UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2004/2005

Oktober 2004

EEE 228E – ISYARAT DAN SISTEM

Masa : 3 Jam

ARAHAN KEPADA CALON:-

Sila pastikan kertas peperiksaan ini mengandungi **SEPULUH (10)** muka surat beserta **Lampiran (10 muka surat)** bercetak dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan.

Agihan markah diberikan di sut sebelah kanan soalan berkenaan.

Semua soalan hendaklah dijawab di dalam **Bahasa Malaysia** atau **Bahasa Inggeris** atau kombinasi **kedua-duanya**.

...2/-

1. (a) Salah satu fungsi terpenting dalam bidang isyarat dan sistem ialah isyarat eksponen e^{st}, yang mana s adalah kompleks secara am, diberikan sebagai

One of the most important functions in the area of signals and systems is the exponential signal e^{st}, where s is complex in general, given by

$$s = \sigma + j\omega$$

Fungsi e^{st}, meliputi satu kelas besar fungsi-fungsi. Bincangkan empat kes khas e^{st} secara penuh.

The function e^{st} encompasses a large class of functions. Discuss the four special cases of e^{st} fully.

(12%)

(b) Suatu sinusoid $e^{\sigma t}$ cosωt boleh diungkapkan sebagai jumlah eksponen e^{st} dan e^{-st}. Lokasikan dalam satah frekuensi kompleks (satah-s) frekuensi-frekuensi sinusoid berikut:

A sinusoid $e^{\sigma t}$ $\cos\omega t$ can be expressed as a sum of exponentials e^{st} and e^{-st}. Locate in the complex frequency plane (s-plane) the frequencies of the following sinusoids:

[i] e^{-2t} cos (5t + θ)

[ii] cos (5t + θ)

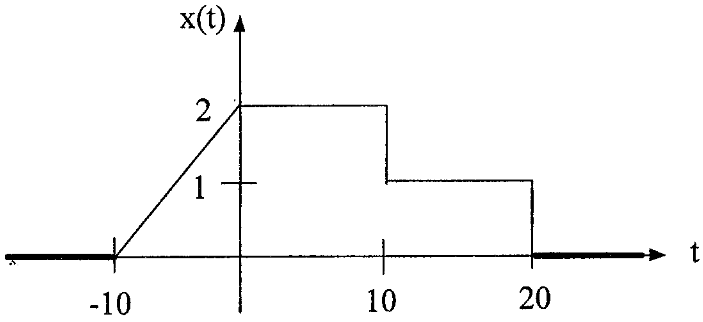
[iii] e^{-8t}

[iv] 8

(8%)

2. (a) Pertimbangkan isyarat yang ditunjukkan dalam Rajah 1.

Consider the signal shown in Figure 1.



Rajah 1
Figure 1

(b) Plot

[i] $x(3+t)$

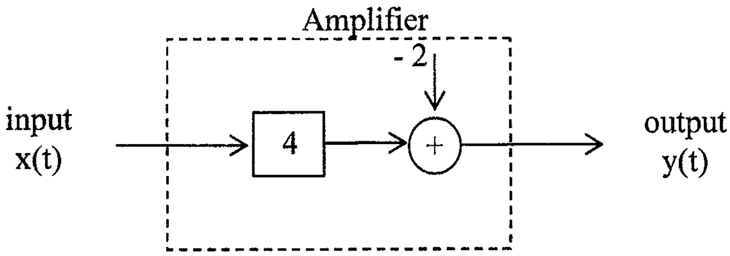
[ii] $x\left(-t/3\right)$

[iii] $x(2-t)$ (10%)

(b) Katakan isyarat x(t) dalam Rajah 1 dikenakan kepada suatu penguat yang mempunyai gandaan 4 dan memperkenalkan pincangan (nilai arus terus) -2 seperti yang ditunjukkan dalam Rajah 2. Cari dan plot isyarat keluaran penguat, y(t).

