
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2006/2007

October/November 2006

EEE 228E - SIGNAL AND SYSTEM [ISYARAT DAN SISTEM]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of NINE pages of printed material and TEN pages APPENDIX before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat bercetak dan SEPULUH muka surat LAMPIRAN sebelum anda memulakan peperiksaan].

This paper contains SIX questions. THREE questions in Section A and THREE questions in Section B.

[Kertas soalan ini mengandungi ENAM soalan. TIGA soalan dalam Bahagian A dan TIGA soalan dalam Bahagian B].

Instructions: Answer FIVE questions.

[Arahan: Jawab LIMA soalan].

Use two answer booklets which is provided where the answer for questions in Section A are in one answer booklet and for Section B in another answer booklet. Answer to any question must start on a new page.

[Gunakan dua buku jawapan yang berasingan supaya jawapan-jawapan bagi soalan Bahagian A adalah dalam satu buku jawapan dan jawapan bagi Bahagian B dalam buku jawapan yang lain. Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru].

All questions must be answered in English. However, ONE question can be answered in bahasa Malaysia.

[Jawab semua soalan dalam bahasa Inggeris. Walau bagaimanapun, SATU soalan dibenarkan dijawab dalam bahasa Malaysia].

PART A

BAHAGIAN A

1. (a) The sum of continuous periodic signals is not necessarily periodic. However, the sum of discrete periodic signals is periodic.
Hasil tambah isyarat berterusan berkala tidak semestinya berkala. Bagaimanapun, hasil tambah isyarat diskret berkala adalah berkala.

Justify this statement. Under what condition is the sum of continuous periodic signals periodic?

Tentukan kenyataan ini. Dalam keadaan bagaimana hasil tambah isyarat berterusan berkala adalah berkala?

Is the following signal periodic? Justify? If so, find its fundamental frequency and period.

Adakah isyarat di bawah berkala? Jelaskan? Jika ya, carikan frekuensi asas dan kalanya.

$$x(t) = 2 \cos(15t + 30^\circ) + 4 \sin(20t + 60^\circ)$$

(30%)

- (b) Define the unit step and unit ramp functions. Describe the relationship between them.

Definisikan fungsi langkah unit dan fungsi rampa unit. Perihalkan hubungan antaranya.

(30%)

- (c) Write an expression for the signals shown in Figure 1 in terms of the unit step function only.

Tuliskan persamaan bagi isyarat yang ditunjukkan dalam Rajah 1 dalam sebutan fungsi langkah unit sahaja.

...3/-

and sketch
dan lakarkan

$$z(t) = \int_{-\infty}^{\infty} y(t)dt$$

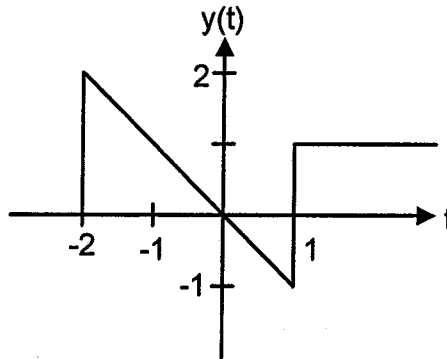


Figure 1
Rajah 1

(40%)

2. (a) What is meant by linear interpolation. Explain fully.

Apakah yang dimaksudkan dengan interpolasi lurus. Jelaskan sepenuhnya.

(5%)

- (b) A linear time invariant:-

Suatu sistem lurus masa tak varian dijelaskan oleh:-

$$y''(t) + 4y'(t) = x''(t) + x'(t) + x(t)$$

Draw the first canonical form simulation diagram for the system.

Lukiskan rajah simulasi bentuk berkanun pertama bagi sistem tersebut.

(20%)

- (d) Find and sketch the convolution sum $z[k] = c[k] * p[k]$ of the signals shown in Figure 3.

Cari dan lakar hasil konvolusi $z[k] = c[k] * p[k]$ bagi isyarat-isyarat ditunjukkan dalam Rajah 3.

