Stagnation point flow towards a melting shrinking sheet in an upper convected Maxwell fluid

Yian Yian Lok, Mohamad Faisal Abdul Karim, and Ioan Pop

Citation: AIP Conf. Proc. **1522**, 614 (2013); doi: 10.1063/1.4801181
View online: http://dx.doi.org/10.1063/1.4801181
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1522&Issue=1
Published by the American Institute of Physics.

Stagnation Point Flow towards a Melting Shrinking Sheet in an Upper Convected Maxwell Fluid

Lok Yian Yian[a], Mohamad Faisal Abdul Karim[a] and Ioan Pop[b]

[a]*Mathematics Section, School of Distance Education, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia*
[b]*Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, CP 253, Romania*

Abstract. An analysis of the steady stagnation point flow towards a melting shrinking sheet in an upper convected Maxwell (UCM) fluid has been studied. A similarity transformation is used to reduce the governing partial differential equations to third-order nonlinear ordinary differential equation which are then solved numerically using an implicit finite difference method, namely the Keller-box method. The influences of the melting and shrinking parameters on the flow and heat transfer characteristics are studied. Representative results for the reduced skin friction coefficient and the reduced heat flux from the surface of sheet (local Nusselt number) as well as the velocity and temperature profiles are presented. It is found that the solutions are non-unique for some values of the shrinking parameter.

Keywords: Melting, stagnation point flow, shrinking sheet, Maxwell fluid
PACS: 44.90.+c, 47.11.-j, 47.15.-x, 47.50.-d

INTRODUCTION

Many industrial fluids are non-Newtonian or rheological in their flow characteristics (such as molten plastics, polymers, suspension, foods, slurries, paints, glues, printing inks, blood). That is, they might exhibit dynamic deviation from Newtonian behavior depending upon the flow configuration and/or the rate of deformation. These fluids often obey non-linear constitutive equations and the complexity in the equations is the main culprit for the lack of exact analytical solutions. Therefore the significance of the results reported in the above works is limited as far as the polymer industry is concerned. Visco-elastic fluids are one of the non-Newtonians fluids which exhibit both viscous and elastic characteristics. These kinds of fluids are able to keep memory of their past deformations hence become the focus interest of many researchers. Many fluid models have been suggested to describe the behaviour of viscoelastics fluids; one of them is the upper convected Maxwell fluid (UCM) fluid which takes into account the stress relaxation that exists in the flow. Obviously for the theoretical results to be of any industrial importance, more general visco-elastic fluid models such as upper convected Maxwell model (UCM fluid) or Oldroyd B model should be invoked.

Recently, the flow due to a shrinking sheet has gained considerable interest. For such flow, the sheet is shrunk towards a fixed point which would cause a velocity away from the sheet. Miklavčič and Wang [1] found non-unique solutions for the problem of shrinking sheet of viscous flow with certain suction rates. Besides, the shrinking problem near a stagnation point too will give non-unique solutions [2]. Therefore, there are three conditions for shrinking flow to exist physically, i.e. either imposed adequate suction on the boundary [1], or added stagnation flow which contains the vorticity [2] or imposition of suction to stagnation point flow [3].

The aim of this study is to investigate the steady two-dimensional stagnation flow of an upper convected Maxwell fluid impinging on a shrinking sheet in the presence of melting effect. As far as we are concerned, such analysis has not yet reported in open literature. It should be mentioned here that Hayat et al. [4] investigated the mass transfer in the MHD flow of UCM fluid over a porous shrinking sheet with chemically reactive species and solved the nonlinear system of ordinary differential equations by using homotopy analysis method. We mentioned also the paper by Hayat et al. [5] which considered the problem of boundary layer flow and melting heat transfer in the stagnation point flow of an UCM fluid but for stretching sheet case while Bachok et al. [6] and Yacob et al. [7] studied the melting heat transfer in boundary layer stagnation point flow towards both stretching and shrinking sheet in viscous fluid and micropolar fluid, respectively. The study on melting heat transfer has its applications in magma solidification, the melting of permafrost and the thawing of frozen grounds [8]. In this paper, we use the UCM model which given by Sadeghy et al. [9], and Kumar and Nath [10]. The governing equations are analyzed by

Proceedings of the 20th National Symposium on Mathematical Sciences
AIP Conf. Proc. 1522, 614-619 (2013); doi: 10.1063/1.4801181

boundary layer approximations and similarity transformations, and the resulting nonlinear equations are solved numerically using Keller-box method for some values of governing parameters.

