UNIVERSITI SAINS MALAYSIA

First Semester Examination
2014/2015 Academic Session

December 2014 / January 2015

# EKC 361 – Process Dynamics and Control
# *[Dinamik dan Kawalan Proses]*

Duration : 3 hours
*[Masa : 3 jam]*

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Please check that this examination paper consists of THIRTEEN pages of printed material and FOUR page of Appendix before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak dan EMPAT muka surat Lampiran sebelum anda memulakan peperiksaan ini.]*

**Instruction:** Answer **ALL** (5) questions.

*[**Arahan**: Jawab **SEMUA** (5) soalan.]*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]*

Answer ALL questions.

1. [a] A process furnace heats a process stream from near ambient temperature to a desired temperature of 300 °C. The process stream outlet temperature is regulated by manipulating the flow rate of fuel gas to the furnace, as shown in Figure Q.1.[a].

[i] Discuss the objectives of this control strategy.

[ii] Identify the measured output.

[iii] Identify the manipulated input.

[iv] Identify the possible disturbances.

[v] Is this a continuous or batch process?

[vi] Is this a feed-forward or feedback controller?

*[6 marks]*

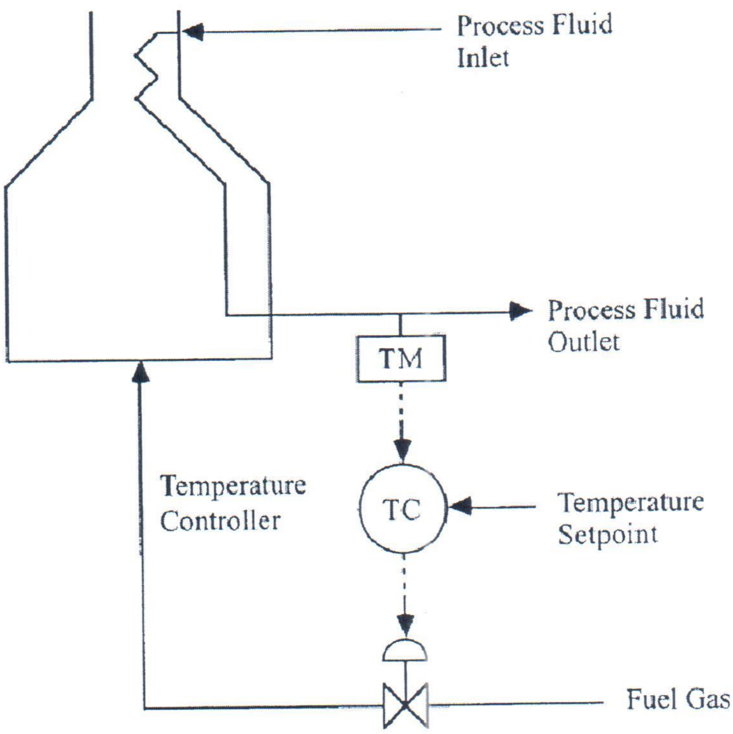



Figure Q.1.[a]: Furnace

*Jawab SEMUA soalan.*

1. [a] *Satu relau memanaskan satu aliran proses daripada suhu hampir persekitaran ke suhu yang diingini iaitu 300 °C. Suhu aliran keluar tersebut diatur dengan mengolah kadar aliran gas bahan api kepada relau tersebut seperti yang ditunjukkan dalam Rajah S.1.*

   [i] *Bincangkan objektif-objektif strategi kawalan tersebut.*

   [ii] *Kenalpasti keluaran terukur.*

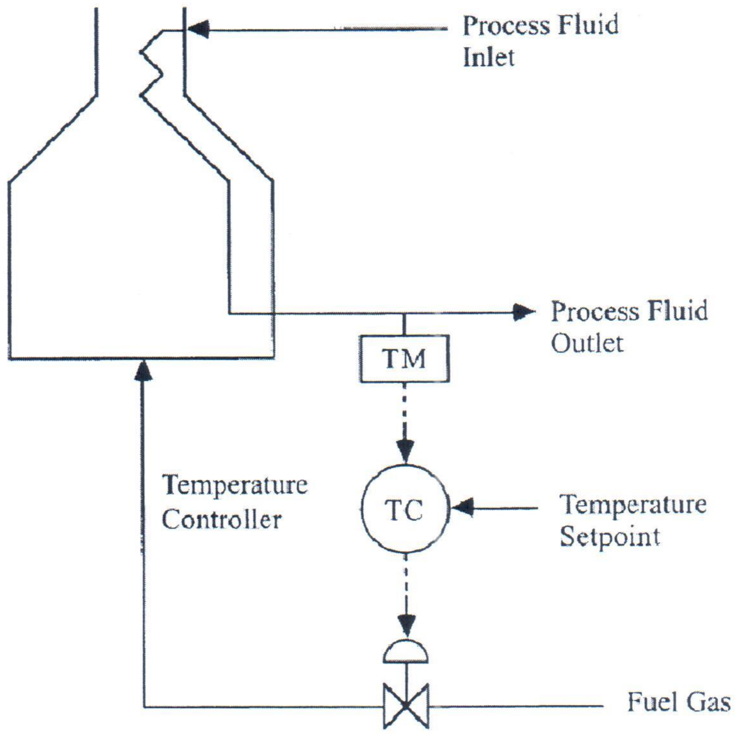
   [iii] *Kenalpasti masukan olahan.*

   [iv] *Kenalpasti gangguan-gangguan yang mungkin.*

   [v] *Adakah proses ini berterusan atau kelompok?*

   [vi] *Adakah ia pengawal suap-depan atau suap-balik?*

[6 markah]




*Rajah S.1.[a]: Relau*

[b] The caustic concentration of the mixing tank shown in Figure Q.1.[b] is measured using a conductivity cell. The total volume of solution in the tank is constant at 7 $ft^3$ and the density ($\rho$ = 70 $lb/ft^3$) can be considered to be independent of concentration. Let $c_m$ denote the caustic concentration measured by the conductivity cell. The dynamic response of the conductivity cell to a step change (at $t$ = 0) of 3 $lb/ft^3$ in the actual concentration (passing through the cell) is also shown in Figure Q.1.[b].

