
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

EEE 228 – SIGNAL AND SYSTEM
[Isyarat Dan Sistem]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material and SEVEN pages APPENDIX before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat bercetak dan TUJUH muka surat LAMPIRAN sebelum anda memulakan peperiksaan ini].

This paper contains SIX questions.

[Kertas soalan ini mengandungi ENAM soalan].

Instructions: Answer **FIVE** (5) questions. If a candidate answer more than five questions, only the first five answered will be examined and awarded marks.

Arahan: Jawab **LIMA** soalan. Jika calon menjawab lebih daripada lima soalan hanya lima soalan pertama mengikut susunan dalam skrip jawapan akan diberi markah].

Answer to any question must start on a new page.

[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru].

You may answer a question either in bahasa Malaysia or in English.

[Anda dibenarkan menjawab soalan sama ada dalam bahasa Malaysia atau bahasa Inggeris].

1. Terangkan secara ringkas:-

Briefly explain:-

- (a) Interpolasi lurus.
Linear interpolation.
- (b) Teori pensampelan.
Sampling Theorem.
- (c) Pembahagian frekuensi multiplex.
Frequency Division Multiplexing.
- (d) Pengesanan segerak atau pengesanan jelas.
Synchronous or coherent detection.

(100%)

2. (a) Berikan ungkapan matematik bagi isyarat yang ditunjukkan dalam Rajah 2.

Give the mathematical expression for the signal shown in Figure 2.

(25%)

(b) Tuliskan persamaan bagi isyarat ini dalam sebutan fungsi langkah unit sahaja.

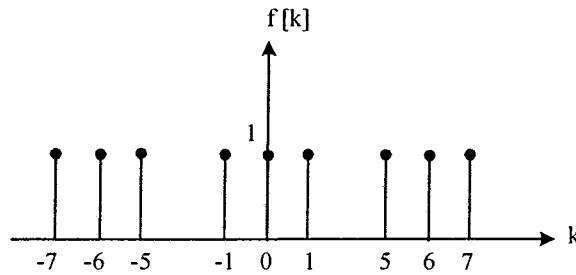
Write an expression for this signal in terms of the unit step function only.

(25%)

...3/-

4. (a) Carikan Siri Fourier Diskret Masa bagi isyarat di bawah.

Find the Discrete-Time Fourier Series (DTFS) of the following signal.

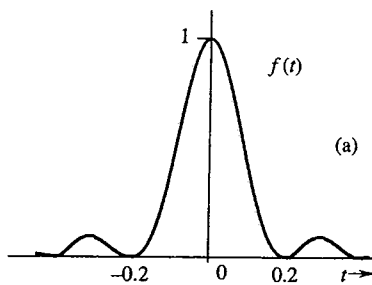


Rajah 4(a)
Figure 4(a)

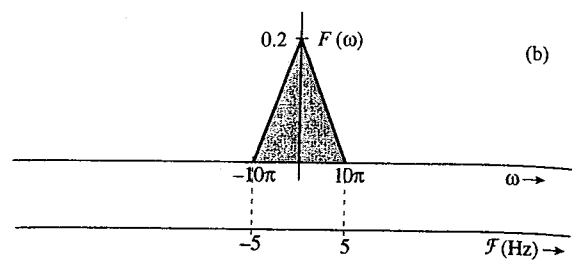
(60%)

- (b) Satu isyarat ditunjukkan pada Rajah 4(b). Spektranya ditunjukkan pada Rajah 4(c).

A signal $f(t) = \text{sinc}^2(5\pi t)$ is as shown in Figure 4(b). Its corresponding spectrum, $F(\omega) = 0.2\Delta\left(\frac{\omega}{20\pi}\right)$ is shown in Figure 4(c).



Rajah 4(b)
Figure 4(b)



Rajah 4(c)
Figure 4(c)

- (i) Apakah f_N , kadar Nyquist dan t_N , julat Nyquist bagi $f(t)$?

What is f_N , the Nyquist rate and t_N , the Nyquist interval for $f(t)$?

