
UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2004/2005

May 2005

EEE 228E – SIGNAL AND SYSTEM [*Isyarat Dan Sistem*]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of SIX pages of printed material and TEN pages APPENDIX before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat bercetak dan SEPULUH mukasurat LAMPIRAN sebelum anda memulakan peperiksaan].

This paper contains SIX questions.

[Kertas soalan ini mengandungi ENAM soalan].

Instructions: Answer **FIVE** questions. If a candidate answer more than five questions, only the first five answered will be examined and awarded marks.

[Arahan: Jawab LIMA soalan. Jika calon menjawab lebih daripada lima soalan hanya lima soalan pertama mengikut susunan dalam skrip jawapan akan diberi markah].

Answer to any question must start on a new page.

[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru].

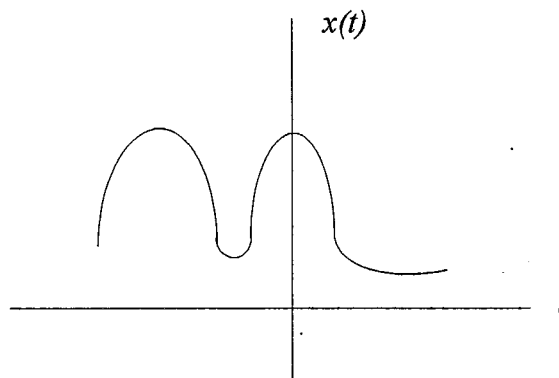
All questions must be answered in bahasa Malaysia. However, ONE question can be answered in English

[Jawab semua soalan dalam bahasa Malaysia. Walau bagaimanapun, SATU soalan dibenarkan dijawab dalam bahasa Inggeris].

1. (a) Suatu isyarat masa-selanjara $x(t)$ ditunjukkan dalam Rajah 1. Lakarkan $x(t-t_0)$ untuk $t_0 < 0$. Operasi ini melibatkan suatu transformasi asas isyarat pada paksi masa. Apakah operasi ini? Jika t_0 bernilai positif, apakah transformasi isyarat ini? Terangkan dengan penuh.

A continuous-time signal $x(t)$ is shown in Figure 1. Sketch $x(t-t_0)$ for $t_0 < 0$. This operation involves a basic signal transformation of the time axis. What is this operation? If t_0 is positive, what is the signal transformation? Explain fully.

(10%)



Rajah 1
Figure 1

(b) Diberi
$$x(t) = \begin{cases} \cos(t), & \text{jika } t < 0 \\ \sin(t), & \text{jika } t \geq 0 \end{cases}$$

Given
$$x(t) = \begin{cases} \cos(t), & \text{if } t < 0 \\ \sin(t), & \text{if } t \geq 0 \end{cases}$$

Tentukan sama ada isyarat yang diberikan ini berkala. Jelaskan jawapan anda.

Determine whether or not the given signal is periodic. Justify your answer.

(10%)

...3/-

2. (a) Pertimbangkan suatu isyarat eksponen kompleks umum $x(t) = Ce^{rt} \cos(\omega_0 t + \theta)$. Lakarkan dan terangkan isyarat $x(t)$ untuk

Consider a general complex exponential signal $x(t) = Ce^{rt} \cos(\omega_0 t + \theta)$. Sketch and explain the signal $x(t)$ for

[i] $r > 0$

[ii] $r < 0$

(10%)

- (b) Pertimbangkan sistem
Consider the system

$$y[n] = 2x[n] + 3$$

Tentukan sama ada sistem ini lurus. Jelaskan jawapan anda.
Determine if the system is linear. Justify your answer.

(10%)

3. (a) Banyak sistem nyata dibina sebagai saling sambungan beberapa subsistem. Lukiskan:

Many real systems are built as interconnections of several subsystems. Draw:

[i] saling-sambungan suapbalik
Feedback interconnection

[ii] saling-sambungan siri-selari
series-parallel interconnection

(8%)

...4/-

- (b) Andaikan $x[n]$ dan $v[n]$ bersamaan dengan denyut segiempat $p[n]$ yang didefinisikan sebagai

$$p(n) = \begin{cases} 1 & , 0 \leq n \leq 9 \\ 0 & , \text{nilai lain } n \end{cases}$$

Plot $x[n] * v[n]$

Suppose that $x[n]$ and $v[n]$ are equal to the rectangular pulse $p[n]$ defined by

$$p(n) = \begin{cases} 1 & , 0 \leq n \leq 9 \\ 0 & , \text{all other } n \end{cases}$$

Plot $x[n] * v[n]$

(12%)

4. (a) Terangkan dengan lengkap sambutan masukan-kosong dan sambutan keadaan kosong. Berikan satu contoh.

