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Citation: AIP Conference Proceedings **1602**, 317 (2014); doi: 10.1063/1.4882505
View online: http://dx.doi.org/10.1063/1.4882505
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1602?ver=pdfcov
Published by the AIP Publishing

Unsteady Separated Stagnation-Point Flow with Suction towards a Stretching Sheet

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Abstract. The problem of unsteady boundary layer separated stagnation-point flow towards a porous stretching sheet is considered. By using a similarity transformation, the governing equations are reduced to a system of ordinary differential equations which are then solved numerically. The effects of suction and stretching parameters on the flow characteristics are studied. It is observed that the solutions admit two types of solutions, one is the attached flow solution and the other is reverse flow solution.

Keywords: Unsteady separated flow; Boundary layer; Stretching sheet.
PACS: 47.15.Cb; 47.32.Ff; 47.85.L-

INTRODUCTION

The study of unsteady boundary layer flow has gained considerable attention due to the fact that many boundary layers, which occur in practice, are unsteady. One of the important features of unsteady flow is the phenomenon of separation. Boundary layer separation occurs when the portion of the boundary layer that closest to the wall reverses in flow direction.

Some studies have been conducted to investigate the relationship between the features of separation for steady flow over moving walls and unsteady flow over fixed walls. Telionis [1], and Sears and Telionis [2,3] have shown that the the point of vanishing wall shear (skin friction) does not coincide with the separation point. Telionis and Tsahalis [4], and Tsahalis [5] have overcame the difficulty of integrating the boundary layer equations by employing an upwind differencing scheme that is unconditionally stable to study the problem on impulsively moving cylinder and upstream-moving wall, respectively. Ingham [6] presented a numerical method for calculating the unsteady separation laminar flow over a circular cylinder using the method of series truncation. Contradict to Telionis and Tsahalis [4], he found that it is possible for a singularity to occur at a finite time for the unsteady boundry-layer equations.

All the above mentioned studies considered the unsteady boundary layer flows either over a fixed plate; or over a moving but rigid plate or cylinder. In this paper, the problem of unsteady separated stagnation-point flow over a stretching sheet is considered. The sheet is flexible and it can be stretched from the origin of the sheet. An effective boundary value problem solver, i.e. bvp4c program in Matlab that developed by Shampine et al. [7] and Kierzenka [8] is used to obtain similarity solutions for some values of governing parameters. In fluid dynamics, similarity solution is exceptionally important in order to simplify a model by looking at physical effects that present in a system; consequently the transformation variables can be derived from the scaling of governing equations. Ma and Hui [9] first used the method of Lie group transformation to derive all possible group-invariant similarity solutions, including the solutions for the unsteady separated stagnation-point flow over fixed wall. Many new solutions were found which are also solutions to the full Navier-Stokes equations, therefore the solutions remain valid in the boundary layer region even when flow separation or reversal occurs. The analysis method of [9] will be applied to the current problems over a stretching sheet.

The flow due to a stretching sheet has important applications in industries, such as manufacturing of polymer sheets, filaments, continuous casting of metals, etc. There are quite a number of studies on unsteady flow over a stretching sheet in Newtonian and non-Newtonian fluids, see for example [10-14], but none of them investigated the problem of unsteady separated stagnation-point flow. It has to be mentioned here that Dholey and Gupta [15] studied the unsteady separated stagnation-point flow of moving plate with suction. Its surface is moving with speed

Proceedings of the 3rd International Conference on Mathematical Sciences
AIP Conf. Proc. 1602, 317-322 (2014); doi: 10.1063/1.4882505

that is solely depended on time, whereas in our case, the sheet is stretched from the origin of the surface, where the velocity is in terms of time and spatial (the direction is along the surface).

MATHEMATICAL FORMULATION

Consider the unsteady two-dimensional laminar boundary layer equations over a porous stretching sheet near a stagnation-point as shown in Figure 1. Cartesian coordinates (x, y) are taken such that the x-axis is measured along the sheet and the y-axis is normal to it. It is assumed that the free stream velocity is $u_e(x,t) = (a/t)x$ where a is a positive constant and t is time. The sheet is stretched with a velocity $u_w(x,t) = (c/t)x$ where c > 0 is the stretching rate while the mass flux velocity is $v_w(x,t)$ which will be defined later. Under these assumptions, the simplified two-dimensional boundary layer equations of this problem can be written as (see Ma and Hui [9] and Dholey and Gupta [15])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The initial and boundary conditions are

$$\begin{aligned} t < 0: &\quad u = 0, \quad v = 0 \quad \text{for any } x, y \\ t \geq 0: &\quad u = u_w(x,t), \quad v = v_w(x,t) \quad \text{at} \quad y = 0 \\ &\quad u = u_e(x,t) \qquad \text{as} \quad y \to \infty \end{aligned} \tag{3}$$

where u and v are the velocity components along the x– and y– axes, and ν is the kinematic viscosity of the fluid.

