UNIVERSITI SAINS MALAYSIA

First Semester Examination
2014/2015 Academic Session

December 2014 / January 2015

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of ELEVEN pages of printed material and SEVEN page of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEBELAS muka surat yang bercetak dan TUJUH muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **ALL** (4) questions.

*[**Arahan**: Jawab **SEMUA** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Answer ALL questions.

1. A thin layer of water-hexane is left to flow down an inclined plate so that the solvent (hexane) could naturally evaporate at a certain rate. The plate has a length, L and width, W. The angle of the plate with the vertical gravitational line is given as γ.

[a] Sketch a schematic diagram showing the flow of the liquid mixture down an inclined plane.

[5 marks]

[b] With an aid of the diagram sketched in [a], derive the velocity profile of the liquid flow v_z.

[6 marks]

[c] Using the same diagram re-derive the velocity profile by changing x by a coordinate $\bar{x}$ measured away from the plate, that is $\bar{x} = 0$ at the plate surface, and $\bar{x} = \delta$ at the liquid-gas interface. Show that the velocity distribution is given by;

$$v_z = \frac{\rho g \delta^2}{\mu}\left[\left(\frac{\bar{x}}{\delta}\right) - \frac{1}{2}\left(\frac{\bar{x}}{\delta}\right)^2\right]\cos\gamma$$

[7 marks]

[d] Using the equation derived in [c], find the average velocity of the system.

[7 marks]

2. [a] In the mass transport theory, the mass flux j_A is generally given as,

$$j = -\rho D \nabla \omega$$

Show that for a binary mixture of two components A and B, only ONE diffusivity is needed to describe the diffusional behaviour of a binary mixture.

[5 marks]

[b] Estimate the value of diffusivity, D_{AB} for the system of methane-ethane at 293K and 1 atm using the following methods;

[i] the correlation developed between the kinetics theory and the corresponding-state argument given by;

$$\frac{pD_{AB}}{(p_{cA}p_{cB})^{1/3}(T_{cA}T_{cB})^{5/12}\left(\frac{1}{M_A}+\frac{1}{M_B}\right)^{1/2}} = a\left(\frac{T}{\sqrt{T_{cA}T_{cB}}}\right)^b$$

Jawab SEMUA soalan.

1. *Satu lapisan nipis air-heksana dibiarkan mengalir ke bawah melalui satu plat condong di mana pelarut (heksana) boleh terpeluwap secara semulajadi berdasarkan kadar tertentu. Plat mempunyai panjang,* L *dan lebar,* W. *Sudut di antara plat dan garisan menegak graviti diberi sebagai* γ.

[a] *Lakarkan gambarajah skematik yang menunjukkan aliran campuran cecair menuruni plat condong.*

[5 markah]

[b] *Dengan berpandukan gambarajah yang telah dilakarkan di [a], terbitkan profil halaju bagi aliran cecair,* v_z.

[6 markah]

[c] *Dengan menggunakan gambarajah yang sama, terbitkan semula profil halaju dengan menggantikan x dengan koordinat* $\bar{x}$ *yang diukur menjauhi plat, di mana* $\bar{x} = 0$ *pada permukaan plat dan* $\bar{x} = \delta$ *pada cecair-gas antara fasa. Tunjukkan bahawa halaju taburan diberi sebagai;*

$$v_z = \frac{\rho g \delta^2}{\mu}\left[\left(\frac{\bar{x}}{\delta}\right) - \frac{1}{2}\left(\frac{\bar{x}}{\delta}\right)^2\right]\cos\gamma$$

[7 markah]

[d] *Dengan menggunakan persamaan yang diterbitkan pada [c], cari halaju purata bagi sistem ini.*

[7 markah]

2. [a] *Dalam teori pengangkutan jisim, fluks jisim* j_A *umumya diberi sebagai;*

$$j = -\rho D \nabla \omega$$

Tunjukkan bahawa bagi satu campuran perduaan komponen A dan B, hanya SATU kemeresapan diperlukan bagi memperihalkan mengenai kelakuan pemeresapan campuran perduaan.