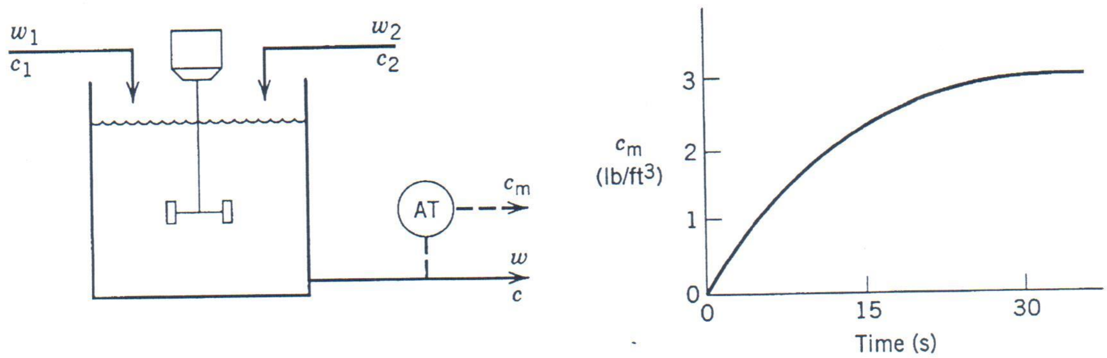



Figure Q.1.[b]: Mixing Tank

[i] Determine the transfer function $C_m(s)/C_1(s)$ assuming the flow rates are equal and constant: ( $w_1 = w_2 = 5$ lb/min).

*[6 marks]*

[ii] Determine the response for a step change in $c_1$ from 14 to 17 $lb/ft^3$.

*[3 marks]*

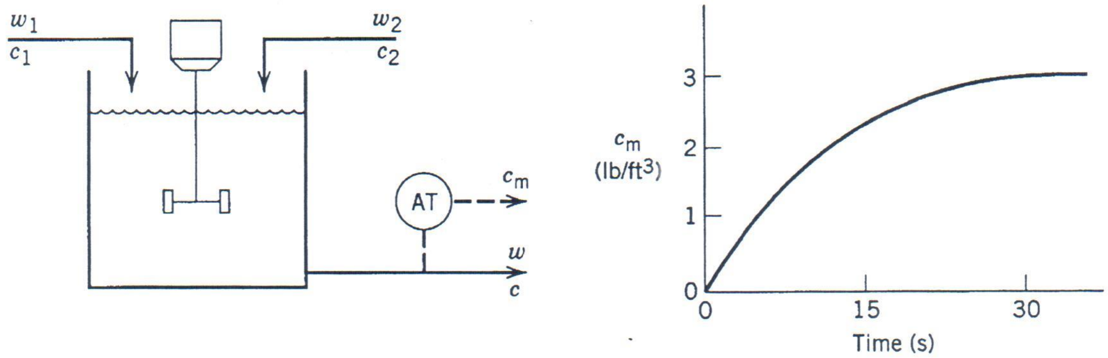
[iii] If the transfer function $C_m(s)/C(s)$ were approximated by 1 (unity), determine the step response of the system for the same input change.

*[3 marks]*

[iv] Compare the answer in [ii] and [iii] in term of the dynamics of the conductivity cell.

*[1 marks]*

[b] *Kepekatan kaustik bagi tangki pencampur yang ditunjukkan dalam Rajah S.1.[b] diukur menggunakan sel keberaliran. Jumlah isipadu larutan dalam tangki tersebut adalah malar pada 7 kaki$^3$ dan ketumpatannya ($\rho$ = 70 lb/kaki$^3$) boleh dianggap tidak bergantung kepada kepekatan. Biar* $c_m$ *sebagai kepekatan kaustik yang diukur oleh sel keberaliran. Sambutan dinamik bagi sel keberaliran dalam kepekatan sebenar (yang melepasi sel tersebut) terhadap satu perubahan langkah (pada* $t = 0$*) sebanyak 3 lb/kaki$^3$ juga ditunjukkan dalam Rajah S.1.[b].*




*Rajah S.1.[b]: Tangki Pencampur*

[i] *Tentukan fungsi pindah* $C_m(s)/C_1(s)$ *dengan menganggap kadar-kadar aliran adalah sama dan malar: (* $w_1 = w_2 = 5$ *lb/min).*

[6 markah]

[ii] *Tentukan sambutan bagi satu perubahan langkah dalam* $c_1$ *daripada 14 kepada 17 lb/kaki$^3$.*

[3 markah]

[iii] *Jika fungsi pindah* $C_m(s)/C(s)$ *dianggarkan bersamaan dengan 1, tentukan sambutan langkah sistem tersebut bagi perubahan masukan yang sama.*

[3 markah]

[iv] *Bandingkan jawapan dalam [ii] dan [iii] dari segi dinamik bagi sel keberaliran tersebut.*

[1 markah]

[c] Figure Q.1.[c] shows the response of different processes to a step change in input. Give an indication of the form(s) of possible transfer function(s) for each process.