Suppose that the signal $x(t)$ of Figure 1 is applied to an amplifier that has a gain of 4 and introduces a bias (dc value) of -2 as shown in Figure 2. Find and plot the amplifier output signal, y(t).

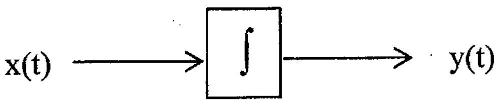
(10%)



Rajah 2
Figure 2

3. (a) Pertimbangkan pengkamir dalam Rajah 3. Sistem ini mempunyai sambutan dedenyut h(t) = u(t). Menggunakan pengkamiran konvolusi, cari sambutan sistem kepada masukan, $x(t) = e^{5t}u(t)$. Lakarkan isyarat masukan, $x(t)$.

Consider the integrator in Figure 3. This system has the impulse response h(t) = u(t). Using the convolution integral, find the system response to the input, $x(t) = e^{5t}u(t)$. *Sketch the input signal,* $x(t)$.



Rajah 3
Figure 3

(6%)

(b) Diberi
Given

$$x(t) = 2\left[u(t) - u(t-2)\right]$$

[i] Ungkapkan $x(t)$ dalam bentuk fungsi segiempat.

Express $x(t)$ in the form of the rectangular function.

(3%)

[ii] Guna jadual jelmaan Fourier dan jadual operasi dalam Lampiran A, cari jelmaan Fourier untuk $x(t)$.

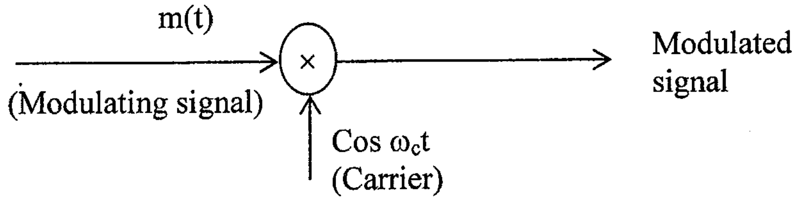
Use the table of Fourier transforms and table of operations in Appendix A, find the Fourier transform of $x(t)$.

(6%)

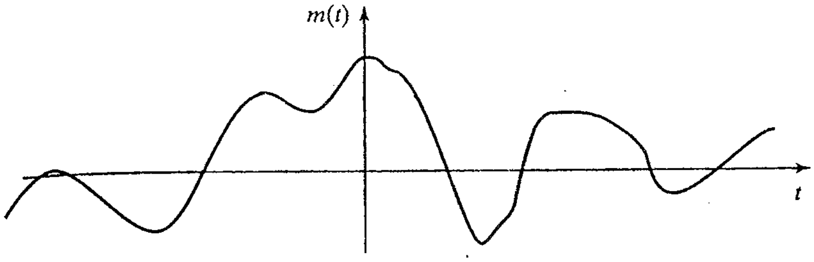
(c) DSB/SC – AM dicapai dengan mendarabkan isyarat maklumat (mesej), m(t) dengan suatu isyarat sinus dipanggil isyarat pembawa cos $\omega_c t$, yang berada pada frekuensi yang dikehendaki untuk penghantaran radio secara berkesan. Proses ini ditunjukkan dalam Rajah 4(a). Untuk isyarat mesej yang ditunjukkan dalam Rajah 4(b), lakarkan isyarat termodulat.

DSB/SC – AM is accomplished by multiplying the information (message) signal, m(t) by a sinusoidal signal called the carrier signal cos $\omega_c t$, which is at the desired frequency for efficient radio transmission. This process is illustrated in Figure 4(a). For the message signal shown in Figure 4(b), sketch the modulated signal.

(5%)



Rajah 4(a)
Figure 4(a)



Rajah 4(b)
Figure 4(b)

4. (a) Tentukan kekausalan untuk sistem-sistem dengan sambutan dedenyut berikut:

Determine the causality for the systems with the following impulse responses.