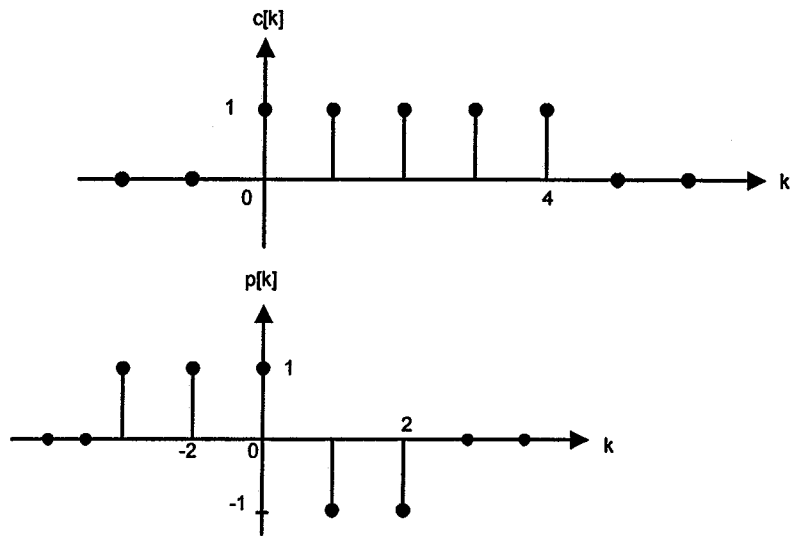


Figure 3
Rajah 3

(40%)

3. Consider the system block diagram as shown in Figure 4.
 Pertimbangkan rajah blok sistem ditunjukkan dalam Rajah 4.

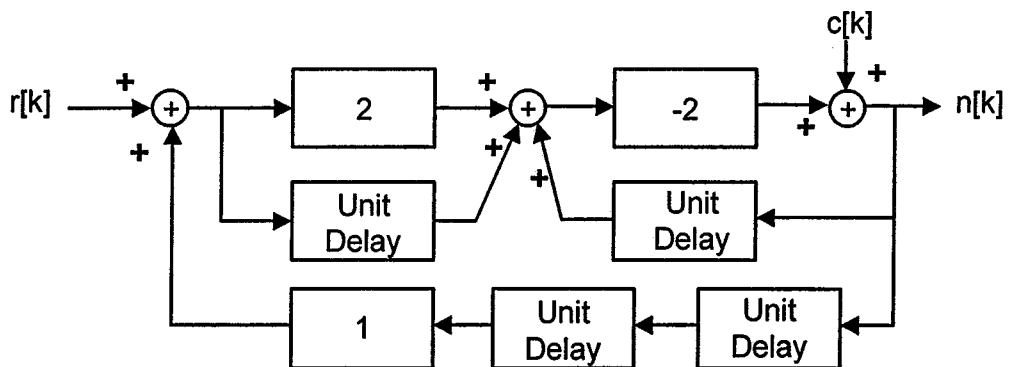


Figure 4
Rajah 4

...6/-

- (a) Describe the system using a suitable difference equation.

Jelaskan sistem tersebut menggunakan persamaan kebezaan yang sesuai.

(15%)

- (b) Determine the impulse response, $h[k]$, of the system for $0 \leq k \leq 4$.

Tentukan sambutan dedenyut, $h[k]$ bagi sistem tersebut untuk $0 \leq k \leq 4$.

(30%)

- (c) Express $y[k]$ in terms of $h[k]$ when:-

Nyatakan $y[k]$ dalam sebutan $h[k]$ apabila:-

$$x_1[k] = \begin{cases} 3 & k = 0 \\ 1 & k = 1 \\ -2 & k = 2 \\ 0 & \text{elsewhere} \end{cases}$$

and

dan

$$x_2[k] = \begin{cases} -1 & k = 0 \\ 2 & k = 1 \\ 0 & k = 2 \\ 0 & \text{elsewhere} \end{cases}$$

(40%)

- (d) Determine the values of $y[k]$ for $2 \leq k \leq 4$.

Tentukan nilai-nilai bagi $y[k]$ $2 \leq k \leq 4$.