*[6 marks]*

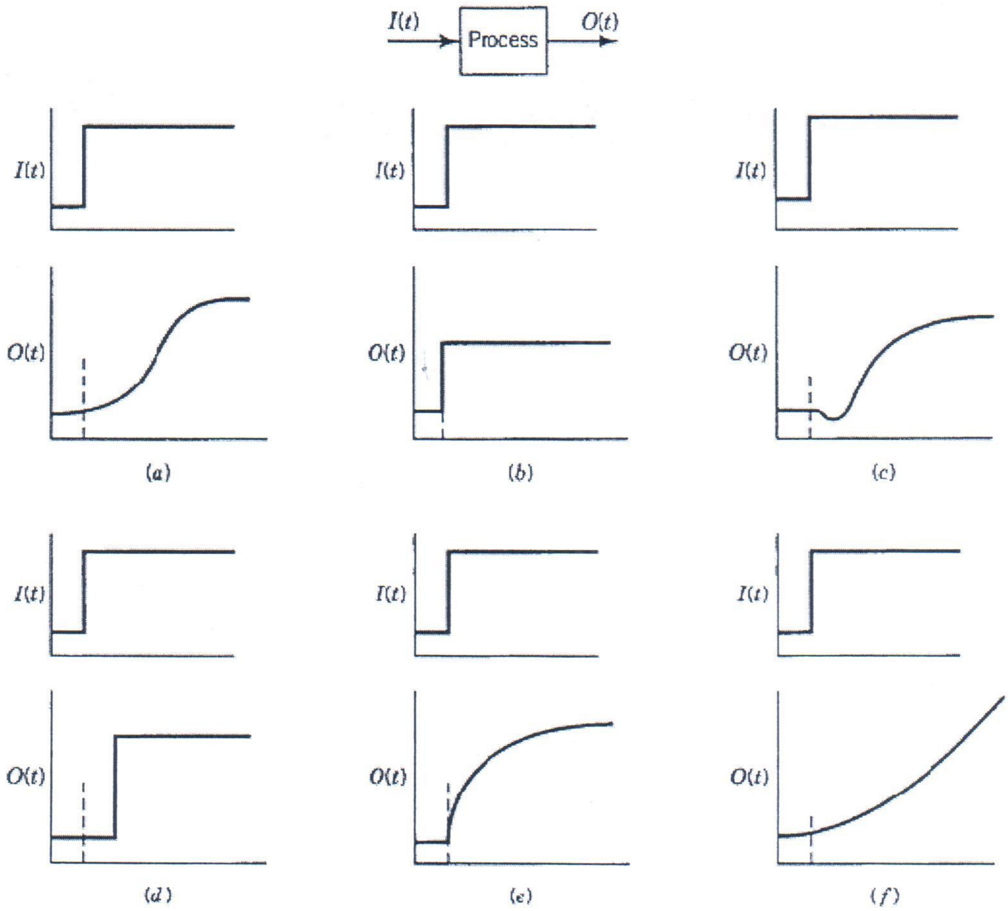


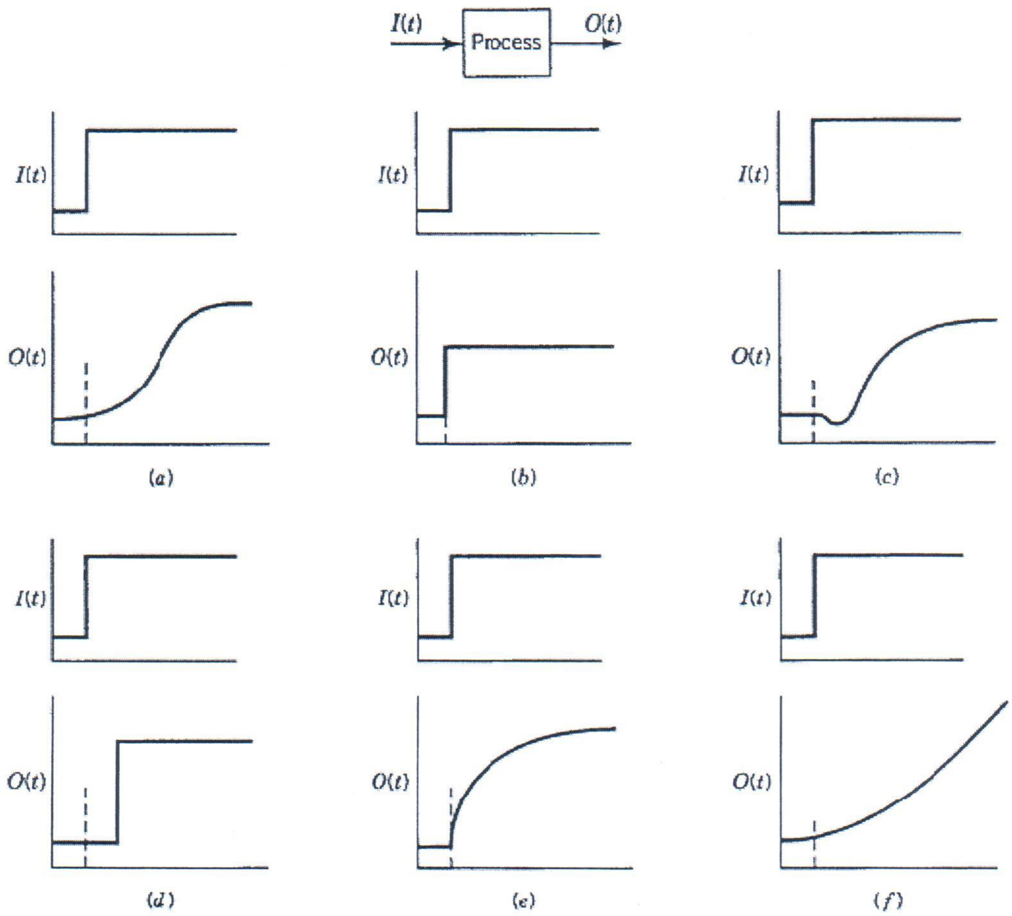

Figure Q.1.[c]: The response of different processes to a step change in input

2. [a] As a process engineer, you decide to develop a first-order plus time delay model of a process using a step test. The process is initially at steady state, with an input flow rate of 5 gpm and an output of 0.75 mol/L. You make a step increase of 0.5 gpm at 3:00 p.m. and do not observe any changes until 3:07 p.m. At 3:20 p.m., the value of the output is 0.8 mol/L. You become distracted and do not have a chance to look at the output variable again, until you leave for a break at 6:30 p.m. You note that the output has ceased to change and has achieved a new steady-state value of 0.85 mol/L. Determine the values of the process parameters with the correct units.

