UNIVERSITI SAINS MALAYSIA

First Semester Examination
2015/2016 Academic Session

December 2015 / January 2016

EEE 512 – Advanced Digital Signal and Image Processing

Duration : 3 hours

Please check that this examination paper consists of **EIGHT (8)** pages printed before you begin the examination.

Instructions: This question paper consists of **SIX (6)** questions. Answer **FIVE (5)** questions. All questions carry the same marks.

1. (a) Study the image $f(x, y)$ shown in Figure 1(a)

Figure 1(a)

Without detailed mathematical calculation, explain the effect of filtering Figure 1(a) using (i) first derivatives, $\left[\left(\frac{\partial f}{\partial x}\right), \left(\frac{\partial f}{\partial y}\right)\right]$, and (ii) second derivatives, $\left[\left(\frac{\partial^2 f}{\partial x^2}\right), \left(\frac{\partial^2 f}{\partial y^2}\right)\right]$.

(40 marks)

(b) An $M \times N$ size image f contains only zero pixel value at all locations except at the center as shown in Figure 1(b)(i), while Figure 1(b)(ii) shows the 2×2 mask *h*

$$
f = \begin{bmatrix}
0 & 0 & 0 & . & . & . & . & & & & 0 & 0 & 0 \\
0 & & & & & & & & & & & & 0 \\
0 & & & & & & & & & & & & 0 \\
0 & & & & . & . & . & . & & & & & 0 \\
. & & & . & . & . & . & . & . & & & & . \\
. & & & . & . & 0 & 0 & 0 & . & . & & & . \\
.. & & & . & . & 0 & 1 & 0 & . & . & & & .. \\
 & & & . & . & 0 & 0 & 0 & . & . & & & \\
 & & & & . & . & . & . & . & . & & & \\
 & & & & & . & . & . & & & & & \\
0 & & & & & & & & & & & & 0 \\
0 & & & & & & & & & & & & 0 \\
0 & 0 & 0 & . & . & . & & & & & 0 & 0 & 0
\end{bmatrix}
$$

(i)

$$
h = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

(ii)

Figure 1(b)

(i) Calculate the result of convolution, $(f \otimes h)$.

(15 marks)

(ii) Calculate the result of convolution, $\big((f \otimes h) \otimes h\big)$.

(15 marks)

(iii) From (i) and (ii), explain the result of repeatedly convolving *f* with *h*.

(30 marks)

2. (a) Construct a 4×4 Fourier matrix. Show that this matrix is a unitary matrix.

(40 marks)

(b) Consider a 4×4 image as shown in Figure 2(b)

$$f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2(b)

(i) Perform Discrete Fourier Transform (DFT) of f.

(15 marks)

(ii) Reconstruct f using the first TWO basis images.

(15 marks)

(iii) Repeat (ii) using the expansion of vector outer product.

(30 marks)

<u>Given:</u>

The $N \times N$ unitary matrix for Fourier transformation is defined as:

$$U_{xy} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi xy}{N}}$$

3. (a) Write an expression for a wavelet $\psi_{2,3}(x)$ in terms of the Haar scaling function. Hence plot $\psi_{2,3}(x)$.

(40 marks)

(b) Consider a 1×4 image as shown below :

$$f(x) = [4, -2, 1, 2]$$

(i) Perform discrete wavelet series expansion of $f(x)$ using Haar functions and for $j_0 = 1$.

(40 marks)

(ii) Use the result from (i) to calculate $f(1)$

(20 marks)

Given:

The wavelet series expansion is defined as:

$$f(x) = \frac{1}{\sqrt{M}} \sum_k W_\varphi(j_0, k)\, \varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j,k) \psi_{j,k}(x)$$

and,

$$W_\varphi(j_0, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \varphi_{j_0,k}(x)$$

$$W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \psi_{j,k}(x)$$

The wavelet functions are defined as :

$$\psi_{j,k}(x) = 2^{\frac{j}{2}} \psi\left(2^j x - k\right)$$

$$\psi(x) = \sum_n h_\psi(n) \sqrt{2} \varphi(2x - n)$$

$$\psi(x) = \begin{cases} 1 & ; \quad 0 \le x < 0.5 \\ -1 & ; \quad 0.5 \le x < 1.0 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

The Haar scaling functions are defined as :

$$\varphi(x)=\begin{cases}1 & ; \quad 0\le x<1\\ 0 & ; \quad \text{elsewhere}\end{cases}$$

$$\varphi_{j,k}(x)=2^{\frac{j}{2}}\varphi\left(2^{j}x-k\right)$$

The scaling function coefficients for the Haar function are given by:

$$h_{\varphi}(n)=\left\{\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\} \quad \text{for} \quad n=0,1$$

The scaling function coefficients for the Haar wavelet are given by:

$$h_{\psi}(n)=\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right\} \quad \text{for} \quad n=0,1$$

4. (a) After sampling an analogue signal, you get a finite length sequence $x[n]$ = {0,0,0,4,2,2,2,-5,2}. Express this sequence as a summation of unit steps.