Explain fully the zero-input response and the zero-state response. Give an example.

(10%)

- (b) Cari dan lakarkan jelmaan Fourier untuk $f(t) = \text{rect} \left(\frac{t}{\tau} \right)$ menggunakan definisi asas.

Find and sketch the Fourier transform of $f(t) = \text{rect} \left(\frac{t}{\tau} \right)$ using the basic definition.

(10%)

...5/-

5. (a) Jelmaan Fourier diskret, DFT dalam bentuk segiempat diberikan oleh

The rectangular form of the discrete Fourier Transform DFT is given by

$$X_r = \begin{cases} 6 & , r = 0 \\ -1 - j & , r = 1 \\ 0 & , r = 2 \\ -1 + j & , r = 3 \end{cases}$$

Kira jelmaan Fourier diskret songsang, IDFT

Compute the inverse discrete Fourier transform, IDFT.

(10%)

- (b) Sambutan dedenyut unit suatu sistem LTID ialah $h[n] = [-(0.2)^n + 4(0.8)^n]u[n]$. Cari sambutan $y[n]$ sistem ini jika masukan $f[n] = 4^{-n}u[n]$.

The unit impulse response of an LTID system is $h[n] = [-(0.2)^n + 4(0.8)^n]u[n]$. Find the response $y[n]$ of this system if the input $f[n] = 4^{-n}u[n]$.

(10%)

A Short Table of Fourier Transforms

$f(t)$	$F(\omega)$	
1 $e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2 $e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3 $e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4 $te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5 $t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6 $\delta(t)$	1	
7 1	$2\pi\delta(\omega)$	
8 $e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9 $\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10 $\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11 $u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12 $\text{sgn } t$	$\frac{2}{j\omega}$	
13 $\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14 $\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15 $e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16 $e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17 $\text{rect}(\frac{t}{\tau})$	$\tau \text{sinc}(\frac{\omega\tau}{2})$	
18 $\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}(\frac{\omega}{2W})$	
19 $\Delta(\frac{t}{\tau})$	$\frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	
20 $\frac{W}{2\pi} \text{sinc}^2(\frac{Wt}{2})$	$\Delta(\frac{\omega}{2W})$	
21 $\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22 $e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^t f(x) dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞ , then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots$$

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \quad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots \quad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots + \binom{n}{k} x^k + \dots + x^n$$

$$\approx 1 + nx \quad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^k r^m = \frac{r^{k+1} - 1}{r - 1} \quad r \neq 1$$

$$\sum_{m=M}^N r^m = \frac{r^{N+1} - r^M}{r - 1} \quad r \neq 1$$

$$\sum_{m=0}^k \left(\frac{a}{b}\right)^m = \frac{a^{k+1} - b^{k+1}}{b^k(a-b)} \quad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

$$a + jb = re^{j\theta} \quad r = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \frac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \frac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos \left(x \pm \frac{\pi}{2} \right) = \mp \sin x$$

$$\sin \left(x \pm \frac{\pi}{2} \right) = \pm \cos x$$

$$2 \sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3 \cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a \cos x + b \sin x = C \cos(x + \theta)$$

$$\text{in which } C = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1} \left(\frac{-b}{a} \right)$$

B.7-7 Indefinite Integrals

$$\int u dv = uv - \int v du$$

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax \qquad \int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x \sin ax dx = \frac{1}{a^2} (\sin ax - ax \cos ax)$$

$$\int x \cos ax dx = \frac{1}{a^2} (\cos ax + ax \sin ax)$$

$$\int x^2 \sin ax dx = \frac{1}{a^3} (2ax \sin ax + 2 \cos ax - a^2 x^2 \cos ax)$$

$$\int x^2 \cos ax dx = \frac{1}{a^3} (2ax \cos ax - 2 \sin ax + a^2 x^2 \sin ax)$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = - \left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)} \right] \qquad a^2 \neq b^2$$