[5 markah]

[b] *Anggarkan nilai kemeresapan,* D_{AB} *bagi sistem metana-etana pada 293 K dan 1 atm dengan menggunakan kaedah-kaedah:*

[i] *perkaitan yang dibina di antara teori kinetik dan hujah keadaan-sepadan yang diberikan oleh;*

$$\frac{pD_{AB}}{(p_{cA}p_{cB})^{1/3}(T_{cA}T_{cB})^{5/12}(\frac{1}{M_A}+\frac{1}{M_B})^{1/2}} = a\left(\frac{T}{\sqrt{T_{cA}T_{cB}}}\right)^b$$

...4/-

where p represents the pressure in atm, D_{AB} is the diffusivity between two components, T represents the temperature in Kelvin and M is the components' relative molecular mass. The dimensionless constants where a = 2.745 x 10^{-4} and b = 1.823 are values obtained from experimental observation. (The Lennard-Jones parameter and properties table may be used).

[10 marks]

[ii] the Chapman-Enskog relation given by;

$$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B) \quad \text{and} \quad \varepsilon_{AB} = \sqrt{\varepsilon_A \varepsilon_B}$$

with the diffusivity given by;

$$D_{AB} = 0.0018583\sqrt{T^3\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}\frac{1}{p\sigma_{AB}^2\Omega_{D,AB}}$$

[10 marks]

3. A liquefied gas is stored in a spherical container vented to the atmosphere, having inner wall radius, r_0 and outer wall radius, r_1. The container is well-insulated and the thermal conductivity of the insulation is k. Temperatures at the inner and outer walls are T_0 and T_1, respectively.

[a] By applying energy balance on an elemental solid of spherical shell of thickness Δr, show that the variation of temperature T along the radial position r within the wall can be written as:

$$\frac{d}{dr}\left(r^2 k\frac{dT}{dr}\right) = 0$$

[6 marks]

[b] Given thermal conductivity of the insulation varies linearly with temperature, and can be expressed as:

$$\frac{k - k_0}{k_1 - k_0} = \frac{T - T_0}{T_1 - T_0} = \theta$$

Note that the subscripts 0 and 1 refer to the values at boundaries, inner wall and outer wall, respectively. Also, θ is defined as dimensionless temperature and can be conveniently used to solve for the constants of integration that appear when solving the above differential equation.

[i] State <u>TWO</u> boundary conditions showing the value of θ at the boundaries.

[2 marks]

di mana p *mewakili tekanan dalam atm,* D_{AB} *adalah kemeresapan di antara dua komponen,* T *mewakili suhu dalam Kelvin dan* M *adalah jisim molekul relatif bagi suatu komponen. Pemalar tanpa-dimensi* a $= 2.745 \times 10^{-4}$ *dan* b $= 1.823$ *adalah nilai-nilai yang diperolehi daripada cerapan ujikaji. (Parameter Lennard-Jones dan jadual ciri boleh digunakan).*

[10 markah]

[ii] *perkaitan Chapman-Enskog diberikan oleh;*

$$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B) \quad dan \quad \varepsilon_{AB} = \sqrt{\varepsilon_A \varepsilon_B}$$

dengan kemeresapan diberikan oleh;

$$D_{AB} = 0.0018583\sqrt{T^3\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}\frac{1}{p\sigma_{AB}^2\Omega_{D,AB}}$$

[10 markah]

3. *Suatu gas cecair disimpan di dalam bekas berbentuk sfera yang dilepaskan ke atmosfera, mempunyai jejari dinding dalam,* r_0 *dan jejari dinding luar,* r_1. *Bekas itu ditebat dengan baik dan keberaliran haba penebat adalah* k. *Suhu pada dinding dalam dan luar adalah masing-masing* T_0 *dan* T_1.

[a] *Dengan mengaplikasikan keseimbangan tenaga pada satu elemen pejal kelompang sfera berketebalan* Δr, *tunjukkan bahawa variasi suhu* T *di sepanjang kedudukan jejarian* r *dalam dinding boleh ditulis sebagai:*

$$\frac{d}{dr}\left(r^2 k \frac{dT}{dr}\right) = 0$$

[6 markah]

[b] *Diberi keberaliran haba penebat berubah secara linear dengan suhu, dan boleh diungkapkan sebagai:*

$$\frac{k - k_0}{k_1 - k_0} = \frac{T - T_0}{T_1 - T_0} = \theta$$

Subskrip 0 dan 1 merujuk kepada nilai-nilai di sempadan, masing-masing untuk dinding dalam dan dinding luar. Juga, θ *ditakrifkan sebagai suhu tak berdimensi dan dapat digunakan untuk memudahkan penyelesaian bagi pemalar pengamiran yang muncul apabila menyelesaikan persamaan pembezaan di atas.*

[i] *Nyatakan* <u>*DUA*</u> *keadaan sempadan yang menunjukkan nilai* θ *pada batas-batas tersebut.*

[2 markah]

[ii] Derive an expression for temperature profile within the walls, showing the variation of θ with respect to r.