*[5 marks]*

[c] *Rajah S.1.[c] menunjukkan sambutan proses berbeza terhadap seunit tukar langah pada masukan. Nyatakan bentuk kemungkinan fungsi pindah bagi setiap proses.*

[6 markah]




*Rajah S.1.[c]: Sambutan proses berbeza terhadap seunit tukar langah pada masukan*

2. [a] *Sebagai seorang jurutera proses, anda membuat keputusan untuk membina satu model tertib pertama dengan masa lengah bagi satu proses menggunakan satu ujian langkah. Proses tersebut pada awalnya dalam keadaan mantap dengan kadar aliran masukan 5 gpm dan keluaran 0.75 mol/L. Pada jam 3.00 p.m., anda melakukan penambahan langkah sebanyak 0.5 gpm dan anda perhatikan tiada sebarang perubahan sehingga jam 3:07 p.m. Pada jam 3:20 p.m., nilai keluaran ialah 0.8 mol/L. Anda rasa terganggu dan tiada peluang melihat pembolehubah keluaran tersebut sehingga anda pergi untuk berehat pada jam 6:30 p.m. Anda dapati bahawa keluaran telah berhenti berubah dan mencapai keadaan mantap yang baru iaitu pada nilai 0.85 mol/L. Tentukan nilai-nilai proses parameter dengan unit yang betul.*

[5 markah]

[b] [i] Develop the transfer functions $\frac{H_2}{Q}$ and $\frac{H_3}{Q}$ for the three-tank system shown in Figure Q.2.[b] $H_2$, $H_3$ and $Q$ are deviation variables. Tank 1 and Tank 2 are interacting

*[12 marks]*

[ii] For a unit-step change in $q$ (i.e. $Q = \frac{1}{s}$), determine $H_3(0)$, $H_3(\infty)$ and sketch $H_3(t)$ versus $t$.

*[2 marks]*

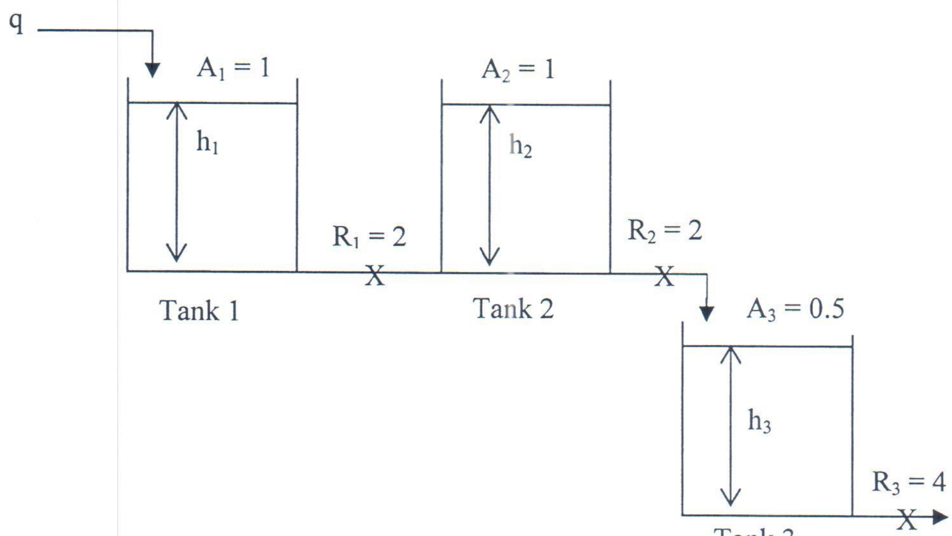



Figure Q.2.[b] : Three-tank system.

[c] Construct the block diagram representing the following transfer functions. In each case, do not conduct any algebraic manipulations to simplify the transfer functions, but use the rules of block diagram algebra to simplify the diagram if possible.

$$Y(s) = \frac{K_1}{(\tau_1 s + 1)} X(s) + \frac{K_2}{(\tau_2 s + 1)} X(s)$$

$$Y(s) = \frac{1}{(\tau s + 1)} [K_1 F_1(s) - K_2 F_2(s)]$$

$$Y_1(s) = G_1(s) X(s) + G_3(s) Y_2(s)\,;\; Y_2(s) = G_2(s) Y_1(s)$$

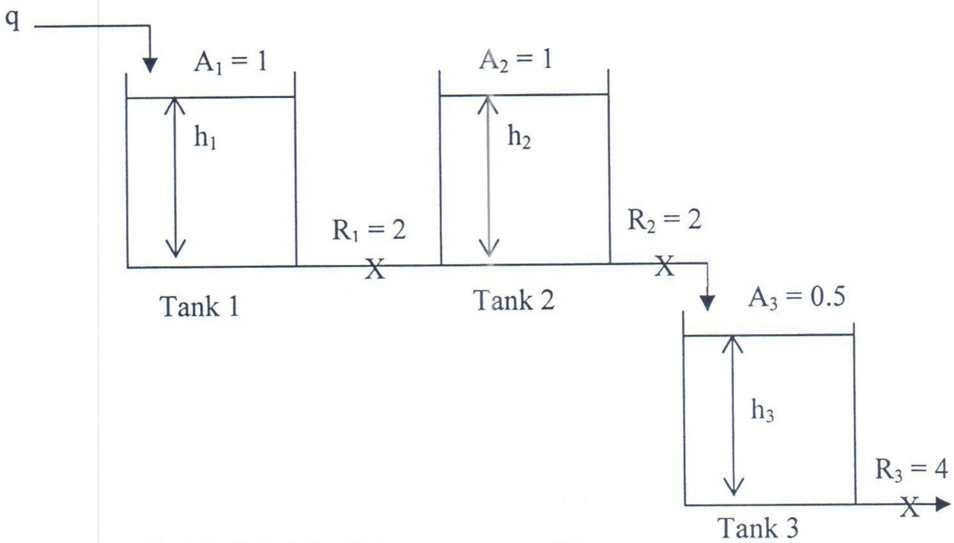
*[6 marks]*

[b] [i] *Bina fungsi pindah* $\frac{H_2}{Q}$ *dan* $\frac{H_3}{Q}$ *bagi sistem tiga tangki yang ditunjukkan dalam Rajah S.2.[b].* $H_2$, $H_3$ *dan* Q *adalah pembolehubah-pembolehubah sisihan. Tangki 1 dan Tangki 2 adalah saling tindak.*

