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Unsteady Boundary Layer Separated Stagnation-Point Flow towards a Permeable Shrinking Sheet

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Abstract. A study of the unsteady separated stagnation-point flow with constant suction towards a shrinking sheet is presented. A similarity transformation reduces the governing partial differential equation to the third order nonlinear ordinary differential equation which the terms of unsteady effect are clearly shown. The problem is solved numerically where the influences of shrinking and suction parameters on flow are studied. It is found that two solutions exist, one representing an attached flow while the other a reverse flow. It is found that adequate suction is necessary for the solutions to exist.

Keywords: Separated flow; Stagnation-point; Shrinking sheet; Suction.
PACS: 47.15.Cb; 47.32.Ff; 47.85.L-

INTRODUCTION

In fluid dynamics, similarity solution is exceptionally important in order to simplify a model by examining the physical effects that are presented in a system; consequently transformation variables can be derived from the scaling of governing equations. The similarity solutions of the steady boundary layer equations are well-studied, however, not for the case of unsteady flow. Williams and Johnson [1] obtained a semisimilar solution of unsteady two-dimensional incompressible boundary layer equations; however, their boundary-layer equations are not valid when the unsteady separation phenomenon occurred. Ma and Hui [2] obtained a better solution by using the method of Lie group transformation to derive all possible group-invariant similarity solutions for the two-dimensional unsteady boundary layer equations. One of the six group-invariant solutions is unsteady separated stagnation-point flow over a fixed wall. In contrast to the solutions by Williams and Johnson [1], Ma and Hui [2] found that their solutions were remaining valid in the boundary layer region even when flow separation or reversal occurs.

In this paper, the specific problem of unsteady separated stagnation-point flow is considered where the surface is moving (shrunk) towards the origin of the permeable surface with constant suction. The flow due to a shrinking sheet has its important applications in industries such as manufacturing of polymer sheets, filaments and wires. The shrinking sheet phenomenon with a suction effect was first investigated by Miklavĉiĉ and Wang [3]. Since then, there have been quite a number of studies which are related to permeable shrinking flow, either in different fluid media or different boundary conditions. However, we noticed that there is limited study on the unsteady flow over shrinking sheet near a stagnation point. Fan et al. [4] studied the unsteady stagnation flow and heat transfer problem towards a shrinking sheet but they did not study the case of separation flow. Recently, Dholey and Gupta [5] studied the unsteady separated stagnation-point flow of moving plate with suction where the velocity is depended on time. For our case, the velocity of the shrinking sheet is a function of two independent variables, i.e. time and spatial.

MATHEMATICAL FORMULATION

Consider the unsteady laminar boundary layer for flow over a permeable shrinking sheet near a stagnation-point as shown in Figure 1. Cartesian coordinates are used with the *x*-axis is measured along the surface and the *y*-axis is normal to it. The free stream velocity is $u_e(x,t) = (a/t)x$ where a is a positive constant and t is time. The velocity on the boundary, $u_w(x,t) = (c/t)x$ is moving towards a fixed point at the origin where $c < 0$ is the shrinking rate. Under these assumptions the basic equations of this problem can be written as (see Ma and Hui [2] and Dholey and Gupta [5])

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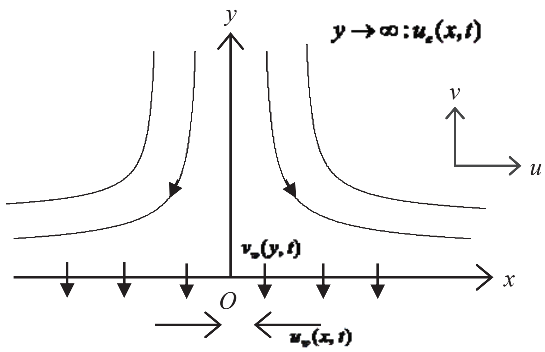

FIGURE 1. Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The boundary conditions are

$$\begin{aligned} t < 0: &\quad u = 0, \quad v = 0 \quad \text{for any } x, y \\ t \geq 0: &\quad u = u_w(x,t), \quad v = v_w(y,t) \quad \text{at} \quad y = 0 \\ &\quad u = u_e(x,t) \qquad \text{as} \quad y \to \infty \end{aligned} \tag{3}$$

Here, u and v are the velocity components along the x- and y- axes, υ is the kinematic viscosity of the fluid, and $v_w(x,t)$ is the mass flux velocity that will be defined later.