[7 marks]

[iii] By taking the inner wall as reference, develop an expression for the steady-state heat transfer rate through the wall of the container.

[5 marks]

[c] The simplified expression for steady state rate of heat transfer through the wall of an insulated spherical container is given by:

$$Q = 4\pi r_0 r_1 k_a \frac{(T_1 - T_0)}{(r_1 - r_0)}$$

The average thermal conductivity of the insulation is denoted by k_a. Estimate the rate of evaporation of liquid oxygen (in kg/h) from a spherical container of 1.8 m inside diameter covered with a 30 cm thick annular evacuated jacket filled with particulate insulation.

Temperature at inner surface of insulation	-183°C
Temperature at outer surface of insulation	0°C
Boiling point of O_2	-183°C
Heat of vaporization of O_2	214 kJ/kg
Average thermal conductivity of insulation	1.4×10^{-3} W/m·K

[5 marks]

4. [a] Derive the following continuity relationship for a single phase fluid flow:

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right)$$

where ρ is the fluid density, t is time and v_x, v_y, and v_z, are the velocities in the x, y and z-directions respectively. To what form does the above equation reduce into for an incompressible (constant density) fluid?

The above continuity equation is very useful in deriving the famous Navier-Stokes equation (in x-direction) in the form given below;

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + vz \frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

The above equation is <u>ONLY</u> applicable to describe the transport of fluid in laminar flow.

[ii] *Terbitkan satu ungkapan bagi profil suhu dalam dinding, dengan menunjukkan variasi θ terhadap* r.

[7 markah]

[iii] *Dengan mengambil kira dinding dalam sebagai rujukan, bangunkan satu ungkapan kadar pemindahan haba melalui dinding bekas pada keadaan mantap.*

[5 markah]

[c] *Ungkapan yang dipermudahkan untuk kadar pemindahan haba pada keadaan mantap melalui dinding bekas berbentuk sfera yang bertebat diberikan oleh:*

$$Q = 4\pi r_0 r_1 k_a \frac{(T_1 - T_0)}{(r_1 - r_0)}$$

Keberaliran haba purata bagi penebat ditandakan sebagai k_a. *Anggarkan kadar penyejatan cecair oksigen (dalam kg/j) daripada bekas berbentuk sfera berdiameter dalam 1.8 m yang dilitupi dengan jaket anulus yang dikosongkan dengan ketebalan 30 sm dipenuhi dengan partikel penebat.*

Suhu di permukaan dalam penebat	*-183 ° C*
Suhu di permukaan luar penebat	*0 ° C*
Takat didih O_2	*-183 ° C*
Haba pengewapan O_2	*214 kJ / kg*
Keberaliran haba purata bagi penebat	1.4×10^{-3} *W/m·K*

[5 markah]

4. *[a]* *Terbitkan persamaan keselanjaran berikut bagi aliran cecair satu fasa.*

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) \right)$$

di mana ρ *adalah ketumpatan cecair,* t *adalah masa dan* v_x, v_y, *dan* v_z, *adalah masing-masing halaju pada arah* x, y *dan* z. *Kepada bentuk apakah persamaan diatas dapat dikurangkan bagi cecair tidak boleh mampat (ketumpatan tetap)?*

Persamaan keselanjaran di atas berguna untuk menerbitkan persamaan terkenal Navier-Stokes (pada arah-x) di mana bentuknya diberi di bawah;

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + vz \frac{\partial v_x}{\partial z} \right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial \rho}{\partial x} + \rho g_x$$

Persamaan ini <u>*HANYA*</u> *digunakan untuk memperihalkan pengangkutan cecair pada aliran laminar.*

With the given Navier-Stokes equation of flow (in x-direction), derive the Reynolds-Averaged Navier-Stokes equation by defining the fluctuation velocity v_x as;

$$v_x' = v_x - \bar{v}_x$$

State all the assumptions used and the time-smooth averaged velocities with 0 values.

