UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2006/2007

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EEE 223 – TEORI ELEKTROMAGNET

Masa : 3 Jam

Sila pastikan kertas peperiksaan ini mengandungi **SEPULUH** **(10)** muka surat beserta **Lampiran 3 muka surat** bercetak dan **ENAM** **(6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA** **(5)** soalan.
Soalan ini mengandungi **Bahagian A** dan **Bahagian B**.

Gunakan dua buku jawapan yang diberikan supaya jawapan-jawapan bagi soalan-soalan **Bahagian A** adalah di dalam satu buku jawapan dan bagi **Bahagian B** di dalam buku jawapan yang lain.

Agihan markah diberikan di sudut sebelah kanan soalan berkenaan.
Semua soalan hendaklah dijawab di dalam Bahasa Inggeris.

Note:
Symbols have their usual meanings.
Vectors are represented by bold face letters.
Use SI system of units
Take $\varepsilon_0 = 8.85 \times 10^{-12}$ *F/m,* $\mu_0 = 4 \times 10^{-7}$ *H/m*
Assume appropriate data where not given

Bahagian A
Part A

1. (a) Di dalam talian penghantaran yang mempunyai kehilangan diuja oleh masukan sinus, pemalar perambatan $\gamma = \alpha + j\beta$ diberi sebagai $\sqrt{(R + j\omega L)(G + j\omega C)}$. α ialah pemalar pelemahan dan β ialah pemalar fasa. Juga R, L, G dan C adalah, masing-masing, perintang, aruhan, kealiran dan kemuatan bagi setiap unit panjang talian tersebut. Jika talian adalah tanpa kehilangan dan mempunyai 80 cm panjang serta galangan keciriannya 50 Ω, kemuatan 100pF/m dan beroperasi pada ω = 600MHz, nyatakan

For a lossy transmission line excited by sinusoidal inputs, the propagation constant $\gamma = \alpha + j\beta$ is given by $\sqrt{(R + j\omega L)(G + j\omega C)}$. α is the attenuation constant and β is the phase constant. Also R, L, G and C are, respectively, the resistance, inductance, conductance and capacitance per unit length of the line. If the line is lossless and has a length of 80 cm a characteristic impedance of 50 Ω, 100pF/m capacitance and is operating at ω = 600MHz, determine

(i) Jumlah aruhan bagi talian tersebut.
Total inductance of the line.

(ii) Pemalar pelemahan bagi talian tersebut.
Attenuation constant of the line.

(iii) Pemalar fasa bagi talian tersebut.
Phase constant of the line.

(iv) Halaju fasa dalam talian tersebut.
Phase velocity on the line.

(v) Galangan masukan bagi talian tersebut bila litar dipintas pada penghujung bebanan.

The input impedance of the line when it is short circuited at the load end.

Nota ringkas bagi talian hantaran tanpa kehilangan:-

Note that for a lossless transmission line:-

Halaju fasa dan

$$Z_{in} = Z_0 \frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}$$; *phase velocity* $v_p = \omega / \beta$, *and* $Z_0 = \sqrt{L/C}$

Setiap simbol mempunyai maksud tersendiri.

Symbols have their usual meaning.

(50%)

(b) Filamen arus tak terhingga membawa arus 3 Amps dan terletak di atas paksi-x. Dengan menggunakan Hukum Biot–Savart, cari keamatan medan magnet **H** dalam koordinat Cartesian pada titik P(-1,3,2). Hukum Biot-Savart dalam ungkapan matematik adalah

*An infinite current filament carries a current of 3 Amps and lies along the x-axis. Using Biot-Savart Law, find the magnetic field intensity **H** in Cartesian coordinates at a point P(-1,3,2). Biot-Savart Law is mathematically expressed as*

$$\mathbf{dH} = \frac{I\mathbf{dL} \times \mathbf{a}_r}{4\pi R^2}$$

Setiap simbol mempunyai maksud tersendiri

Symbols have their usual meaning.