BASIC EQUATIONS

Consider the steady stagnation point flow towards a shrinking sheet in an upper convected Maxwell fluid with melting effects. It is assumed that the velocity of the shrinking sheet is $u_w(x) = cx$, while the flow velocity outside boundary layer (inviscid fluid) is $u_e(x) = ax$, where a (> 0) and c are constants, with $c > 0$ for a stretching sheet and $c < 0$ for a shrinking sheet, respectively. The flow taking place in the space $y \le 0$, where x and y are the Cartesian coordinates measured along the surface of the sheet and normal to it, respectively. It is also assumed that T_m and C_m are the constant melting temperature and melting concentration of the solid surface, while T_∞ and C_∞ are the constant temperature and constant concentration of the ambient fluid. Applying the boundary layer approximations, the basic equations for the problem under consideration can be written as [9-10]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{d u_e}{d x} + \nu\frac{\partial^2 u}{\partial y^2} + k_0\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &u = u_w(x) = cx \quad T = T_m, \quad &&\text{at} \quad y = 0 \\ &u = u_e(x) = ax, \quad T = T_\infty, \quad &&\text{as} \quad y \to \infty \end{aligned} \tag{4}$$

along with the melting condition [11]

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho\left[\gamma + C_S\,(T_m - T_0)\right]v(x,0) \tag{5}$$

Here u and v are the velocity components in the x and y directions, respectively, T is the fluid temperature, k_0 is the relaxation time, α is the thermal diffusivity, ρ is the density, υ is the kinematic viscosity, k is thermal conductivity, γ is the latent heat of the fluid, C_s is the heat capacity of the solid surface and T_0 is the melting temperature of the solid surface. Equation (5) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature T_0 to its melting temperature T_m [8, 11].

In order to solve equations (1)–(3) with the boundary conditions (5), we consider the following similarity variables

$$u = ax\,f'(\eta), \quad v = -(a\nu)^{1/2}\,f(\eta), \quad \theta(\eta) = (T - T_m)/(T_\infty - T_m), \quad \eta = (a/\nu)^{1/2}\,y \tag{6}$$

where primes denote differentiation with respect to η. Substituting (6) into equations (2)–(3), we obtain the following ordinary differential equations

$$f''' + f f'' + 1 - f'^2 + K(f^2 f''' - 2 f f' f'') = 0 \tag{7}$$

$$\theta'' + \Pr f \theta' = 0 \tag{8}$$

and the boundary conditions (4) and (5) become

$$\begin{aligned} &f'(0) = c/a = \lambda, \quad \Pr f(0) + M\theta'(0) = 0, \quad \theta(0) = 0, \\ &f'(\eta) \to 1, \quad \theta(\eta) \to 1, \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{9}$$

The dimensionless parameters appearing in these equations are the Deborah or Weissenger number K [12], the stretching ratio λ, the Prandtl number Pr, the thermal diffusivity α and the melting number M, which are defined as

$$K = a k_0, \quad \lambda = \frac{c}{a}, \quad \Pr = \frac{\nu}{\alpha}, \quad \alpha = \frac{k}{\rho C_p}, \quad M = \frac{C_p (T_\infty - T_m)}{\gamma + C_S (T_m - T_0)} \tag{10}$$

Quantities of physical interest in this problem are the skin friction coefficient C_f the local Nusselt number Nu_x which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2(x)}, \quad Nu_x = \frac{x q_w}{k(T_m - T_\infty)} \tag{11}$$

where τw is the wall skin friction or shear stress and q_w is the heat flux from the plate which are given by

$$\tau_w = \mu(1+K)\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

Substituting (7) into (14) and using (13), we get

$$\mathrm{Re}_x^{1/2} C_f = (1+K) f''(0), \quad \mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0) \tag{13}$$

where $\mathrm{Re}_x = u_e(x) x / \nu$ is the local Reynolds number.