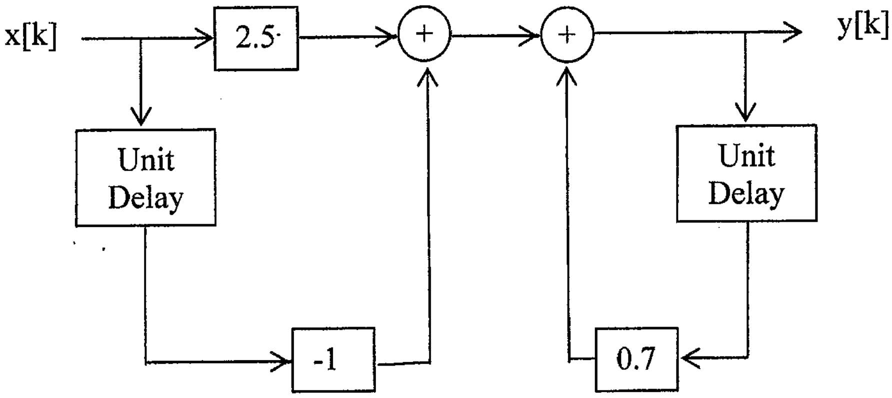
[i] $h[k] = k\,e^{-k}\;u[k]$

[ii] $h[k] = e^{-k}\;u[-k]$

(4%)

(b) Pertimbangkan perwakilan gambarajah blok sistem dalam Rajah 5.

Consider the system block diagram representation of Figure 5.



Rajah 5
Figure 5

[i] Cari persamaan bezaan sistem ini.

Find the difference equation of the system.

(4%)

[ii] Tentukan sambutan dedenyut h[k], $0 \le k \le 4$ untuk sistem ini.

Determine the impulse response h[k], $0 \le k \le 4$ for the system.

(5%)

[iii] Katakan masukan sistem ini diberikan oleh

Suppose that the system input is given by

$$x[k] = \begin{cases} 1 & , \quad n = -2 \\ -3 & , \quad n = 0 \\ 2 & , \quad n = 1 \end{cases}$$

dan x[k] ialah kosong untuk semua nilai k yang lain. Ungkapkan keluaran y[k] sebagai fungsi h[k].

and x[k] is zero for all other values of k. Express the output y[k] as a function of h[k].

(7%)

5. (a) Pertimbangkan suatu sistem dengan masukan $x[k]$ dan keluaran y[k] yang dispesifikasikan oleh persamaan.

Consider the system with an input x[k] and output y[k] specified by the equation.

$$y[k] = \sin\left(\frac{\pi k}{4}\right) x[k]$$

[i] Adakah sistem ini lelurus? Jelaskan jawapan anda.

Is this system linear? Justify your answer.

[ii] Adakah sistem ini tak-ubah masa? Jelaskan jawapan anda.

Is this system time invariant? Justify your answer.

(10%)

(b) Voltan pada nod ke-k suatu tangga berintangan dalam Rajah 6 ialah v[k] (k = 0,1,2,...,N). Tunjukkan bahawa v[k] memenuhi persamaan bezaan tertib- dua ini.

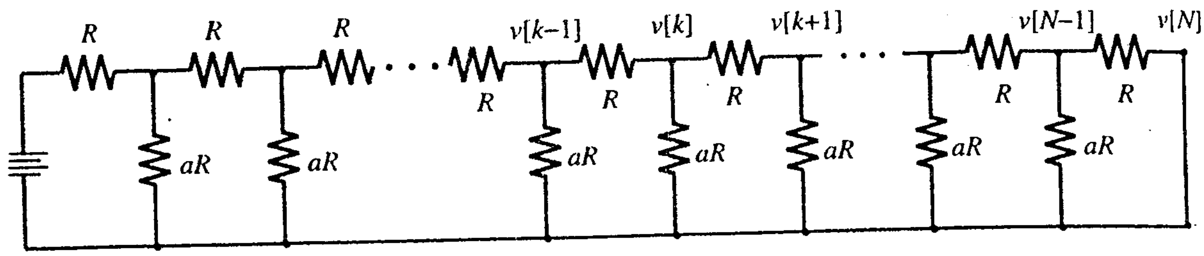
The voltage at the kth node of a resistive ladder in Figure 6 is v[k]. (k = 0,1,2,...,N). Show that v[k] satisfies the second-order difference equation.

$$v\,[k+2] - A\, v\,[k+1] + v\,[k] = 0\ ,\ A = 2 + \frac{1}{a}$$

Panduan : Pertimbangkan persamaan nod pada nod ke–k dengan voltan v [k].