Following Ma and Hui [2], and Dholey and Gupta [5], we introduce the following similarity transformations

$$u = \frac{a}{t}x\, f'(\eta), \quad v = -a\sqrt{\frac{\nu}{t}}\, f(\eta), \quad \eta = y\sqrt{\frac{1}{\nu t}} \tag{4}$$

so that equations (1) and (2) can be reduced to

$$f''' + \left(\frac{\eta}{2} + a f\right) f'' + \left(1 - a f'\right) f' + a - 1 = 0 \tag{5}$$

where primes denote differentiation with respect to η. We notice that equation (5) is the combination of Hiemenz flow and unsteady effect terms.

Using the transformation variables in (4) too, the boundary conditions (3) now become

$$f(0) = s, \quad f'(0) = \lambda, \quad \text{and} \quad f'(\eta) \to 1 \quad \text{as} \quad y \to \infty \tag{6}$$

Here, $\lambda = c/a$ is the velocity ratio parameter with $\lambda < 0$ corresponds to the case of shrinking sheet while the mass flux velocity is defined as

$$v_w(y,t) = -a\sqrt{\frac{\nu}{t}}\, s \tag{7}$$

where $s = f(0)$ is the constant mass flux with $s > 0$ for the case of suction.

RESULTS AND DISCUSSION

The third order nonlinear ordinary differential equation (5) subject to boundary conditions (6) has been solved numerically using the bvp4c program in Matlab. This is a finite difference code that implements the three-stage Lobatto IIIa formula, which is a collocation method with forth-order accuracy. Examples of solving boundary value problems with bvp4c can be found in a book by Shampine et al. [6] or through an online tutorial by Kierzenka [7]. For the sake of simplicity, we consider fixed value of $a = 1$ throughout the computation of this problem. Besides, the maximum residual is set to 10^{-5}.

The variation of the velocity $f'(\eta)$ with η is depicted in Figure 2 for some values of shrinking parameter λ and considering fixed value of suction parameter $s = 1$. It is observed that for the case $a = 1$, two solutions exist, one representing an attached flow, the other a reverse flow. Both solutions show that the far field boundary condition ($f'(\eta) \to 1$ as $\eta \to \infty$) is approached asymptotically, with positive velocity gradient at the leading edge of the shrinking sheet for the case of attached flow while negative velocity gradient is observed near to the wall for the case of reverse flow. The boundary layer thickness of the reverse flow is greater than the boundary layer thickness of the attached flow.

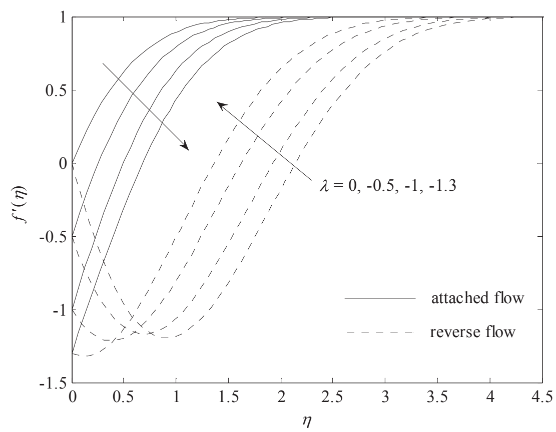

FIGURE 2. Velocity profile for some values of shrinking parameter when $a = 1$ and $s = 1$.

