2,2)$ to $(0,2)$

garis lurus dari titik $(2,0)$ *ke* $(2,2)$ *dan kemudian dari* $(2,2)$ *ke* $(0,2)$

From your answer in part (i), (ii) and (iii), is the integral independent of the path? Explain your answer.

Dari jawapan anda dalam bahagian (i), (ii) dan (iii), adakah kamiran yang didapati bebas laluan? Jelaskan jawapan anda.

(60 marks/*markah*)

4. (a) Using Residue theorem, evaluate the following:

Dengan menggunakan teorem baki, nilai yang berikut:

i) $\oint_C \frac{z^3-z^2+z-1}{z^3+4z}dz$, $C: |z|=1$

ii) $\oint_C \frac{(z-1)}{(z^2-4)(z+1)^2}dz$, C is the triangle with vertices at $-\frac{5}{2}+j, -\frac{5}{2}-j, 3+0j$

$\oint_C \frac{(z-1)}{(z^2-4)(z+1)^2}dz$, C *adalah segitiga dengan bucu* $-\frac{5}{2}+j, -\frac{5}{2}-j, 3+0j$

(50 marks/*markah*)

(b) Find the image from the z −plane to the w-plane for the following:
Dapatkan imej pada satah – z ke satah – w untuk yang berikut:

i) The interior of the circle $|z-2|=2$ under the bilinear transformation
Bahagian dalaman bulatan $|z-2|=2$ *dibawah transformasi dwilinear*

$$w=\frac{z}{2z-8}$$

ii) The circle $|z-j2|=2$ under the transformation
Bulatan $|z-j2|=2$ *dibawah transformasi*

$$w=\frac{1}{z}$$

(50 marks/*markah*)

5. (a) Determine whether the following series is convergent or divergent using the indicated test.

Tentukan sama ada yang berikut adalah menumpu atau mencapah dengan menggunakan ujian yang dinyatakan.

i) using ratio test
dengan menggunakan ujian nisbah

$$\sum_{n=0}^{\infty}\frac{(100+75j)^n}{n!}$$

ii) using root test
dengan menggunakan ujian punca

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n}}(4-j)^n$$

(20 marks/*markah*)

(b) Find the first three non zero terms of the Taylor series expansion of the function $f(z) = \frac{z}{z+1}$ at the point $z = j$. Hence, determine the radius of convergence.

Dapatkan tiga sebutan pertama bukan sifar bagi pengembangan siri Taylor bagi fungsi $f(z) = \frac{z}{z+1}$ *pada titikt* $z = j$. *Seterusnya, tentukan jejari penumpuan.*

(30 marks/*markah*)

(c) Find the Laurent series expansion of the following:

Dapatkan pengembangan siri Laurent bagi yang berikut:

i) $f(z) = \frac{1}{z^2+4}$ for $|z - 2j| > 4$

$f(z) = \frac{1}{z^2+4}$ *untuk* $|z - 2j| > 4$

ii) $f(z) = \frac{e^z}{z(1-z)}$ for $z = 1$

$f(z) = \frac{e^z}{z(1-z)}$ *untuk* $z = 1$

(50 marks/*markah*)

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APPENDIX
LAMPIRAN

$z = x + jy$

$e^{jz} = \cos z + j \sin z$

$\cosh z = \frac{1}{2}\,(e^{z} + e^{-z})$

$\sinh z = \frac{1}{2}\,(e^{z} + e^{-z})$

$\sin(jz) = j \sinh z$

$\tan(jz) = j \tanh z$

$\cos(jz) = \cosh z$

$cosh^2 z - sinh^2 z = 1$

$\cosh jz = \frac{1}{2}\left(e^{jz} - e^{-jz}\right) = \cos z$

$\sinh jz = \frac{1}{2}\left(e^{jz} - e^{-jz}\right) = jsin\ z$

$\tanh jz = j \tan z$

$coz\ z = \cos x\ \ coshy - j\ sinx\ sinhy$

$sin\ z = sinx\ coshy + j\ cosx\ sinhy$

$\cosh z = \cosh x \cos y + j \sinh x \sin y$

$\sinh z = \cos y \sinh x + j\ sin\ y \cosh x$

Maclaurin's series $f(z) = \sum_{n=0}^{\alpha} \frac{f^{(n)}(0)}{n!}\ z^n$

Taylor's series $f(z) = \sum_{n=0}^{\alpha} \frac{f^{(n)}(z_0)}{n!}\ (z - z_0)^n$

Residue of $f(z)$ at z_0

$$\text{Res}\,[f(z), z_0] = \frac{1}{(m-1)!} \lim_{z \to z_0} \left[\frac{d^{m-1}}{dz^{m-1}}\ (z - z_0)^m\ f(z)\right]$$

Residue Theorem $\oint_C\ f(z)dz = 2\pi j \sum_{k=1}^{n} \text{Res}\, f(z)$