[10 marks]

[b] In a chemical process where carbon dioxide is formed as one of the by-products, the gas mixture needs to be treated before it can be recycled. Methyldiethanol-amine (MDEA) is a popular absorption agent, which can absorp carbon dioxide from the mixture.

In the bubble column, carbon dioxide dissolves in MDEA irreversibly and attaches to the bond with the rate given as;

$$r_{abs} = k''' C_{CO_2}$$

Prove that the ratio of carbon dioxide gas to that of the initial value is given by;

$$\frac{C_{CO_2}}{C_{CO_{2,0}}} = \frac{\sin\phi \cosh\phi\zeta + (\beta - \cosh\phi)\sinh\phi\zeta}{\sinh\phi}$$

where $\beta = \dfrac{C_{CO_{2,\delta}}}{C_{CO_2,0}}$ and δ is the liquid film thickness.

[8 marks]

[c] The general form of energy equation can be used to derive expression for the use in an adiabatic frictionless process and other useful processes related to heat transport.

$$\rho \hat{C}_P \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial ln\rho}{\partial lnT}\right)_P \frac{DP}{Dt}$$

[i] Briefly explain the meaning of substantial time derivative $\frac{D}{Dt}$ in the above equation.

[1 mark]

Berdasarkan persamaan aliran (arah-x) Navier-Stokes yang diberi, terbitkan persamaan Reynolds-Averaged Navier-Stokes dengan menakrifkan halaju turun-naik v_x sebagai;

$$v_x' = v_x - \bar{v}_x$$

Nyatakan semua anggapan-anggapan yang digunakan dan halaju-halaju purata masa-terlicin dengan nilai-nilai 0.

[10 markah]

[b] *Dalam satu proses kimia di mana karbon dioksida dibebaskan sebagai salah satu produk sampingan, campuran gas perlu dirawat sebelum ia dapat dikitar semula. Metildietanol-amina (MDEA) adalah satu agen penyerap popular yang boleh menyerap karbon dioksida daripada campuran tersebut.*

Dalam satu turus gelembung, karbon dioksida terlarut di dalam MDEA secara tidak berbalik dan terlekat pada ikatan dengan kadar yang diberikan oleh;

$$r_{abs} = k''' C_{CO_2}$$

Buktikan bahawa nisbah gas karbon dioksida kepada nilai asal gas tersebut diberikan oleh;

$$\frac{C_{CO_2}}{C_{CO_{2,0}}} = \frac{\sin\phi \cosh\phi\zeta + (\beta - \cosh\phi)\sinh\phi\zeta}{\sinh\phi}$$

di mana $\beta = \dfrac{C_{CO_{2,\delta}}}{C_{CO_2,0}}$ *dan* δ *adalah ketebalan cecair filem*

[8 markah]

[c] *Persamaan tenaga dalam bentuk umum boleh digunakan bagi menerbitkan ungkapan untuk digunakan dalam satu proses adiabatik tanpa geseran dan proses-proses yang berguna lain yang berkaitan dengan pengangkutan haba:*

$$\rho \hat{C}_P \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial ln\rho}{\partial lnT}\right)_P \frac{DP}{Dt}$$

[i] *Terangkan secara ringkas maksud masa terbitan nyata* $\frac{D}{Dt}$ *dalam persamaan di atas.*

[1 markah]

[ii] Develop an equation for the relationship of local pressure to temperature in a stream of ideal gas in which the momentum flux τ and the heat flux q are negligible.

The specific heat capacity of the gas can be assumed constant and independent of temperature.

Ideal gas law $PM_r = \rho RT$ *[6 marks]*

[ii] *Bangunkan satu persamaan untuk menunjukkan hubungan tekanan lokal kepada suhu dalam satu aliran gas unggul di mana fluks momentum* τ *dan fluks haba* q *boleh diabaikan.*

Muatan haba tentu gas boleh dianggap malar dan tak bergantung kepada suhu.