Hint : Consider the node equation at the kth node with voltage v[k].

(10%)



Rajah 6
Figure 6

6. (a) Diberi jelmaan-z unilateral berikut:

Given the following unilateral z-transform:

$$X[z] = \frac{0.4\, z^2}{(z-1)\,(z-0.6)}$$

[i] Cari jelmaan-z songsang dengan kembangan pecahan separa.

Find the inverse z-transform by partial-fraction expansions.

(5%)

[ii] Tentukan $x[k]$ untuk tiga nilai bukan kosong yang pertama dengan kembangkan X[z] ke dalam suatu siri kuasa menggunakan pembahagian panjang.

Determine $x[k]$ for the first three nonzero values by expanding $X[z]$ into a power series using long division.

(5%)

(b) Pertimbangkan suatu sistem dengan masukan $x[\mathrm{k}]$ yang dihuraikan oleh persamaan bezaan berikut.

Consider a system with input $x[\mathrm{k}]$ described by the following difference equation.

$$y[k] - 1.5\, y[k-1] + 0.5\, y[k-2] = x[k]$$

$$x[k] = \begin{cases} 1 & , k = 1 \\ 0 & , otherwise \end{cases}$$

Andaikan semua keadaan awal bernilai kosong.

Assume that all initial conditions are zero.

[i] Cari fungsi pindah sistem.

Find the system transfer function.

(3%)

[ii] Cari y[k] menggunakan jelmaan-z.

Find y[k], using the z-transform.

(3%)

[iii] Guna ciri nilai terakhir untuk mengira y[∞]. Adakah ciri nilai terakhir ini memberi nilai y[∞] yang betul?

Use the final-value property to evaluate y[∞]. Is the final-value property give the correct value of y[∞]?

(4%)

- oooOooo -

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\displaystyle\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2+b^2}, \qquad \theta = \tan^{-1}\left(\tfrac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \tfrac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \tfrac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x-y) + \sin(x+y)]$$

$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\dfrac{-b}{a}\right)$

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x \sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x \cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2 \sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2 \cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax \cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$	$\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{d}{dx}\sin ax = a\cos ax$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}\cos ax = -a\sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx}\ln(ax) = \frac{1}{x}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}\log(ax) = \frac{\log e}{x}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}e^{bx} = be^{bx}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any **quadratic** equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TABLE 2.1: Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\frac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=o}^{n}\frac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\frac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\frac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\frac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right] u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right] u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\|\gamma_2\| > \|\gamma_1\|$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right] u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right] u[k]$	
10	$\|\gamma_1\|^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R}\left[\|\gamma_1\|^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right] u[k]$	γ_2 real
			$R = \left[\|\gamma_1\|^2 + \gamma_2^2 - 2\|\gamma_1\|\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\frac{(\|\gamma_1\|\sin\beta)}{(\|\gamma_1\|\cos\beta - \gamma_2)}\right]$	

Table 11.1: (Unilateral) z-Transform Pairs

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\dfrac{z}{z-1}$
3	$ku[k]$	$\dfrac{z}{(z-1)^2}$
4	$k^2u[k]$	$\dfrac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\dfrac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\dfrac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\dfrac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\dfrac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\dfrac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos\beta k\, u[k]$	$\dfrac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin\beta k\, u[k]$	$\dfrac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$	$\dfrac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k] \qquad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\dfrac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

Table 11.2

$\mathcal{Z}$- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2F[z] - z^2f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.