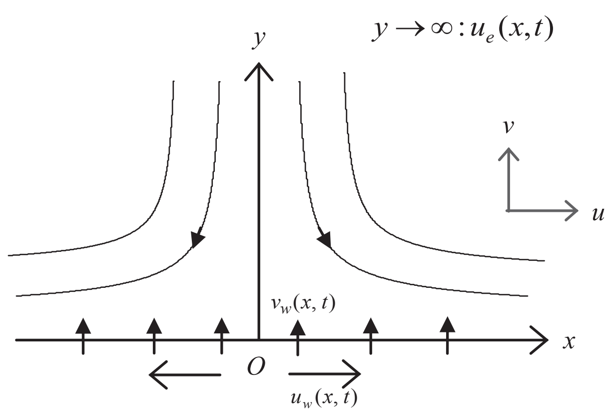

FIGURE 1. Physical model and coordinate system.

Following Ma and Hui [9], and Dholey and Gupta [15], we introduce the following similarity transformations:

$$u = \frac{a}{t}x f'(\eta), \quad v = -a\sqrt{\frac{\nu}{t}}\, f(\eta), \quad \eta = y\sqrt{\frac{1}{\nu t}} \tag{4}$$

so that equations (1) and (2) can be reduced to

$$f''' + \left(\frac{\eta}{2} + a f\right) f'' + (1 - a f') f' + a - 1 = 0 \tag{5}$$

where primes denote differentiation with respect to η.
The mass flux velocity $v_w(x,t)$ is defined as

$$v_w(x,t) = -a\sqrt{\frac{\nu}{t}}\, s \tag{6}$$

where $s = f(0)$ is the constant mass flux with $s > 0$ for suction.
Using the transformation variables in (4), the boundary conditions (3) now become

$$f(0) = s, \quad f'(0) = \lambda, \quad \text{and} \quad f'(\eta) \to 1 \quad \text{as} \quad y \to \infty \tag{7}$$

Here, $\lambda = c/a$ is the velocity ratio parameter with $\lambda > 0$ corresponding to stretching sheet.

It should be noticed that for the steady-state flow case, the similarity solutions (4) should be written as

$$u = a x f'(\eta), \quad v = -\sqrt{a\nu}\; f(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}} \tag{8}$$

and assuming $u_e(x) = a x$, $u_w(x) = c x$ and $v_w(x) = -s\sqrt{a\nu}$.
Thus, equation (5) reduces to Hiemenz flow [16] which describes the steady stagnation-point flow

$$f''' + f f'' - f'^2 + 1 = 0 \tag{9}$$

and subject to the same boundary conditions (7).

It is interesting to note that equation (5) can be written as

$$f''' + a\,(f f'' + 1 - f'^2) + \left(\frac{\eta}{2} f'' + f' - 1\right) = 0 \tag{10}$$

which clearly shown that the unsteady effect arises from the terms $\left(\frac{\eta}{2} f'' + f' - 1\right)$. By taking $a = 1$, equation (10) reduces to equation (41) established by Ma and Hui [9].

NUMERICAL COMPUTATION

The third-order nonlinear ordinary differential equation (5) subject to boundary conditions (7) has been solved numerically using the Matlab program bvp4c. The bvp4c is a finite difference method code that implements the three-stage Lobatto IIIa formula. Tutorial and examples of solving boundary value problem using bvp4c program can be found in the book by Shampine et al. [7] or through an online tutorial by Kierzenka [8]. The program is simple yet efficient in solving boundary value problems. The differential equation (5) is first reduced to a system of equations, and guess of initial values is needed to start the computation. The mesh selection and error control are based on the residual of the continuous solution. Here, we set the relative error tolerance to 10^{-5}.