*[12 markah]*

[ii] *Bagi satu unit tukar langkah, dalam* q *(iaitu* $Q = \frac{1}{s}$*), tentukan* $H_3(0)$, $H_3(\infty)$ *dan lakarkan* $H_3(t)$ *lawan* t.

*[2 markah]*




*Rajah S.2.[b] : Sistem tiga tangki.*

*[c]* *Bina gambarajah blok yang mewakili fungsi pindah berikut. Dalam setiap kes, jangan lakukan sebarang manipulasi algebra untuk memudahkan fungsi pindah tersebut, sebaliknya gunakan peraturan algebra bagi gambarajah blok untuk meringkaskan gambarajah tersebut jika boleh.*

$$Y(s) = \frac{K_1}{(\tau_1 s + 1)} X(s) + \frac{K_2}{(\tau_2 s + 1)} X(s)$$

$$Y(s) = \frac{1}{(\tau s + 1)} [K_1 F_1(s) - K_2 F_2(s)]$$

$$Y_1(s) = G_1(s) X(s) + G_3(s) Y_2(s) \; ; \; Y_2(s) = G_2(s) Y_1(s)$$

*[6 markah]*

3. Consider PI control of a second order process of

$$G_p = \frac{2}{(0.1s+1)(0.2s+1)} \text{ and } G_m = G_v = G_d = 1.$$

[a] Determine the value of $K_{cu}$.

*[5 marks]*

[b] Show the stability of closed loop system with controller setting of $K_c = 0.5$ and $\tau_I = 0.3$ using Bode plot. Log-log graph is provided in the Appendix.

*[10 marks]*

[c] Determine the $K_{cm}$, maximum value of $K_c$ that can be used with $\tau_I = 0.2$ min and still achieve closed loop stability.

*[10 marks]*

4. The closed-loop system in Figure Q.4 has the following transfer functions:

$$G_p(s) = \frac{1}{s+1} \quad G_d(s) = \frac{2}{(s+1)(5s+1)}$$

$$G_v = G_m = G_t = 1$$

Design a feedforward controller based on a dynamic analysis.

Plate Q.4.: Sample of test question

3. *Pertimbangkan kawalan PI bagi proses tertib ke dua*

$$G_p = \frac{2}{(0.1s+1)(0.2s+1)} \text{ dan } G_m = G_v = G_d = 1.$$

[a] *Tentukan nilai* $K_{cu}$.

[5 markah]

[b] *Tunjukkan kestabilan sistem gelung tertutup dengan penalaan kawalan,* $K_c = 0.5$ *dan* $\tau_I = 0.3$ *menggunakan plot Bode. Geraf log-log disediakan dalam Apendik.*

[10 markah]

[c] *Tentukan nilai maksimum* $K_c$ *iaitu* $K_{cm}$ *yang boleh digunakan dengan* $\tau_I = 0.2$ *min dan masih boleh mengekalkan kestabilan gelung tertutup.*

[10 markah]

4. *Sistem gelung tertutup dalam Gambarajah S.4 mempunyai fungsi pindah berikut:*

$$G_p(s) = \frac{1}{s+1} \quad G_d(s) = \frac{2}{(s+1)(5s+1)}$$
$$G_v = G_m = G_t = 1$$