(15%)

...7/-

PART B**BAHAGIAN B**

4. (a) Find the Fourier transforms of
Carikan jelmaan Fourier untuk

(i) $e^{-at}u(t)$

(ii) $f(t) = \text{rect}\left(\frac{t}{\tau}\right)$

(40%)

- (b) Proof the following Fourier transform properties :
Buktikan ciri-ciri jelmaan Fourier berikut:

(i) Scaling, $f(t) \Leftrightarrow \frac{1}{|a|} F\left(\frac{w}{a}\right)$
Skalaan

(ii) Time-Shifting, $f(t-t_0) \Leftrightarrow F(w)e^{-jw t_0}$
Anjakan-masa

(40%)

- (c) Sketch $\varphi_{AM}(t)$ for modulation indices of $\mu = 0.5$ (50% modulation) and $\mu = 1$ (100% modulation), when

Lakarkan modulasi $\varphi_{AM}(t)$ untuk $\mu = 0.5$ (modulasi 50%) and $\mu = 1$ (modulasi 100%), bila

$$m(t) = B\cos\omega_m t.$$

(20%)

...8/-

5. (a) A sampler with sampling interval $T = 0.001$ second samples continuous-time sinusoids of the following frequencies:

Satu pensampel dengan selang pensampelan $T = 0.001$ saat mensampel isyarat sinus yang berterusan dengan masa untuk frekuensi-frekuensi berikut:

- (i) 400 Hz
- (ii) 1 KHz
- (iii) 1.4 KHz
- (iv) 1.6 KHz
- (v) 3.522 KHz

Determine the aliased frequencies of the resulting sampled signals.

Tentukan frekuensi aliasnya pada isyarat yang telah disampelkan.

(30%)

- (b) For a system specified by the equation.

Untuk sesuatu sistem yang telah ditentukan oleh persamaan.

$$(E^2 - 3E + 2)y[k] = (E + 2)f[k]$$

Find the forced response for the input $f[k] = (3)^k u[k]$.

Carikan respon pada masukkan $f[k] = (3)^k u[k]$.

(20%)

- (c) Using the Discrete-Time Fourier Transform (DTFT) method, find the zero-state response $y[k]$ of a causal system with frequency response.

Dengan menggunakan cara masa diskret jelmaan fourier (DTFT), carikan respon keadaan sifar $y[k]$ untuk 'causal system' dengan frekuensi responnya.

$$H(\Omega) = \frac{e^{j\Omega} + 0.32}{e^{j2\Omega} + e^{j\Omega} + e^{j\Omega} 0.16}$$

...9/-

A Short Table of Fourier Transforms

$f(t)$	$F(\omega)$	
1 $e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2 $e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3 $e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4 $te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5 $t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6 $\delta(t)$	1	
7 1	$2\pi\delta(\omega)$	
8 $e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9 $\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10 $\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11 $u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12 $\text{sgn } t$	$\frac{2}{j\omega}$	
13 $\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14 $\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15 $e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16 $e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17 $\text{rect}_t(\frac{t}{\tau})$	$\tau \text{sinc}(\frac{\omega\tau}{2})$	
18 $\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}(\frac{\omega}{2W})$	
19 $\Delta(\frac{t}{\tau})$	$\frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	
20 $\frac{W}{2\pi} \text{sinc}^2(\frac{Wt}{2})$	$\Delta(\frac{\omega}{2W})$	
21 $\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22 $e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^t f(x) dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterminate form $0/0$ or ∞/∞ , then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \quad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots \quad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots + \binom{n}{k} x^k + \dots + x^n$$

$$\approx 1 + nx \quad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^k r^m = \frac{r^{k+1} - 1}{r - 1} \quad r \neq 1$$

$$\sum_{m=M}^N r^m = \frac{r^{N+1} - r^M}{r - 1} \quad r \neq 1$$

$$\sum_{m=0}^k \left(\frac{a}{b}\right)^m = \frac{a^{k+1} - b^{k+1}}{b^k(a-b)} \quad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

$$a + jb = re^{j\theta} \quad r = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \frac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \frac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos \left(x \pm \frac{\pi}{2} \right) = \mp \sin x$$

$$\sin \left(x \pm \frac{\pi}{2} \right) = \pm \cos x$$

$$2 \sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3 \cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a \cos x + b \sin x = C \cos(x + \theta)$$

$$\text{in which } C = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1} \left(\frac{-b}{a} \right)$$