$$\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3} (a^2 x^2 - 2ax + 2)$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2} dx = \frac{1}{2} \ln(x^2 + a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx} f(u) = \frac{d}{du} f(u) \frac{du}{dx}$	$\frac{d}{dx} a^{bx} = b(\ln a) a^{bx}$
$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	$\frac{d}{dx} \sin ax = a \cos ax$
$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	$\frac{d}{dx} \cos ax = -a \sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx} \tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx} \ln(ax) = \frac{1}{x}$	$\frac{d}{dx} (\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx} \log(ax) = \frac{\log e}{x}$	$\frac{d}{dx} (\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx} e^{bx} = be^{bx}$	$\frac{d}{dx} (\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$$\pi \approx 3.1415926535$$

$$e \approx 2.7182818284$$

$$\frac{1}{e} \approx 0.3678794411$$

$$\log_{10} 2 = 0.30103$$

$$\log_{10} 3 = 0.47712$$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TABLE 2.1: Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t - T)$	$f(t - T)$
2	$e^{\lambda t} u(t)$	$u(t)$	$\frac{1 - e^{\lambda t}}{-\lambda} u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2} u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$te^{\lambda t} u(t)$
6	$te^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$\frac{1}{2} t^2 e^{\lambda t} u(t)$
7	$t^n u(t)$	$e^{\lambda t} u(t)$	$\frac{n! e^{\lambda t}}{\lambda^{n+1}} u(t) - \sum_{j=0}^n \frac{n! t^{n-j}}{\lambda^{j+1} (n-j)!} u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m! n!}{(m+n+1)!} t^{m+n+1} u(t)$
9	$te^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_2 t} - e^{\lambda_1 t} + (\lambda_1 - \lambda_2) te^{\lambda_1 t}}{(\lambda_1 - \lambda_2)^2} u(t)$
10	$t^m e^{\lambda t} u(t)$	$t^n e^{\lambda t} u(t)$	$\frac{m! n!}{(n+m+1)!} t^{m+n+1} e^{\lambda t} u(t)$
11	$t^m e^{\lambda_1 t} u(t)$	$t^n e^{\lambda_2 t} u(t)$	$\sum_{j=0}^m \frac{(-1)^j m! (n+j)! t^{m-j} e^{\lambda_1 t}}{j! (m-j)! (\lambda_1 - \lambda_2)^{n+j+1}} u(t)$ $\lambda_1 \neq \lambda_2$ $+ \sum_{k=0}^n \frac{(-1)^k n! (m+k)! t^{n-k} e^{\lambda_2 t}}{k! (n-k)! (\lambda_2 - \lambda_1)^{m+k+1}} u(t)$
12	$e^{-\alpha t} \cos(\beta t + \theta) u(t)$	$e^{\lambda t} u(t)$	$\frac{\cos(\theta - \phi) e^{\lambda t} - e^{-\alpha t} \cos(\beta t + \theta - \phi)}{\sqrt{(\alpha + \lambda)^2 + \beta^2}} u(t)$ $\phi = \tan^{-1}[-\beta/(\alpha + \lambda)]$
13	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} u(t) + e^{\lambda_2 t} u(-t)}{\lambda_2 - \lambda_1} \quad \text{Re } \lambda_2 > \text{Re } \lambda_1$
14	$e^{\lambda_1 t} u(-t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_2 - \lambda_1} u(-t)$

TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$
1	$\delta[k-j]$	$f[k]$	$f[k-j]$
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma} \right] u[k]$
3	$u[k]$	$u[k]$	$(k+1)u[k]$
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2} \right] u[k] \quad \gamma_1 \neq \gamma_2$
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2 - \gamma_1} \gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2 - \gamma_1} \gamma_2^k u[-(k+1)] \quad \gamma_2 > \gamma_1 $
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1 \gamma_2}{(\gamma_1 - \gamma_2)^2} \left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1 - \gamma_2}{\gamma_2} k\gamma_1^k \right] u[k] \quad \gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6} k(k-1)(k+1)u[k]$
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k - 1) + k(1-\gamma)}{(1-\gamma)^2} \right] u[k]$
10	$ \gamma_1 ^k \cos(\beta k + \theta) u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R} \left[\gamma_1 ^{k+1} \cos[\beta(k+1) + \theta - \phi] - \gamma_2^{k+1} \cos(\theta - \phi) \right] u[k] \quad \gamma_2 \text{ real}$ $R = [\gamma_1 ^2 + \gamma_2^2 - 2 \gamma_1 \gamma_2 \cos \beta]^{1/2}$ $\phi = \tan^{-1} \left[\frac{(\gamma_1 \sin \beta)}{(\gamma_1 \cos \beta - \gamma_2)} \right]$