*Reka bentuk suatu kawalan suap depan berdasarkan analisa dinamik.*

*Plat S.4.: Sampel soalan ujian*

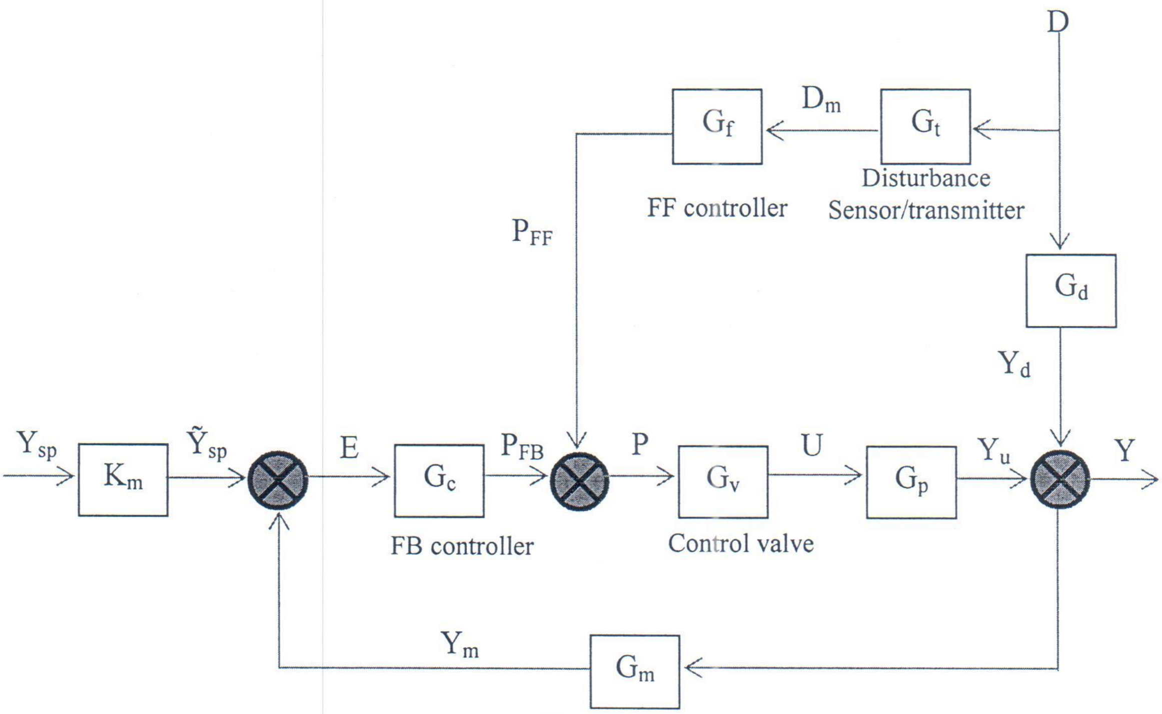



Figure Q.4.

Figure Q.4. is the block diagram supplemented for the sample of test question in Plate Q.4.

For the specified problem above, <u>describe</u> the steps followed in order to set up and solve the problem. The answer should not be more than a page. Consider the Grading Criteria (Hanson & Williams, 2008) as below when preparing the step.

[a] Has the student provided sufficient detail that I could reproduce the approach to the solution?

[b] Has the student demonstrated an understanding of what is being done in the solution process?

[c] Is the description written such that I can understand what the student means?

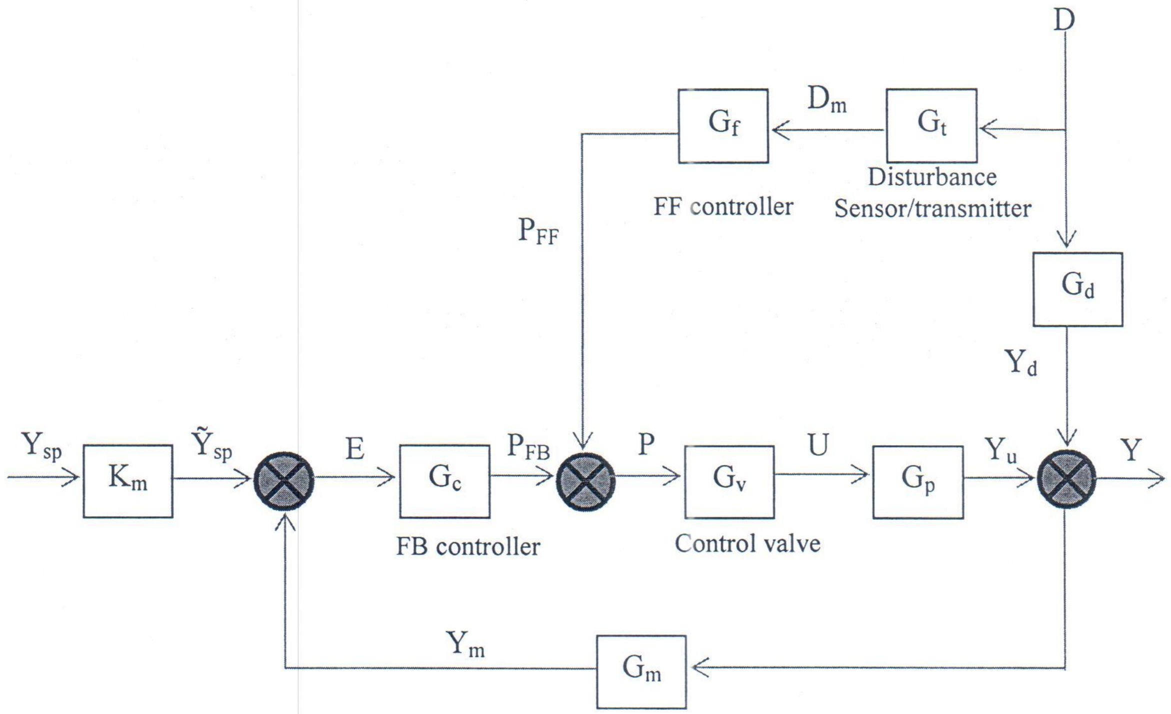
[d] Is the description focused on the approach to the solution of this problem, not the specific numbers of the solution?

Hanson, J.H. and Williams, J.M. (2008). Using writing assignment to improve self-assessment and communication skills in an engineering statics course. Journal of Engineering Education 97(4): 515-529.

*[12 marks]*

5. Derive the closed loop transfer function for set-point changes.

*[13 marks]*




*Gambarajah S.4.*

*Gambarajah S.4. adalah gambarajah blok dibekalkan untuk sampel soalan ujian dalam Plat S.4.*

*Bagi masalah tertentu di atas perihalkan langkah-langkah persediaan dan penyelesaian masalah. Jawapan tidak boleh melebihi satu muka surat. Pertimbangkan kriteria penggredan (Hanson & Williams, 2008) di bawah apabila menyediakan langkah-langkah itu.*

[a] *Adakah pelajar menyediakan perincian yang mencukupi supaya saya boleh menggunakan kaedah penyelesaiannya?*

[b] *Adakah pelajar menunjukkan kefahaman atas apa yang dibuat dalam proses penyelesaian?*

[c] *Adakah penerangan yang ditulis membolehkan saya memahami apa yang pelajar maksudkan?*

[d] *Adakah penerangan memfokuskan terhadap pendekatan penyelesaian masalah, bukannya jumlah tertentu penyelesaian?*

Hanson, J.H. and Williams, J.M. (2008). Using writing assignment to improve self-assessment and communication skills in an engineering statics course. Journal of Engineering Education 97(4): 515-529.