B.7-7 Indefinite Integrals

$$\int u dv = uv - \int v du$$

$$\int f(x)g(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax \quad \int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \quad \int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x \sin ax dx = \frac{1}{a^2} (\sin ax - ax \cos ax)$$

$$\int x \cos ax dx = \frac{1}{a^2} (\cos ax + ax \sin ax)$$

$$\int x^2 \sin ax dx = \frac{1}{a^3} (2ax \sin ax + 2 \cos ax - a^2 x^2 \cos ax)$$

$$\int x^2 \cos ax dx = \frac{1}{a^3} (2ax \cos ax - 2 \sin ax + a^2 x^2 \sin ax)$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \quad a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = - \left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)} \right] \quad a^2 \neq b^2$$

$$\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \quad a^2 \neq b^2$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3} (a^2 x^2 - 2ax + 2)$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2} dx = \frac{1}{2} \ln(x^2 + a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx} f(u) = \frac{d}{du} f(u) \frac{du}{dx}$	$\frac{d}{dx} a^{bx} = b(\ln a) a^{bx}$
$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	$\frac{d}{dx} \sin ax = a \cos ax$
$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	$\frac{d}{dx} \cos ax = -a \sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx} \tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx} \ln(ax) = \frac{1}{x}$	$\frac{d}{dx} (\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx} \log(ax) = \frac{\log e}{x}$	$\frac{d}{dx} (\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx} e^{bx} = be^{bx}$	$\frac{d}{dx} (\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$$\pi \approx 3.1415926535$$

$$e \approx 2.7182818284$$

$$\frac{1}{e} \approx 0.3678794411$$

$$\log_{10} 2 = 0.30103$$

$$\log_{10} 3 = 0.47712$$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TABLE 2.1: Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t - T)$	$f(t - T)$
2	$e^{\lambda t} u(t)$	$u(t)$	$\frac{1 - e^{\lambda t}}{-\lambda} u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2} u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$te^{\lambda t} u(t)$
6	$te^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$\frac{1}{2} t^2 e^{\lambda t} u(t)$
7	$t^n u(t)$	$e^{\lambda t} u(t)$	$\frac{n! e^{\lambda t}}{\lambda^{n+1}} u(t) - \sum_{j=0}^n \frac{n! t^{n-j}}{\lambda^{j+1} (n-j)!} u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m! n!}{(m+n+1)!} t^{m+n+1} u(t)$
9	$te^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_2 t} - e^{\lambda_1 t} + (\lambda_1 - \lambda_2) t e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)^2} u(t)$
10	$t^m e^{\lambda t} u(t)$	$t^n e^{\lambda t} u(t)$	$\frac{m! n!}{(n+m+1)!} t^{m+n+1} e^{\lambda t} u(t)$
11	$t^m e^{\lambda_1 t} u(t)$	$t^n e^{\lambda_2 t} u(t)$	$\sum_{j=0}^m \frac{(-1)^j m! (n+j)! t^{m-j} e^{\lambda_1 t}}{j! (m-j)! (\lambda_1 - \lambda_2)^{n+j+1}} u(t)$ $\lambda_1 \neq \lambda_2$ $+ \sum_{k=0}^n \frac{(-1)^k n! (m+k)! t^{n-k} e^{\lambda_2 t}}{k! (n-k)! (\lambda_2 - \lambda_1)^{m+k+1}} u(t)$
12	$e^{-\alpha t} \cos(\beta t + \theta) u(t)$	$e^{\lambda t} u(t)$	$\frac{\cos(\theta - \phi) e^{\lambda t} - e^{-\alpha t} \cos(\beta t + \theta - \phi)}{\sqrt{(\alpha + \lambda)^2 + \beta^2}} u(t)$ $\phi = \tan^{-1}[-\beta / (\alpha + \lambda)]$
13	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} u(t) + e^{\lambda_2 t} u(-t)}{\lambda_2 - \lambda_1} \quad \text{Re } \lambda_2 > \text{Re } \lambda_1$
14	$e^{\lambda_1 t} u(-t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_2 - \lambda_1} u(-t)$

TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$
1	$\delta[k-j]$	$f[k]$	$f[k-j]$
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma} \right] u[k]$
3	$u[k]$	$u[k]$	$(k+1)u[k]$
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2} \right] u[k] \quad \gamma_1 \neq \gamma_2$
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2 - \gamma_1} \gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2 - \gamma_1} \gamma_2^k u[-(k+1)] \quad \gamma_2 > \gamma_1 $
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1 \gamma_2}{(\gamma_1 - \gamma_2)^2} \left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1 - \gamma_2}{\gamma_2} k\gamma_1^k \right] u[k] \quad \gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6} k(k-1)(k+1)u[k]$
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k - 1) + k(1-\gamma)}{(1-\gamma)^2} \right] u[k]$
10	$ \gamma_1 ^k \cos(\beta k + \theta) u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R} \left[\gamma_1 ^{k+1} \cos[\beta(k+1) + \theta - \phi] - \gamma_2^{k+1} \cos(\theta - \phi) \right] u[k] \quad \gamma_2 \text{ real}$ $R = [\gamma_1 ^2 + \gamma_2^2 - 2 \gamma_1 \gamma_2 \cos \beta]^{1/2}$ $\phi = \tan^{-1} \left[\frac{(\gamma_1 \sin \beta)}{(\gamma_1 \cos \beta - \gamma_2)} \right]$

Table 11.1: (Unilateral) z-Transform Pairs

$f[k]$	$F[z]$
1 $\delta[k - j]$	z^{-j}
2 $u[k]$	$\frac{z}{z - 1}$
3 $ku[k]$	$\frac{z}{(z - 1)^2}$
4 $k^2u[k]$	$\frac{z(z + 1)}{(z - 1)^3}$
5 $k^3u[k]$	$\frac{z(x^2 + 4z + 1)}{(z - 1)^4}$
6 $\gamma^{k-1}u[k - 1]$	$\frac{1}{z - \gamma}$
7 $\gamma^k u[k]$	$\frac{z}{z - \gamma}$
8 $k\gamma^k u[k]$	$\frac{\gamma z}{(z - \gamma)^2}$
9 $k^2\gamma^k u[k]$	$\frac{\gamma z(z + \gamma)}{(z - \gamma)^3}$
10 $\frac{k(k - 1)(k - 2) \cdots (k - m + 1)}{\gamma^{m-1} m!} \gamma^k u[k]$	$\frac{z}{(z - \gamma)^{m+1}}$
11a $ \gamma ^k \cos \beta k u[k]$	$\frac{z(z - \gamma \cos \beta)}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
11b $ \gamma ^k \sin \beta k u[k]$	$\frac{z \gamma \sin \beta}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12a $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{rz[z \cos \theta - \gamma \cos(\beta - \theta)]}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12b $r \gamma ^k \cos(\beta k + \theta)u[k]$ $\gamma = \gamma e^{j\theta}$	$\frac{(0.5re^{j\theta})z}{z - \gamma} + \frac{(0.5re^{-j\theta})z}{z - \gamma^*}$
12c $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{z(Az + B)}{z^2 + 2az + \gamma ^2}$
$r = \sqrt{\frac{A^2 \gamma ^2 + B^2 - 2AaB}{ \gamma ^2 - a^2}}$ $\beta = \cos^{-1} \frac{-a}{ \gamma }, \theta = \tan^{-1} \frac{Aa - B}{A\sqrt{ \gamma ^2 - a^2}}$	

Table 11.2
Z- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k - m]u[k - m]$	$\frac{1}{z^m} F[z]$
	$f[k - m]u[k]$	$\frac{1}{z^m} F[z] + \frac{1}{z^m} \sum_{k=1}^m f[-k]z^k$
	$f[k - 1]u[k]$	$\frac{1}{z} F[z] + f[-1]$
	$f[k - 2]u[k]$	$\frac{1}{z^2} F[z] + \frac{1}{z} f[-1] + f[-2]$
	$f[k - 3]u[k]$	$\frac{1}{z^3} F[z] + \frac{1}{z^2} f[-1] + \frac{1}{z} f[-2] + f[-3]$
Left-shift	$f[k + m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k + 1]u[k]$	$zF[z] - zf[0]$
	$f[k + 2]u[k]$	$z^2 F[z] - z^2 f[0] - zf[1]$
	$f[k + 3]u[k]$	$z^3 F[z] - z^3 f[0] - z^2 f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z \frac{d}{dz} F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z] F_2[z]$
Frequency Convolution	$f_1[k] f_2[k]$	$\frac{1}{2\pi j} \oint F_1[u] F_2\left[\frac{z}{u}\right] u^{-1} du$
Initial value	$f[0]$	$\lim_{z \rightarrow \infty} z F[z]$
Final value	$\lim_{N \rightarrow \infty} f[N]$	$\lim_{z \rightarrow 1} (z - 1) F[z]$ poles of $(z - 1) F[z]$ inside the unit circle.