Table 11.1: (Unilateral) z-Transform Pairs

$f[k]$	$F[z]$
1 $\delta[k - j]$	z^{-j}
2 $u[k]$	$\frac{z}{z - 1}$
3 $ku[k]$	$\frac{z}{(z - 1)^2}$
4 $k^2u[k]$	$\frac{z(z + 1)}{(z - 1)^3}$
5 $k^3u[k]$	$\frac{z(z^2 + 4z + 1)}{(z - 1)^4}$
6 $\gamma^{k-1}u[k - 1]$	$\frac{1}{z - \gamma}$
7 $\gamma^k u[k]$	$\frac{z}{z - \gamma}$
8 $k\gamma^k u[k]$	$\frac{\gamma z}{(z - \gamma)^2}$
9 $k^2\gamma^k u[k]$	$\frac{\gamma z(z + \gamma)}{(z - \gamma)^3}$
10 $\frac{k(k - 1)(k - 2) \cdots (k - m + 1)}{\gamma^m m!} \gamma^k u[k]$	$\frac{z}{(z - \gamma)^{m+1}}$
11a $ \gamma ^k \cos \beta k u[k]$	$\frac{z(z - \gamma \cos \beta)}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
11b $ \gamma ^k \sin \beta k u[k]$	$\frac{z \gamma \sin \beta}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12a $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{rz[z \cos \theta - \gamma \cos(\beta - \theta)]}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12b $r \gamma ^k \cos(\beta k + \theta)u[k]$ $\gamma = \gamma e^{j\theta}$	$\frac{(0.5re^{j\theta})z}{z - \gamma} + \frac{(0.5re^{-j\theta})z}{z - \gamma^*}$
12c $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{z(Az + B)}{z^2 + 2az + \gamma ^2}$
$r = \sqrt{\frac{A^2 \gamma ^2 + B^2 - 2AaB}{ \gamma ^2 - a^2}}$ $\beta = \cos^{-1} \frac{-a}{ \gamma }, \quad \theta = \tan^{-1} \frac{Aa - B}{A\sqrt{ \gamma ^2 - a^2}}$	

Table 11.2
Z- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$a f[k]$	$a F[z]$
Right-shift	$f[k - m]u[k - m]$	$\frac{1}{z^m} F[z]$
	$f[k - m]u[k]$	$\frac{1}{z^m} F[z] + \frac{1}{z^m} \sum_{k=1}^m f[-k]z^k$
	$f[k - 1]u[k]$	$\frac{1}{z} F[z] + f[-1]$
	$f[k - 2]u[k]$	$\frac{1}{z^2} F[z] + \frac{1}{z} f[-1] + f[-2]$
	$f[k - 3]u[k]$	$\frac{1}{z^3} F[z] + \frac{1}{z^2} f[-1] + \frac{1}{z} f[-2] + f[-3]$
Left-shift	$f[k + m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k + 1]u[k]$	$z F[z] - z f[0]$
	$f[k + 2]u[k]$	$z^2 F[z] - z^2 f[0] - z f[1]$
	$f[k + 3]u[k]$	$z^3 F[z] - z^3 f[0] - z^2 f[1] - z f[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F \left[\frac{z}{\gamma} \right]$
Multiplication by k	$k f[k]u[k]$	$-z \frac{d}{dz} F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z] F_2[z]$
Frequency Convolution	$f_1[k] f_2[k]$	$\frac{1}{2\pi j} \oint F_1[u] F_2 \left[\frac{z}{u} \right] u^{-1} du$
Initial value	$f[0]$	$\lim_{z \rightarrow \infty} F[z]$
Final value	$\lim_{N \rightarrow \infty} f[N]$	$\lim_{z \rightarrow 1} (z - 1) F[z]$ poles of $(z - 1) F[z]$ inside the unit circle.