Hukum gas unggul $PM_r = \rho RT$

[6 markah]

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APPENDICES

Appendix A: Conversion Factors

Given a quantity in these units:	Multiply by:	To get quantity in these units:
Pounds	453.59	Grams
Kilograms	2.2046	Pounds
Inches	2.5400	Centimeters
Meters	39.370	Inches
Gallons (U.S.)	3.7853	Liters
Gallons (U.S.)	231.00	Cubic inches
Gallons (U.S.)	0.13368	Cubic feet
Cubic feet	28.316	Liters
Kelvins	1.800000	Degrees Rankine
Degrees Rankine	0.555556	Kelvins

Table A.1

Given a quantity in this units	Multiply by table value to convert to these units	$N = kg{\cdot}m/s^2$ (Newtons)	$g{\cdot}cm/s^2$	$lb_m{\cdot}ft/s^2$	lb_f
$N = kg{\cdot}m/s^2$	(Newtons)	1	10^5	7.2330	2.24881×10^{-1}
$g{\cdot}cm/s^2$	(dynes)	10^{-5}	1	7.2330×10^{-5}	2.24881×10^{-6}
$lb_m{\cdot}ft/s^2$	(poundals)	1.3826×10^{-1}	1.3826×10^4	1	3.1081×10^{-2}
lb_f		4.4482	4.4482×10^5	32.1740	1

Table A.2

Appendix B: Equation of Motion in Terms of τ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p - [\nabla \cdot \tau] + \rho \mathbf{g}$$

Cartesian coordinates (x, y, z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.1

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.2

Spherical coordinates (r, θ, ϕ):[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi$$

Table B.3

<u>Appendix C</u>: Equation of Motion for a Newtoniam Fluid with Constant ρ and μ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g}$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.1

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.2

Spherical coordinates (r, θ, φ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

Table C.3

Appendix D: Lennard-Jones Potential Parameters and Critical Properties

Substance	Molecular Weight M	Lennard-Jones parameters: σ (Å)	ε/κ (K)	Ref.	Critical properties[g,h]: T_c (K)	p_c (atm)	$\tilde{V}_c$ (cm^3/g-mole)	$\mu_c \times 10^6$ (g/cm · s)	$k_c \times 10^6$ (cal/cm · s · K)
Light elements:									
H_2	2.016	2.915	38.0	*a*	33.3	12.80	65.0	34.7	—
He	4.003	2.576	10.2	*a*	5.26	2.26	57.8	25.4	—
Noble gases:									
Ne	20.180	2.789	35.7	*a*	44.5	26.9	41.7	156.	79.2
Ar	39.948	3.432	122.4	*b*	150.7	48.0	75.2	264.	71.0
Kr	83.80	3.675	170.0	*b*	209.4	54.3	92.2	396.	49.4
Xe	131.29	4.009	234.7	*b*	289.8	58.0	118.8	490.	40.2
Simple polyatomic gases:									
Air	28.964[i]	3.617	97.0	*a*	132.4[i]	37.0[i]	86.7[i]	193.	90.8
N_2	28.013	3.667	99.8	*b*	126.2	33.5	90.1	180.	86.8
O_2	31.999	3.433	113.	*a*	154.4	49.7	74.4	250.	105.3
CO	28.010	3.590	110.	*a*	132.9	34.5	93.1	190.	86.5
CO_2	44.010	3.996	190.	*a*	304.2	72.8	94.1	343.	122.
NO	30.006	3.470	119.	*a*	180.	64.	57.	258.	118.2
N_2O	44.012	3.879	220.	*a*	309.7	71.7	96.3	332.	131.
SO_2	64.065	4.026	363.	*c*	430.7	77.8	122.	411.	98.6
F_2	37.997	3.653	112.	*a*	—	—	—	—	—
Cl_2	70.905	4.115	357.	*a*	417.	76.1	124.	420.	97.0
Br_2	159.808	4.268	520.	*a*	584.	102.	144.	—	—
I_2	253.809	4.982	550.	*a*	800.	—	—	—	—
Hydrocarbons:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
$CH{\equiv}CH$	26.04	4.114	212.	*d*	308.7	61.6	112.9	237.	—
$CH_2{=}CH_2$	28.05	4.228	216.	*b*	282.4	50.0	124.	215.	—
C_2H_6	30.07	4.388	232.	*b*	305.4	48.2	148.	210.	203.
$CH_3C{\equiv}CH$	40.06	4.742	261.	*d*	394.8	—	—	—	—
$CH_3CH{=}CH_2$	42.08	4.766	275.	*b*	365.0	45.5	181.	233.	—
C_3H_8	44.10	4.934	273.	*b*	369.8	41.9	200.	228.	—
$n{-}C_4H_{10}$	58.12	5.604	304.	*b*	425.2	37.5	255.	239.	—
$i{-}C_4H_{10}$	58.12	5.393	295.	*b*	408.1	36.0	263.	239.	—
$n{-}C_5H_{12}$	72.15	5.850	326.	*b*	469.5	33.2	311.	238.	—
$i{-}C_5H_{12}$	72.15	5.812	327.	*b*	460.4	33.7	306.	—	—
$C(CH_3)_4$	72.15	5.759	312.	*b*	433.8	31.6	303.	—	—
$n{-}C_6H_{14}$	86.18	6.264	342.	*b*	507.3	29.7	370.	248.	—
$n{-}C_7H_{16}$	100.20	6.663	352.	*b*	540.1	27.0	432.	254.	—
$n{-}C_8H_{18}$	114.23	7.035	361.	*b*	568.7	24.5	492	259.	—
$n{-}C_9H_{20}$	128.26	7.463	351.	*b*	594.6	22.6	548.	265.	—
Cyclohexane	84.16	6.143	313.	*d*	553.	40.0	308.	284.	—
Benzene	78.11	5.443	387.	*b*	562.6	48.6	260.	312.	—
Other organic compounds:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
CH_3Cl	50.49	4.151	355.	*c*	416.3	65.9	143.	338.	—
CH_2Cl_2	84.93	4.748	398.	*c*	510.	60.	—	—	—
$CHCl_3$	119.38	5.389	340.	*e*	536.6	54.	240.	410.	—
CCl_4	153.82	5.947	323.	*e*	556.4	45.0	276.	413.	—
C_2N_2	52.034	4.361	349.	*e*	400.	59.	—	—	—
COS	60.076	4.130	336.	*e*	378.	61.	—	—	—
CS_2	76.143	4.483	467.	*e*	552.	78.	170.	404.	—
CCl_2F_2	120.91	5.116	280.	*b*	384.7	39.6	218.	—	—