(50%)

2. (a) Vektor upaya magnet, **A** dalam sesuatu kawasan ruang bebas diberi sebagai **A** = $50\rho^2\mathbf{a_z}$ Wb/m. Dengan menggunakan hubungan dimana ikalan **A** adalah sama dengan ketumpatan fluks magnet **B** dan ikalan **H** adalah sama dengan ketumpatan arus **J**, nyatakan nilai-nilai **B**, **H** dan **J**. Ikalan bagi vektor **V** bagi koordinat silinder (r, φ, z) diberi sebagai

*The vector magnetic potential **A** in a certain region of free space is given as **A** = $50\rho^2\mathbf{a_z}$ Wb/m. Using the relation that curl of **A** is equal to the magnetic flux density **B** and the curl of **H** is equal to the current density **J**, determine **B**, **H** and **J**. The curl of a vector **V** in cylindrical (r, φ, z) coordinates is given by*

$$\nabla \times \mathbf{V} = \left[\frac{1}{r}\frac{\partial V_z}{\partial \varphi} - \frac{\partial V_\varphi}{\partial z}\right]\mathbf{a}_r + \left[\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right]\mathbf{a}_\varphi + \frac{1}{r}\left[\frac{\partial (rV_\varphi)}{\partial r} - \frac{\partial V_r}{\partial \varphi}\right]\mathbf{a}_z$$

(50%)

(b) Konduktor lurus panjang tak terhingga dengan keratan rentas bulatan yang mempunyai jejari b membawa arus tetap I. Dengan menggunakan Hukum Litar Ampere nyatakan ketumpatan fluks magnet bagi kedua-dua luaran dan dalaman konduktor tersebut. Hukum Ampere menyatakan bahawa garis kamiran bagi ketumpatan fluks magnet **H** di sekitar laluan tertutup adalah sangat sama dengan arah arus tertutup yang melaluinya secara Matematik,

An infinite long straight conductor with a circular cross-section of radius b carries a steady current I. *Using Ampere's Circuit Law determine the magnetic flux density both inside and outside the conductor. Ampere's Law states that the line integral of the magnetic field intensity **H** about any closed path is exactly equal to the direct current enclosed by the path Mathematically,*

$$\oint \mathbf{H.dL} = I$$

Setiap simbol mempunyai maksud tersendiri.
Symbols have their usual meaning.

(50%)

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3. Talian penghantaran yang tanpa kehilangan mempunyai panjang 3.3λ dan galangan kecirian 50Ω. Talian ditamatkan dengan galangan beban Z_L = (25 + j 50) Ω. Dengan menggunakan carta Smith carikan

A loss-less transmission line is 3.3λ long and has a characteristic impedance of 50 Ω. The line is terminated in a load impedance of Z_L = (25 + j 50) Ω. Use Smith's chart to find

(i) Pekali pembalikan voltan tersebut.
The voltage reflection coefficient. (20%)

(ii) Nisbah voltan gelombang pegun tersebut.
The voltage standing wave ratio. (20%)

(iii) Jarak bagi voltan maksimum pertama dari beban tersebut.
The distance of first voltage maximum from the load. (20%)

(iv) Jarak bagi voltan minimum pertama dari beban tersebut.
The distance of first voltage minimum from the load. (20%)

(v) Galangan masukan bagi talian tersebut.
The input impedance of the line. (20%)

Bahagian B
Part B

4. (a) Diberikan **A**=$\mathbf{a_x}$ + $\mathbf{a_y}$, **B**=$\mathbf{a_x}$ + 2$\mathbf{a_z}$, dan **C** = 2$\mathbf{a_y}$ + $\mathbf{a_z}$. Dapatkan **(A x B) x C** dan bandingkannya dengan **A x (B x C)**. Berikan komen anda.

Given **A**=$\mathbf{a_x}$ + $\mathbf{a_y}$, **B**=$\mathbf{a_x}$ + 2$\mathbf{a_z}$, *and* **C** = 2$\mathbf{a_y}$ + $\mathbf{a_z}$. *Find* **(A x B) x C** *and compare it with* **A x (B x C)**. *Give a comment.*

(25%)

(b) Dapatkan gradien untuk medan skalar $f(x,y,z)=6x^2y^3+e^z$ pada titik P(2,1,0).

Find the gradient of a scalar field $f(x,y,z)=6x^2y^3+e^z$ *at a point P(2,1,0).*

(25%)

(c) Dapatkan persamaan untuk vektor capahan medan di bawah.

Find an expression for the divergence of the vector field.

$$F(r)=\hat{r}\,r\cos(\phi)+\hat{z}(r^2+z^2)$$

(25%)

(d) Ubah bentuk **A**= y$\mathbf{a_x}$ + x$\mathbf{a_y}$ + $\frac{x^2}{\sqrt{x^2+y^2}}$ $\mathbf{a_z}$ dari kordinat Cartesian ke kordinat silinder.