[12 markah]

5. *Terbitkan fungsi pindah gelung tertutup bagi perubahan titik set.*

[13 markah]

Appendix

# Table Laplace Transforms for Various Time-Domain Functions[a]

| $f(t)$ | $F(s)$ |
|---|---|
| 1. $\delta(t)$ (unit impulse) | $1$ |
| 2. $S(t)$ (unit step) | $\dfrac{1}{s}$ |
| 3. $t$ (ramp) | $\dfrac{1}{s^2}$ |
| 4. $t^{n-1}$ | $\dfrac{(n-1)!}{s^n}$ |
| 5. $e^{-bt}$ | $\dfrac{1}{s+b}$ |
| 6. $\dfrac{1}{\tau}e^{-t/\tau}$ | $\dfrac{1}{\tau s+1}$ |
| 7. $\dfrac{t^{n-1}e^{-bt}}{(n-1)!}$ $(n>0)$ | $\dfrac{1}{(s+b)^n}$ |
| 8. $\dfrac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$ | $\dfrac{1}{(\tau s+1)^n}$ |
| 9. $\dfrac{1}{b_1-b_2}(e^{-b_2t}-e^{-b_1t})$ | $\dfrac{1}{(s+b_1)(s+b_2)}$ |
| 10. $\dfrac{1}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$ | $\dfrac{1}{(\tau_1 s+1)(\tau_2 s+1)}$ |
| 11. $\dfrac{b_3-b_1}{b_2-b_1}e^{-b_1t}+\dfrac{b_3-b_2}{b_1-b_2}e^{-b_2t}$ | $\dfrac{s+b_3}{(s+b_1)(s+b_2)}$ |
| 12. $\dfrac{1}{\tau_1}\dfrac{\tau_1-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_1}+\dfrac{1}{\tau_2}\dfrac{\tau_2-\tau_3}{\tau_2-\tau_1}e^{-t/\tau_2}$ | $\dfrac{\tau_3 s+1}{(\tau_1 s+1)(\tau_2 s+1)}$ |
| 13. $1-e^{-t/\tau}$ | $\dfrac{1}{s(\tau s+1)}$ |
| 14. $\sin \omega t$ | $\dfrac{\omega}{s^2+\omega^2}$ |
| 15. $\cos \omega t$ | $\dfrac{s}{s^2+\omega^2}$ |
| 16. $\sin(\omega t+\phi)$ | $\dfrac{\omega\cos\phi+s\sin\phi}{s^2+\omega^2}$ |
| 17. $e^{-bt}\sin \omega t$ ($b, \omega$ real) | $\dfrac{\omega}{(s+b)^2+\omega^2}$ |
| 18. $e^{-bt}\cos \omega t$ ($b, \omega$ real) | $\dfrac{s+b}{(s+b)^2+\omega^2}$ |
| 19. $\dfrac{1}{\tau\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin(\sqrt{1-\zeta^2}\,t/\tau)$ $(0\le\lvert\zeta\rvert<1)$ | $\dfrac{1}{\tau^2s^2+2\zeta\tau s+1}$ |
| 20. $1+\dfrac{1}{\tau_2-\tau_1}(\tau_1 e^{-t/\tau_1}-\tau_2 e^{-t/\tau_2})$ $(\tau_1\ne\tau_2)$ | $\dfrac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$ |
| 21. $1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin[\sqrt{1-\zeta^2}\,t/\tau+\psi]$ <br> $\psi=\tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$, $(0\le\lvert\zeta\rvert<1)$ | $\dfrac{1}{s(\tau^2s^2+2\zeta\tau s+1)}$ |
| 22. $1-e^{-\zeta t/\tau}[\cos(\sqrt{1-\zeta^2}\,t/\tau)$ <br> $+\dfrac{\zeta}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,t/\tau)]$ <br> $(0\le\lvert\zeta\rvert<1)$ | $\dfrac{1}{s(\tau^2s^2+2\zeta\tau s+1)}$ |
| 23. $1+\dfrac{\tau_3-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}+\dfrac{\tau_3-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}$ <br> $(\tau_1\ne\tau_2)$ | $\dfrac{\tau_3 s+1}{s(\tau_1 s+1)(\tau_2 s+1)}$ |
| 24. $\dfrac{df}{dt}$ | $sF(s)-f(0)$ |
| 25. $\dfrac{d^nf}{dt^n}$ | $s^nF(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\cdots$ <br> $-sf^{(n-2)}(0)-f^{(n-1)}(0)$ |
| 26. $f(t-t_0)S(t-t_0)$ | $e^{-t_0s}F(s)$ |

[a]Note that $f(t)$ and $F(s)$ are defined for $t\ge 0$ only.

| Case | Model | $K_cK$ | $\tau_I$ | $\tau_D$ |
|---|---|---|---|---|
| A | $\frac{K}{\tau s+1}$ | $\frac{\tau}{\tau_c}$ | $\tau$ | — |
| B | $\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\frac{\tau_1+\tau_2}{\tau_c}$ | $\tau_1+\tau_2$ | $\frac{\tau_1\tau_2}{\tau_1+\tau_2}$ |
| C | $\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$ | $\frac{2\zeta\tau}{\tau_c}$ | $2\zeta\tau$ | $\frac{\tau}{2\zeta}$ |
| D | $\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$, $\beta>0$ | $\frac{2\zeta\tau}{\tau_c+\beta}$ | $2\zeta\tau$ | $\frac{\tau}{2\zeta}$ |
| E | $\frac{K}{s}$ | $\frac{2}{\tau_c}$ | $2\tau_c$ | — |
| F | $\frac{K}{s(\tau s+1)}$ | $\frac{2\tau_c+\tau}{\tau_c^2}$ | $2\tau_c+\tau$ | $\frac{2\tau_c\tau}{2\tau_c+\tau}$ |
| G | $\frac{Ke^{-\theta s}}{\tau s+1}$ | $\frac{\tau}{\tau_c+\theta}$ | $\tau$ | — |
| H | $\frac{Ke^{-\theta s}}{\tau s+1}$ | $\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$ | $\tau+\frac{\theta}{2}$ | $\frac{\tau\theta}{2\tau+\theta}$ |
| I | $\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$ | $\tau_1+\tau_2-\tau_3$ | $\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$ |
| J | $\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$ | $\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$ | $2\zeta\tau-\tau_3$ | $\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$ |
| K | $\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ | $\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$ | $\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$ | $\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$ |
| L | $\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$ | $\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$ | $2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$ | $\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$ |
| M | $\frac{Ke^{-\theta s}}{s}$ | $\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$ | $2\tau_c+\theta$ | — |
| N | $\frac{Ke^{-\theta s}}{s}$ | $\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$ | $2\tau_c+\theta$ | $\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$ |
| O | $\frac{Ke^{-\theta s}}{s(\tau s+1)}$ | $\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$ | $2\tau_c+\tau+\theta$ | $\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$ |