For comparison purposes, the reserve flow solutions $f'(\eta_s)$ for some values of suction parameter when the plate is fixed ($\lambda = 0$) are presented in Table 1. The values of boundary layer thickness η_s which satisfies $f(\eta_s) = 0$ are also given in Table 1. It can be seen that the results are in very good agreement.

TABLE (1). Comparisons of the values of reverse flow solutions $f'(\eta_s)$ for some values of suction parameter when $a = 1$ and $\lambda = 0$.

s	η_s		$f'(\eta_s)$	
	Dholey and Gupta [5]	Present study	Dholey and Gupta [5]	Present study
0.0	4.2208	4.2207	0.9660	0.9659
0.5	0.8761	0.8760	–0.9372	–0.9361
	3.7375	3.7375	0.9351	0.9349
1.0	1.1381	1.1382	–1.1332	–1.1326
	3.3104	3.3098	0.8827	0.8822
2.0	1.5080	1.5086	–0.8148	–0.8138
	2.5251	2.5249	0.6233	0.6230

Figure 3 show the velocity profiles for some values of suction parameter when $\lambda = -1$ (shrinking sheet). It is found that adequate suction is necessary for the solution to exist. For example, only one solution was observed for $s = 0.4$, no solution for $s \le 0.3$, while dual solutions were obtained for value of $s \ge 0.5$. For the first solution (attached flow), boundary layer thickness decreases as s increases. The second solution (reverse flow) shows opposite effect as compared to the attached flow.

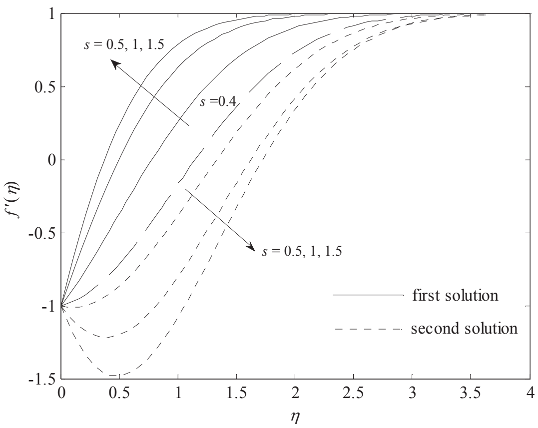

FIGURE 3. Velocity profile for some values of suction parameter when $a = 1$ and $\lambda = -1$.

CONCLUSIONS

We have studied the problem of unsteady separated stagnation-point flow over a shrinking sheet with constant suction. A similarity transformation was employed to reduce the governing partial differential equations into a third-order ordinary differential equation. The bvp4c Matlab program was used to obtain the numerical solutions. Both attached and reverse flow were obtained for the fixed wall ($\lambda = 0$) and also the shrinking sheet ($\lambda < 0$) that considered. It was found that for the attached flow, suction tends to increase the velocity gradient near the wall, i.e. increase the wall shear stress. However, for the case of reverse flow, increasing the suction parameter will increase the boundary layer thickness. We presented only results for the special case $a = 1$, other values of a will be investigated in our next study.

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REFERENCES

1. J. C. Williams and W. D. Johnson, *AIAA Journal* **12**, 1388-1393 (1974).
2. P. K. H. Ma and W. H. Hui, *Journal of Fluid Mechanics* **216**, 537-559 (1990).
3. M. Miklavčič and C. Y. Wang, *Quarterly of Applied Mathematics* **64**, 283–290 (2006).
4. T. Fan, H. Xu and I. Pop, *International Communications in Heat and Mass Transfer* **37**, 1440-1446 (2010).
5. S. Dholey and A. S. Gupta, *Physics of Fluids* **25**, 023601 (2013).
6. L. F. Shampine, I. Gladwell and S. Thompson, *Solving ODEs with Matlab*, Cambridge: Cambridge University Press, 2003.
7. J. Kierzenka, *Tutorial on solving BVPs with BVP4C.* http://www.mathworks.com/matlabcentral/fileexchange/3819-tutorial-on-solving-bvps-with-bvp4c. (Last accessed: 30 September 2013).