Transform **A**= *y*$\mathbf{a_x}$ + *x*$\mathbf{a_y}$ + $\frac{x^2}{\sqrt{x^2+y^2}}$ $\mathbf{a_z}$ *from coordinates Cartesian to cylindrical cordinates.*

(25%)

5. (a) Bermula dari hukum Coulomb, kaitkan kuasa diantara dua titik bercas dengan magnitud cas dan jarak di antara mereka, bina persamaan-persamaan untuk ketumpatan medan elektrik, D dan kekuatan medan elektrik, E, pada jarak r meter dari satu cas yang bermagnitud $+q_1$ Coulomb.

Starting from Coulomb's Law, relating the force between two point charges to the magnitude of the charges and the distance between them, develop expressions for the electric field density, D and electric field strength, E, at a distance of r meter from a charge of magnitude $+q_1$ *Coulomb.*

(20%)

Kemudian, tunjukkan bahawa keupayaan mutlak pada jarak r meter dari satu titik bercas $+q_1$ Coulomb dalam ruang udara adalah diberikan oleh

Hence show that the absolute potential at a distance of r meter from a point charge of + q_1 *Coulomb, in air is given by*

$$V = \frac{q_1}{4\pi\varepsilon_0 r}$$

dimana ε_0 adalah ketelapan untuk ruang bebas.

where ε_0 *is the permittivity of free space.*

(30%)

(b) Satu cas positif bernilai 10 μC diletakkan di dalam planar x-y pada x=0.5m dan y=1m dan cas yang sama turut diletakkan pada x = -0.5m dan y=1m. Satu titik P terletak pada planar y-z dengan kordinatnya adalah y= 1m dan z = 1m. Andaikan bahawa cas-cas tersebut adalah diruang udara, tentukan nilai pada titik P berikut.

A 10 μC positive charge is placed in the x-y plane at x=0.5m and y=1m and an identical point charge is placed at x= -0.5m and y = 1m. A point P lies in the y-z plane with co-ordinates y= 1m and z = 1m. Assuming that the charges are in air, determine, at the point P, the following:

(i) Magnitud dan arah ketumpatan medan elektrik.
The magnitude and direction of the electric flux density.

(20%)

(ii) Magnitud dan arah kekuatan medan elektrik.
The magnitude and direction of the electric field strength.

(10%)

(iii) Magnitud dan arah bagi kuasa ke atas cas positif bernilai 10 μC.
The magnitude and direction of the force on a 10 μC positive charge.

(10%)

(iv) Keupayaan mutlak.
The absolute potential.

(10%)

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6. Bermula daripada prinsip yang pertama, tunjukkan bahawa nilai kapasitan per unit panjang bagi kabel berongga, dengan jejari konduktor dalaman a dan jejari konduktor luaran sebagai b adalah diberikan oleh

Starting from first principles, show that the capacitance of a unit length of coaxial cable, with inner conductor of radius a and outer conductor of radius b, is given by

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(\frac{b}{a})}$$

Dimana ε_0 adalah ketelapan bagi ruang bebas dan ε_r adalah ketelapan bagi dielektrik antara konduktor dalaman dan luaran.

Where ε_0 is the permittivity of free-space and ε_r is the relative permittivity of the dielectric between inner and outer conductors.

(40%)

Sekiranya dielektrik itu mempunyai kehilangan dengan faktor kekonduksian σ, tunjukkan bahawa seunit panjang kabel akan mempunyai rintangan selari yang diberikan oleh

If the dielectric is lossy, with a conductivity of σ, show that a unit length of cable has a shunt resistance given by

$$R = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\sigma}$$

(30%)

Satu panjang kabel berongga mempunyai jejari konduktor dalaman sebanyak 2 mm, jejari konduktor dalaman sebanyak 3 cm. Dielektrik yang digunakan mempunyai $\sigma = 3\times10^{-4}$ S/m dan $\varepsilon_r = 3$. Bandingkan arus kebocoran dan arus kapasitan sekiranya kabel tersebut membawa 1 KV pada frekuensi 50 Hz. Berikan komen kepada keputusan anda.

A length of coaxial cable has an inner conductor of radius 2 mm, an outer conductor of radius 3 cm. The dielectric used has $\sigma = 3\times10^{-4}$ S/m and $\varepsilon_r = 3$. Compare the leakage and capacitive currents if the cable carries 1 KV at a frequency of 50 Hz. Comment on your results.

(30%)

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Table 2-1: Summary of vector relations

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A, $\lvert A \rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}}\,dy\,dz$ $ds_y = \hat{\mathbf{y}}\,dx\,dz$ $ds_z = \hat{\mathbf{z}}\,dx\,dy$	$ds_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $ds_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V} =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Table 2-2: Coordinate transformation relations

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$