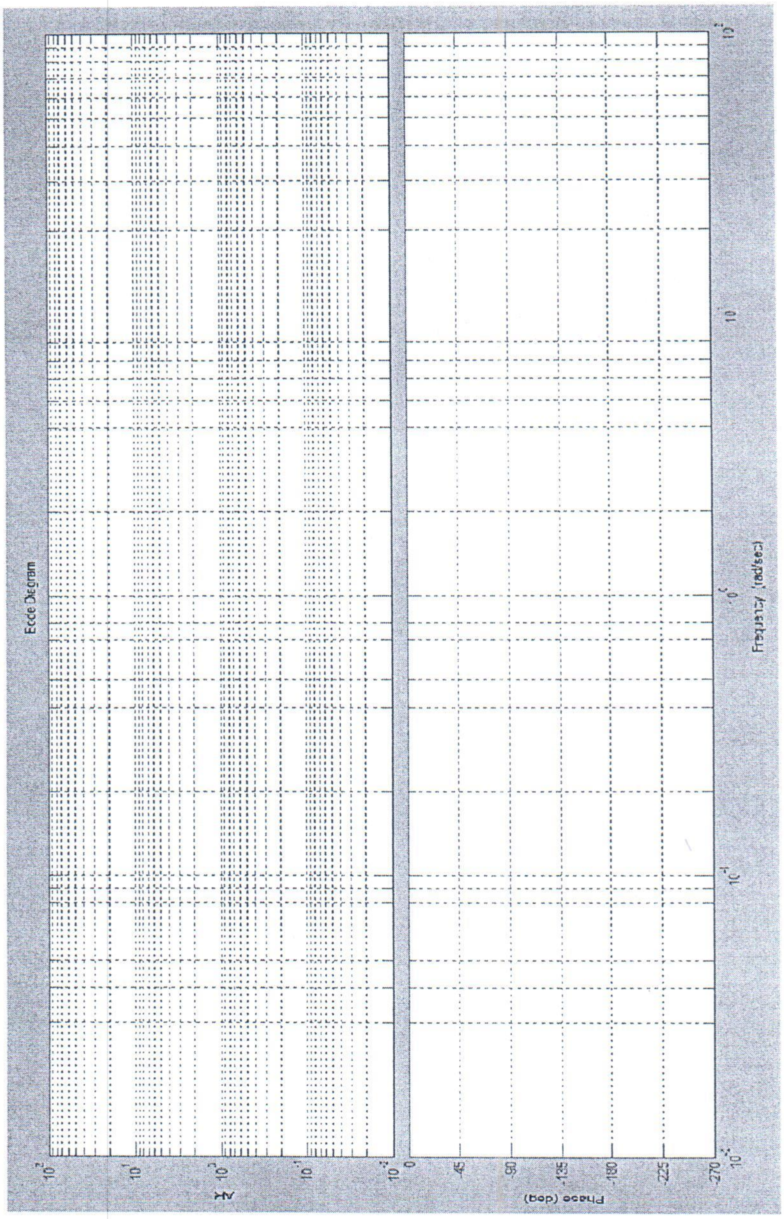
| Parallel Form | Series Form |
|---|---|
| $G_c(s)=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)$ | $G_c(s)=K'_c\left(1+\frac{1}{\tau'_I s}\right)(1+\tau'_D s)^{\dagger}$ |
| $K_c=K'_c\left(1+\frac{\tau'_D}{\tau'_I}\right)$ | $K'_c=\frac{K_c}{2}\left(1+\sqrt{1-4\tau_D/\tau_I}\right)$ |
| $\tau_I=\tau'_I+\tau'_D$ | $\tau'_I=\frac{\tau_I}{2}\left(1+\sqrt{1-4\tau_D/\tau_I}\right)$ |
| $\tau_D=\frac{\tau'_D\tau'_I}{\tau'_I+\tau'_D}$ | $\tau'_D=\frac{\tau_I}{2}\left(1-\sqrt{1-4\tau_D/\tau_I}\right)$ |

†These conversion equations are only valid if $\tau_D/\tau_I \le 0.25$.

| Type of Input | Type of Controller | Mode | A | B |
|---|---|---|---|---|
| Disturbance | PI | P | 0.859 | −0.977 |
| | | I | 0.674 | −0.680 |
| Disturbance | PID | P | 1.357 | −0.947 |
| | | I | 0.842 | −0.738 |
| | | D | 0.381 | 0.995 |
| Set point | PI | P | 0.586 | −0.916 |
| | | I | 1.03[b] | −0.165[b] |
| Set point | PID | P | 0.965 | −0.85 |
| | | I | 0.796[b] | −0.1465[b] |
| | | D | 0.308 | 0.929 |

[a] Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, $\tau/\tau_I$ for the integral mode, and $\tau_D/\tau$ for the derivative mode.

[b] For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$.


Bode Diagram
AR
Phase (deg)
Frequency (rad/sec)