...6/-

- (ii) Lakarkan hasil spektra, $F(\bar{\omega})$ jika $f(t)$ disampel pada kadar Nyquist tersebut. Bagaimana kita boleh mendapatkan semula $F(\omega)$ daripada $F(\bar{\omega})$?

Sketch the resulting spectrum, $F(\bar{\omega})$ if $f(t)$ is sampled at the Nyquist rate.

How can we recover $F(\omega)$ from $F(\bar{\omega})$?

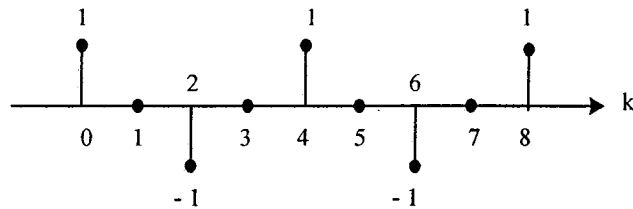
- (iii) Jika $f(t)$ disampel pada $f_s < f_N$, lakarkan hasil spektranya.

If $f(t)$ is sampled at $f_s < f_N$, sketch the resulting spectrum.

(40%)

5. (a) Kirakan DFT dan IDFT empat-titik bagi gelombang ditunjukkan di Rajah 5.

Compute the four-point DFT and IDFT for the waveform shown in Figure 5.



Rajah 5
Figure 5

$$F_r = \sum_{k=0}^{N_0-1} f[k] e^{-j\Omega_0rk}, \quad \Omega_0 = \frac{2\pi}{N_0}$$

$$f[k] = \frac{1}{N_0} \sum_{r=0}^{N_0-1} F_r e^{j\Omega_0rk}$$

(40%)

- (b) Seseorang menerima pinjaman kenderaan sebanyak RM10000 dari sebuah bank pada kadar bunga 1.5% sebulan. Bayaran balik bulannya ialah RM500 dengan bayaran pertamanya sebulan selepas dia menerima pinjaman tersebut.

A person receives an automobile loan of RM10000 from a bank at the interest rate of 1.5% per month. His monthly payment is RM500, with the first payment due one month after he receives the loan.

- (i) Carikan persamaan mengaitkan keluaran (baki pinjaman) dengan masukan (bayaran balik bulanan).

Find the equation relating the output (the balance of the loan) to the input (the monthly payment).

- (ii) Lakarkan perkakasan bagi sistem tersebut.

Sketch the hardware realization for the system.

- (iii) Kirakan bilangan bayaran untuk melangsaikan pinjaman tersebut.

Compute the number of payments required to pay off the loan.

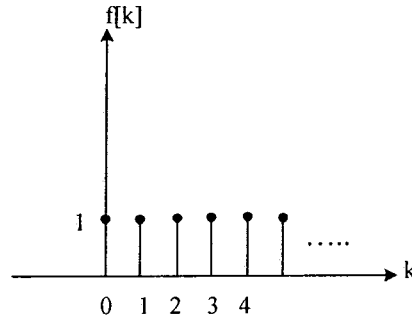
- (iv) Apakah jumlah bayaran terakhir? Nota: Bayaran balik terakhir tidak semestinya RM500.

What is the amount of the last payment? Note: The last payment may not be exactly RM500.

(60%)

6. (a) Carikan jelmaan z bagi isyarat ditunjukkan dalam Rajah 6.

Find the z-transform of the signal shown in Figure 6.



Rajah 6
Figure 6

(30%)

- (b) Selesaikan persamaan kebezaan lurus berikut:-

Solve the following linear difference equation:-

$$y[k + 2] - \frac{5}{6}y[k + 1] + \frac{1}{6}y[k] = 5f[k + 1] - f[k]$$

Diberi keadaan awalan,

Given the initial conditions,

$$y[-1] = 2 ; y[-2] = 0$$

Nyatakan sambutan masukan kosong dan keadaan kosong.

State the zero-input and zero-state responses.