Table D.1

Collision Integrals for use with the Lennard-Jones Potential for the Prediction of Transport Properties of Gases at Low Densities

$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)	$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)
0.30	2.840	2.649	2.7	1.0691	0.9782
0.35	2.676	2.468	2.8	1.0583	0.9682
0.40	2.531	2.314	2.9	1.0482	0.9588
0.45	2.401	2.182	3.0	1.0388	0.9500
0.50	2.284	2.066	3.1	1.0300	0.9418
0.55	2.178	1.965	3.2	1.0217	0.9340
0.60	2.084	1.877	3.3	1.0139	0.9267
0.65	1.999	1.799	3.4	1.0066	0.9197
0.70	1.922	1.729	3.5	0.9996	0.9131
0.75	1.853	1.667	3.6	0.9931	0.9068
0.80	1.790	1.612	3.7	0.9868	0.9008
0.85	1.734	1.562	3.8	0.9809	0.8952
0.90	1.682	1.517	3.9	0.9753	0.8897
0.95	1.636	1.477	4.0	0.9699	0.8845
1.00	1.593	1.440	4.1	0.9647	0.8796
1.05	1.554	1.406	4.2	0.9598	0.8748
1.10	1.518	1.375	4.3	0.9551	0.8703
1.15	1.485	1.347	4.4	0.9506	0.8659
1.20	1.455	1.320	4.5	0.9462	0.8617
1.25	1.427	1.296	4.6	0.9420	0.8576
1.30	1.401	1.274	4.7	0.9380	0.8537
1.35	1.377	1.253	4.8	0.9341	0.8499
1.40	1.355	1.234	4.9	0.9304	0.8463
1.45	1.334	1.216	5.0	0.9268	0.8428
1.50	1.315	1.199	6.0	0.8962	0.8129
1.55	1.297	1.183	7.0	0.8727	0.7898
1.60	1.280	1.168	8.0	0.8538	0.7711
1.65	1.264	1.154	9.0	0.8380	0.7555
1.70	1.249	1.141	10.0	0.8244	0.7422
1.75	1.235	1.128	12.0	0.8018	0.7202
1.80	1.222	1.117	14.0	0.7836	0.7025
1.85	1.209	1.105	16.0	0.7683	0.6878
1.90	1.198	1.095	18.0	0.7552	0.6751
1.95	1.186	1.085	20.0	0.7436	0.6640
2.00	1.176	1.075	25.0	0.7198	0.6414
2.10	1.156	1.058	30.0	0.7010	0.6235
2.20	1.138	1.042	35.0	0.6854	0.6088
2.30	1.122	1.027	40.0	0.6723	0.5964
2.40	1.107	1.013	50.0	0.6510	0.5763
2.50	1.0933	1.0006	75.0	0.6140	0.5415
2.60	1.0807	0.9890	100.0	0.5887	0.5180