RESULTS AND CONCLUSION

The differential equations (7) and (8) subjected to boundary conditions (9) are solved numerically using the Keller box method, i.e. an implicit finite difference method in conjunction with Newton linearization. It has been widely used in solving boundary layer flow problems. In order to validate the results obtained, we have compared our results with those reported by Wang [2] who use the finite difference method; Bachok et al. [6] and Yacob et al. [7] who use the Runge-Kutta-Fehlberg method with shooting technique. The comparisons are shown in Tables 1 and 2 for the case when the Deborah or Weissenger number K equals zero. It is found that the results are in very good agreement therefore we are confident of the accuracy of the results in this study.

TABLE (1). Shrinking sheet case for different values of shrinking parameter when $K = 0$ and $M = 0$

λ	$f''(0)$							
	Wang [2]		**Bachok et al. [6]**		**Yacob et al. [7]**		**Present study**	
	Upper branch	**Lower branch**	**Upper branch**	**Lower branch**	**Upper branch**	**Lower branch**	**Upper branch**	**Lower branch**
-0.25	1.40224		1.4022408		1.402241		1.402241	
-0.50	1.49567		1.4956698		1.495670		1.495670	
-0.75	1.48930		1.4892983		1.489298		1.489298	
-1.00	1.32882	0	1.3288170	0	1.328817	0	1.328817	0
-1.15	1.08223	0.116702	1.0822315	0.1167022	1.082231	0.116702	1.082237	0.116702

TABLE (2). Stretching sheet case for different values of Prandtl number and melting parameter when $K = 0$ and $\lambda = 1$

Pr	**M**	$-\theta'(0)$	
		Bachok et al. [6]	**Present study**
1	0	-0.7978846	-0.79789
	1	-0.5060545	-0.50605
	2	-0.3826383	-0.38264
	3	-0.3119564	-0.31196
7	0	-2.1110042	-2.11101
	1	-1.3388943	-1.33890
	2	-1.0123657	-1.01237
	3	-0.8253591	-0.82536

Figure 1 show the variation of skin friction coefficient, $\mathrm{Re}_x^{1/2} C_f$ and local Nusselt number, $\mathrm{Re}_x^{-1/2} Nu_x$ with the shrinking parameter λ for different values of K when $M = 1$. Here we have used fixed value of Prandtl number, Pr = 0.7 throughout the computation for all figures in the present work. It is observed that for fixed values of λ and M, both skin friction coefficient and local Nusselt number increase as elastic parameter K increases. It should be mentioned here that the surface velocity gradient $f''(0)$ decreases with increasing of K, however, due to the coefficient $(1+K)$ in equation (13), $\mathrm{Re}_x^{1/2} C_f$ in Figure 1(a) increases with increasing of K. Figures 1(a) and (b) clearly show that unique solution is found for $\lambda > -1$ while the existence of dual solutions for shrinking sheet case is found for $\lambda_c \le \lambda < -1$ where solution does not exist for value below the critical value λ_c. In the present study, these critical values are $\lambda_c \approx -1.192$, -1.110 and -1.051 for $K = 0.1$, 0.5 and 1.0, respectively. The curves in Figure 1(a) increase gradually and then decrease to approximate value zero near $\lambda = -1$. From literature papers [7-8], the curves in Figures 1(a) and (b) should end at (-1, 0), but the computation by our method become unstable and inconsistent for value near to $\lambda = -1$, therefore we are able to find the second solutions near to $\lambda = -1$ only but not at $\lambda = -1$.