(40%)

- (c) Carikan tiga sebutan pertama bagi f[k] jika

Find the first 3 terms of f[k] if

$$F[z] = \frac{2z^3 + 13z^2 + z}{z^3 + 7z^2 + 2z + 1}$$

(30%)

A Short Table of Fourier Transforms

$f(t)$	$F(\omega)$	
1 $e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2 $e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3 $e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4 $te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5 $t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6 $\delta(t)$	1	
7 1	$2\pi\delta(\omega)$	
8 $e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9 $\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10 $\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11 $u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12 $\text{sgn } t$	$\frac{2}{j\omega}$	
13 $\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14 $\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15 $e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16 $e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17 $\text{rect}(\frac{t}{\tau})$	$\tau \text{sinc}(\frac{\omega\tau}{2})$	
18 $\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}(\frac{\omega}{2W})$	
19 $\Delta(\frac{t}{\tau})$	$\frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	
20 $\frac{W}{2\pi} \text{sinc}^2(\frac{Wt}{2})$	$\Delta(\frac{\omega}{2W})$	
21 $\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22 $e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^t f(x) dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t - T)$	$f(t - T)$
2	$e^{\lambda t} u(t)$	$u(t)$	$\frac{1 - e^{\lambda t}}{-\lambda} u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2} u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$te^{\lambda t} u(t)$
6	$te^{\lambda t} u(t)$	$e^{\lambda t} u(t)$	$\frac{1}{2} t^2 e^{\lambda t} u(t)$
7	$t^n u(t)$	$e^{\lambda t} u(t)$	$\frac{n! e^{\lambda t}}{\lambda^{n+1}} u(t) - \sum_{j=0}^n \frac{n! t^{n-j}}{\lambda^{j+1} (n-j)!} u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m! n!}{(m+n+1)!} t^{m+n+1} u(t)$
9	$te^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(t)$	$\frac{e^{\lambda_2 t} - e^{\lambda_1 t} + (\lambda_1 - \lambda_2) te^{\lambda_1 t}}{(\lambda_1 - \lambda_2)^2} u(t)$
10	$t^m e^{\lambda t} u(t)$	$t^n e^{\lambda t} u(t)$	$\frac{m! n!}{(n+m+1)!} t^{m+n+1} e^{\lambda t} u(t)$
11	$t^m e^{\lambda_1 t} u(t)$	$t^n e^{\lambda_2 t} u(t)$	$\sum_{j=0}^m \frac{(-1)^j m! (n+j)! t^{m-j} e^{\lambda_1 t}}{j! (m-j)! (\lambda_1 - \lambda_2)^{n+j+1}} u(t)$ $\lambda_1 \neq \lambda_2$ $+ \sum_{k=0}^n \frac{(-1)^k n! (m+k)! t^{n-k} e^{\lambda_2 t}}{k! (n-k)! (\lambda_2 - \lambda_1)^{m+k+1}} u(t)$
12	$e^{-\alpha t} \cos(\beta t + \theta) u(t)$	$e^{\lambda t} u(t)$	$\frac{\cos(\theta - \phi) e^{\lambda t} - e^{-\alpha t} \cos(\beta t + \theta - \phi)}{\sqrt{(\alpha + \lambda)^2 + \beta^2}} u(t)$ $\phi = \tan^{-1}[-\beta/(\alpha + \lambda)]$
13	$e^{\lambda_1 t} u(t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} u(t) + e^{\lambda_2 t} u(-t)}{\lambda_2 - \lambda_1} \quad \text{Re } \lambda_2 > \text{Re } \lambda_1$
14	$e^{\lambda_1 t} u(-t)$	$e^{\lambda_2 t} u(-t)$	$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_2 - \lambda_1} u(-t)$

Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$
1	$\delta[k-j]$	$f[k]$	$f[k-j]$
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma} \right] u[k]$
3	$u[k]$	$u[k]$	$(k+1)u[k]$
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2} \right] u[k] \quad \gamma_1 \neq \gamma_2$
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2 - \gamma_1} \gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2 - \gamma_1} \gamma_2^k u[-(k+1)] \quad \gamma_2 > \gamma_1 $
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1 \gamma_2}{(\gamma_1 - \gamma_2)^2} \left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1 - \gamma_2}{\gamma_2} k \gamma_1^k \right] u[k] \quad \gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6} k(k-1)(k+1)u[k]$
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k - 1) + k(1-\gamma)}{(1-\gamma)^2} \right] u[k]$
10	$ \gamma_1 ^k \cos(\beta k + \theta) u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R} \left[\gamma_1 ^{k+1} \cos[\beta(k+1) + \theta - \phi] - \gamma_2^{k+1} \cos(\theta - \phi) \right] u[k] \quad \gamma_2 \text{ real}$ $R = [\gamma_1 ^2 + \gamma_2^2 - 2 \gamma_1 \gamma_2 \cos \beta]^{1/2}$ $\phi = \tan^{-1} \left[\frac{(\gamma_1 \sin \beta)}{(\gamma_1 \cos \beta - \gamma_2)} \right]$