Table D.2

Appendix E: Some Ordinary Differential Equations and Their Solutions

Equation	Solution
$\frac{dy}{dx} = \frac{f(x)}{g(y)}$	$\int g\,dy = \int f\,dx + C_1$
$\frac{dy}{dx} + f(x)y = g(x)$	$y = e^{-\int f\,dx}\left(\int e^{\int f\,dx} g\,dx + C_1\right)$
$\frac{d^2y}{dx^2} + a^2y = 0$	$y = C_1 \cos ax + C_2 \sin ax$
$\frac{d^2y}{dx^2} - a^2y = 0$	$y = C_1 \cosh ax + C_2 \sinh ax$ or $y = C_3e^{+ax} + C_4e^{-ax}$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) + a^2y = 0$	$y = \frac{C_1}{x} \cos ax + \frac{C_2}{x} \sin ax$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) - a^2y = 0$	$y = \frac{C_1}{x}\cosh ax + \frac{C_2}{x}\sinh ax$ or $y = \frac{C_3}{x}e^{+ax} + \frac{C_4}{x}e^{-ax}$
$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$	Solve the equation $n^2 + an + b = 0$, and get the roots $n = n_+$ and $n = n_-$. Then (a) if n_+ and n_- are real and unequal, $y = C_1\exp(n_+x) + C_2\exp(n_-x)$ (b) if n_+ and n_- are real and equal to n, $y = e^{nx}(C_1x + C_2)$ (c) if n_+ and n_- are complex: $n_\pm = p \pm iq$, $y = e^{px}(C_1 \cos qx + C_2 \sin qx)$
$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$	$y = C_1\int_0^x \exp(-\bar{x}^2)\,d\bar{x} + C_2$
$\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} = 0$	$y = C_1\int_0^x \exp(-\bar{x}^3)\,d\bar{x} + C_2$
$\frac{d^2y}{dx^2} = f(x)$	$y = \int_0^x\int_0^{\bar{x}} f(\bar{\bar{x}})d\bar{\bar{x}}\,d\bar{x} + C_1x + C_2$
$\frac{1}{x}\frac{d}{dx}\left(x\frac{dy}{dx}\right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}}\int_0^{\bar{x}} \bar{\bar{x}}f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x} + C_1 \ln x + C_2$
$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}^2}\int_0^{\bar{x}} \bar{\bar{x}}^2f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x} - \frac{C_1}{x} + C_2$
$\frac{d^2y}{dx^2} = h(y)$	$x = \int_0^y \frac{d\bar{y}}{\sqrt{2\int_0^{\bar{y}} h(\bar{\bar{y}})d\bar{\bar{y}} + C_1}} + C_2$
$x^3\frac{d^3y}{dx^3} + ax^2\frac{d^2y}{dx^2} + bx\frac{dy}{dx} + cy = 0$	$y = C_1x^{n_1} + C_2x^{n_2} + C_3x^{n_3}$, where the n_k are the roots of the equation $n(n-1)(n-2) + an(n-1) + bn + c = 0$, provided that all roots are distinct.

Table E

<u>Appendix E</u>: Some Ordinary Differential Equations and Their Solutions (cont'd)

Error Function:

The error function is defined as

$$\operatorname{erf} x = \frac{\int_0^x \exp(-\bar{x}^2)\, d\bar{x}}{\int_0^\infty \exp(-\bar{x}^2)\, d\bar{x}} = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\bar{x}^2)\, d\bar{x}$$

$$\frac{d}{dx} \operatorname{erf} u = \frac{2}{\sqrt{\pi}} \exp(-u^2) \frac{du}{dx}$$