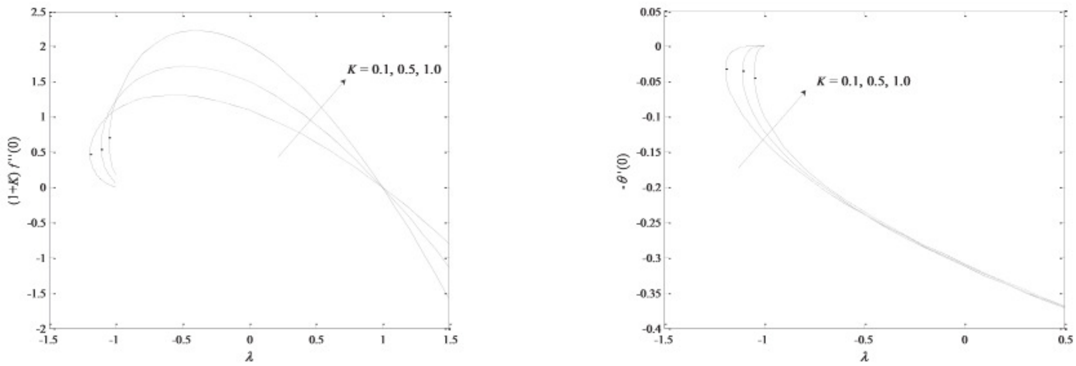

FIGURE 1. (a) Velocity profiles $f'(\eta)$ and (b) temperature profiles $\theta(\eta)$ for different values of K when λ = -1.05 (shrinking case), M = 1 and Pr = 0.7

The effects of governing parameters on velocity and temperature profiles are given in Figures 2 and 3. Dual solutions are also presented in these figures for λ = -1.05 (shrinking case) and Pr = 0.7. As can be seen from Figures 2 and 3, the boundary layer thickness for the upper branch solutions is smaller than that of lower branch solutions. Besides, the boundary layer thickness increases as K and M increases for upper branch solutions, however, opposite trend is observed for lower branch solutions. Physically, increase in K will increase the resistance of fluid motion so that the velocity will decrease result in increase of momentum boundary layer thickness. Meanwhile, increase in M will increase the intensity of melting which act as blowing boundary condition at the shrinking surface [7] hence tends to thicken the boundary layer. It is also noticed that not much effect of melting parameter on the lower branch solutions, see the overlapping dotted lines in Figure 3. The stability analysis of the dual solutions for some boundary layer problems has been performed by some researchers [13-16]. From their studies, they revealed that the solutions along the upper branch solutions are stable while those on the lower branch solution are unstable.

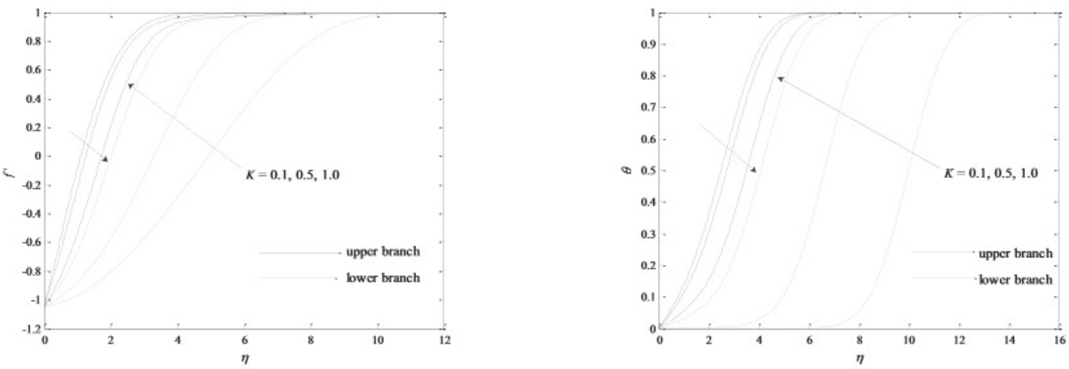

FIGURE 2. (a) Velocity profiles $f'(\eta)$ and (b) temperature profiles $\theta(\eta)$ for different values of K when λ = -1.05 (shrinking case), M = 1 and Pr = 0.7