Z- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k - m]u[k - m]$	$\frac{1}{z^m}F[z]$
	$f[k - m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m} \sum_{k=1}^m f[-k]z^k$
	$f[k - 1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k - 2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k - 3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k + m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k + 1]u[k]$	$zF[z] - zf[0]$
	$f[k + 2]u[k]$	$z^2 F[z] - z^2 f[0] - zf[1]$
	$f[k + 3]u[k]$	$z^3 F[z] - z^3 f[0] - z^2 f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z \frac{d}{dz} F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j} \oint F_1[u]F_2\left[\frac{z}{u}\right] u^{-1} du$
Initial value	$f[0]$	$\lim_{z \rightarrow \infty} F[z]$
Final value	$\lim_{N \rightarrow \infty} f[N]$	$\lim_{z \rightarrow 1} (z - 1)F[z]$ poles of $(z - 1)F[z]$ inside the unit circle.

(Unilateral) z -Transform Pairs

$f[k]$	$F[z]$
1 $\delta[k - j]$	z^{-j}
2 $u[k]$	$\frac{z}{z - 1}$
3 $ku[k]$	$\frac{z}{(z - 1)^2}$
4 $k^2u[k]$	$\frac{z(z + 1)}{(z - 1)^3}$
5 $k^3u[k]$	$\frac{z(z^2 + 4z + 1)}{(z - 1)^4}$
6 $\gamma^{k-1}u[k - 1]$	$\frac{1}{z - \gamma}$
7 $\gamma^k u[k]$	$\frac{z}{z - \gamma}$
8 $k\gamma^k u[k]$	$\frac{\gamma z}{(z - \gamma)^2}$
9 $k^2\gamma^k u[k]$	$\frac{\gamma z(z + \gamma)}{(z - \gamma)^3}$
10 $\frac{k(k - 1)(k - 2) \cdots (k - m + 1)}{\gamma^m m!} \gamma^k u[k]$	$\frac{z}{(z - \gamma)^{m+1}}$
11a $ \gamma ^k \cos \beta k u[k]$	$\frac{z(z - \gamma \cos \beta)}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
11b $ \gamma ^k \sin \beta k u[k]$	$\frac{z \gamma \sin \beta}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12a $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{rz[z \cos \theta - \gamma \cos(\beta - \theta)]}{z^2 - (2 \gamma \cos \beta)z + \gamma ^2}$
12b $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\gamma = \gamma e^{j\beta}$ $\frac{(0.5re^{j\theta})z}{z - \gamma} + \frac{(0.5re^{-j\theta})z}{z - \gamma^*}$
12c $r \gamma ^k \cos(\beta k + \theta)u[k]$	$\frac{z(Az + B)}{z^2 + 2az + \gamma ^2}$
$r = \sqrt{\frac{A^2 \gamma ^2 + B^2 - 2AaB}{ \gamma ^2 - a^2}}$ $\beta = \cos^{-1} \frac{a}{ \gamma }, \theta = \tan^{-1} \frac{Aa - B}{A\sqrt{ \gamma ^2 - a^2}}$	

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\int_{0^-}^t f(\tau) d\tau$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^t f(\tau) d\tau$	$\frac{1}{s}F(s) + \frac{1}{s} \int_{-\infty}^{0^-} f(t) dt$
Time shift	$f(t - t_0)u(t - t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0t}$	$F(s - s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(z) dz$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} sF(s) \quad (\text{poles of } sF(s) \text{ in LHP})$