(20 marks)

(b) You have designed a linear time-invariant (LTI) system as shown in Figure 4(b). In this figure, the impulse responses are:

$$h_1[n]=0.5\delta[n]$$
$$h_2[n]=-0.25\delta[n-1]$$
$$h_3[n]=\delta[n-2]$$
$$h_4[n]=-2(0.5)^{n}u[n]$$

Determine the output $y[n]$, if the input sequence $x[n]$ = {-3,0,2,1,1,4}.

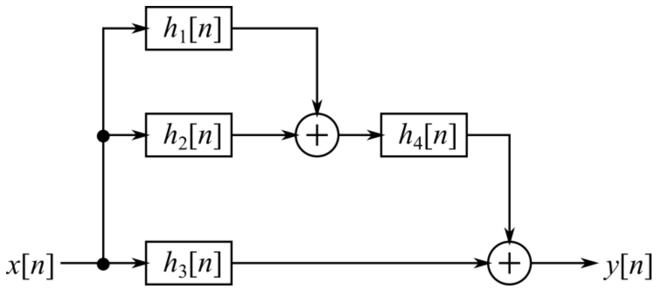

Figure 4(b)

(30 marks)

(c) Find L-1 norm, L-2 norm and L-∞ norm for sequence $x[n] = \{1,3,0,-4,-1\}$

(20 marks)

(d) An output from a digital system $y[n]$ is defined as :

$$y[n] = (x_1[n] + x_2[n]) \times x_3[n]$$

Given that:

$$x_1[n] = 4\cos(0.2\pi n + 0.1\pi)$$
$$x_2[n] = -5\cos(0.7\pi n)$$
$$x_3[n] = 2\sin(0.35\pi n)$$

Determine the period for sequence $y[n]$.

(30 marks)

5. You want to implement the following transfer function:

$$H(z) = 5\left(\frac{1 - 0.2z^{-1} + 0.5z^{-2}}{1 - 0.4z^{-1}}\right)\left(\frac{1 - 0.1z^{-1}}{1 + 0.4z^{-1} - 0.2z^{-2}}\right)$$

(a) Draw a non-canonic filter structure of H(z), using a cascade form.

(20 marks)

(b) Draw a canonic filter structure of H(z), using a cascade form.

(20 marks)

(c) Use parallel form II realization to present the filter in parallel form.

(60 marks)

6. (a) You want to design an analogue low-pass filter. The typical magnitude specification of this type of filter is shown in Figure 6(a). In your design, the minimum attenuation in the stopband is set to 30 dB. On the other hand, the desired peak passband ripple is 0.005 dB. Calculate δ_s and δ_p.

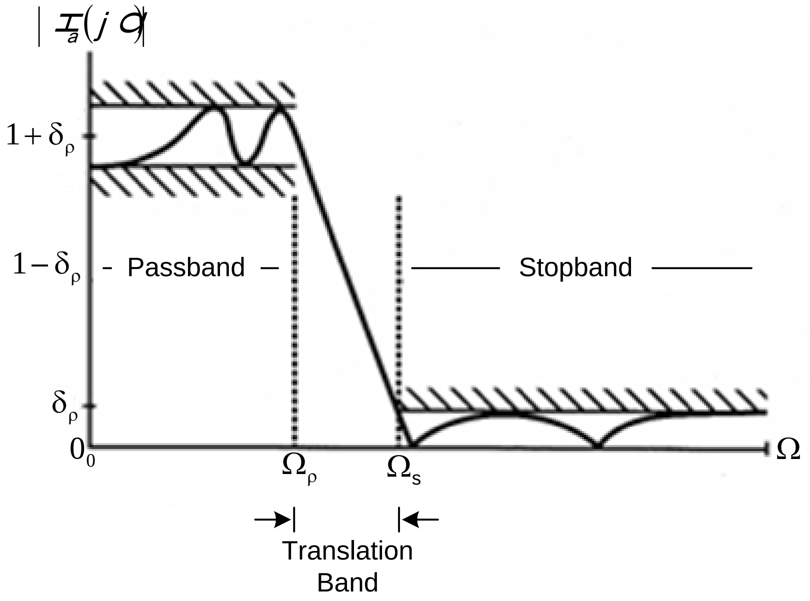

Figure 6(a)

(30 marks)

(b) The system that you want to design requires a lowpass filter that has a flat magnitude at $\Omega = 0$. Therefore, you design an analog Butterworth lowpass filter $H_a(s)$. The filter that you design has the following specifications:

- 1-dB cutoff frequency at 1 kHz.
- A minimum attenuation of 25 dB at 5 kHz.

Determine the lowest order of this filter.

(30 marks)

(c) Given a transfer function for a first order Butterworth highpass analog filter with a 3dB cutoff frequency at Ω_c is:

$$H_{HP}(s) = \frac{s}{s + \Omega_c}$$

By using the simplified bilinear transformation, find the corresponding IIR digital filter.

(20 marks)

(d) Estimate the order of a linear-phase lowpass FIR filter by using Kaiser's formula:

$$N \cong \frac{-20\log_{10}\left(\sqrt{\delta_p \delta_s}\right) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

Given the specifications:

- passband edge F_p = 1.5 kHz
- stopband edge F_s = 2.5 kHz
- peak passband ripple α_p= 0.1dB
- minimum stopband attenuation α_s = 50 dB
- sampling rate F_T = 15 kHz.

(20 marks)

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