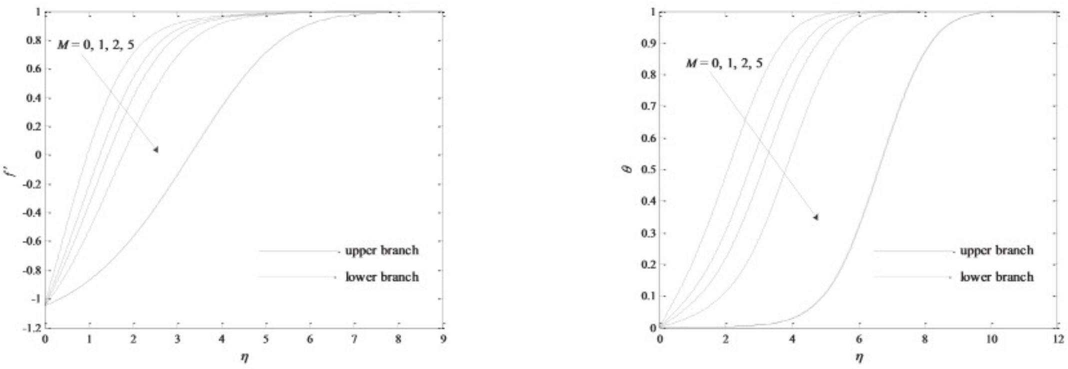

FIGURE 3. (a) Velocity profiles $f'(\eta)$ for different values of M when λ = -1.05 (shrinking case), K = 0.5 and Pr = 0.7 and temperature profiles $\theta(\eta)$ for different values of M when λ = -1.05 (shrinking case), K = 0.5 and Pr = 0.7

CONCLUSIONS

The problem of stagnation flow towards a shrinking and melting sheet in an UCM fluid has been studied numerically. Results for the skin friction coefficient, local Nusselt number, velocity profiles and temperature profiles are presented for some values of governing parameters. Non-unique solutions are found for some values of shrinking parameter. Increasing of elastic parameter and melting parameter will increase both the momentum and thermal boundary layer thickness.

ACKNOWLEDGMENTS

The authors would like to acknowledge Universiti Sains Malaysia for the financial support under the short term research grant no. 304/PJJAUH/6310099.

REFERENCES

1. M. Miklavčič and C. Y.Wang, *Quart. Appl. Math.* **64**, 283–290 (2006).
2. C. Y. Wang, *Int. J. Non-Linear Mech.* **43**, 377–382 (2008).
3. Y. Y. Lok, A. Ishak and I. Pop, *Sains Malaysiana* **40**, 1179–1186 (2011).
4. T. Hayat, Z. Abbas and M. Sajid, *Phys. Lett. A* **358**, 396–403 (2006).
5. T. Hayat, M. Mustafa, S. A. Shehzad and S. Obaidat, *Int. J. Numer. Methods Fluids* **68**, 233–243 (2012).
6. N. Bachok, A. Ishak and I. Pop, *Phys. Lett. A* **374**, 4075–4079 (2010).
7. N. A. Yacob, A. Ishak and I. Pop, *Computers & Fluids* **47**, 16–21 (2011).
8. W.T. Cheng and C.H. Lin, *Int. J. Heat Mass Transfer* **50**, 3026–3034 (2007).
9. K. Sadeghy, H. Hajibeygi and S. M. Taghavi, *Int. J. Non-Linear Mech.* **41**, 1242–1247 (2006).
10. M. Kumari and G. Nath, *Int. J. Non-Linear Mech.* **44**, 1048–1055 (2009).
11 E. M. Epstein and D. H. Cho, *J. Heat Transfer* **98**, 531–533 (1976).
12. R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamic of Polymeric Liquids*, New York: Wiley, 1987.
13. G. Wilks and J. S. Bramley, *Proc. Roy. Soc. Edinburgh* **87A**, 349–358 (1981).
14. J. H. Merkin, *J. Eng. Math.* **20**, 171–179 (1985).
15. J. Paullet and P. Weidman, *Int. J. Non-Linear Mech.* **42**, 1084–1091 (2007).
16. A. postelnicu and I. Pop, *Appl. Math. Comput.* **217**, 4359–4368 (2011).