RESULTS AND DISCUSSION

Table 1 shows the comparison of reverse flow solution $f'(\eta_s)$ with those by Dholey and Gupta [15] for a = 1, λ = 0 (fixed plane), and suction parameter s = 0, 0.5, 1 and 2 where η_s satisfies $f(\eta_s) = 0$. Both results are in good agreement. It has to be mentioned here that we use the method of interpolation to determine the values of η_s which are then used to evaluate $f'(\eta_s)$. The profiles of reverse flow for $s = 0.5$, for example, are given in Figure 2 which clearly shows that there are two values of η_s so that $f(\eta_s) = 0$. For the case of attached flow, $f(\eta)$ begin at 0.5 and then increases unboundedly.

TABLE (1). Comparisons of the values of reverse flow solutions $f'(\eta_s)$ for some values of suction parameter when a = 1 and $\lambda = 0$

s	η_s		$f'(\eta_s)$	
	Dholey and Gupta [15]	Present study	Dholey and Gupta [15]	Present study
0.0	4.2208	4.2207	0.9660	0.9659
0.2		0.5957		–0.6243
		4.0193		0.9555
0.5	0.8761	0.8760	–0.9372	–0.9361
	3.7375	3.7375	0.9351	0.9349
1.0	1.1381	1.1382	–1.1332	–1.1326
	3.3104	3.3098	0.8827	0.8822
2.0	1.5080	1.5086	–0.8148	–0.8138
	2.5251	2.5249	0.6233	0.6230

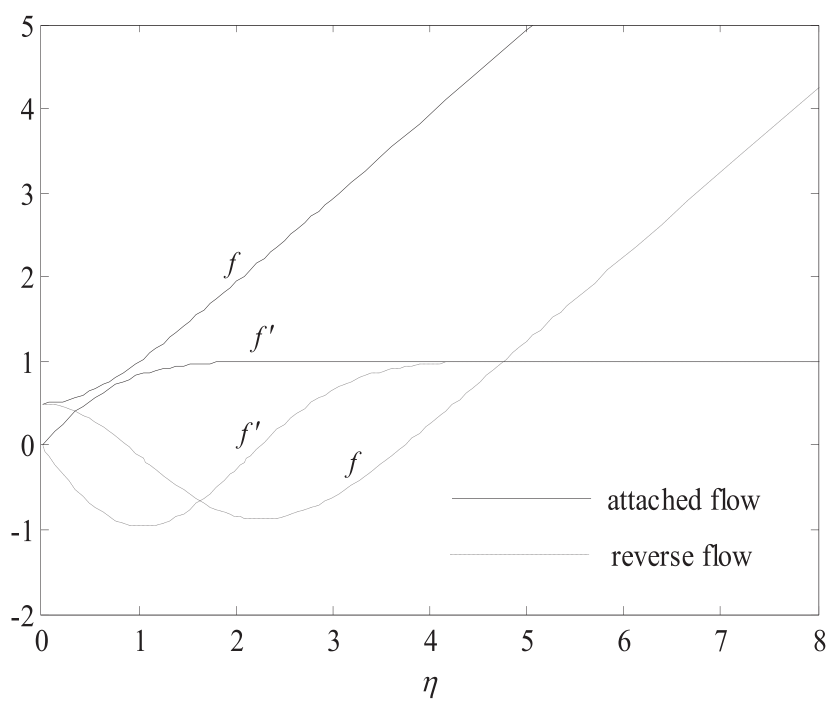

FIGURE 2. The functions of $f(\eta)$ and $f'(\eta)$ for s = 0.5, a = 1 and λ = 0 (fixed plane).

The development of velocity profiles for the case of attached flow is given in Figure 3 for some values of suction parameter when $a = 0.5$. Three different values of stretching parameter $\lambda = a/c$ are considered, i.e. $\lambda = 0.5$, $\lambda = 1$ and $\lambda = 1.5$, which representing the potential flow is less, equal or greater than the velocity of stretching sheet, respectively. The velocity has value unity for $\lambda = 1$, regardless any values of s. Opposite trend of profiles are observed for $\lambda = 0.5$ and $\lambda = 1.5$. The velocities are positive for the case of attached flow. The reverse flow solutions are given in Figure 4. For a fixed value of s, the gradient of velocity near the stretching sheet increases as λ increases. From both Figures 3 and 4, boundary layer thickness decreases as s increases indicate that imposition of suction will slow down the movement of fluid.

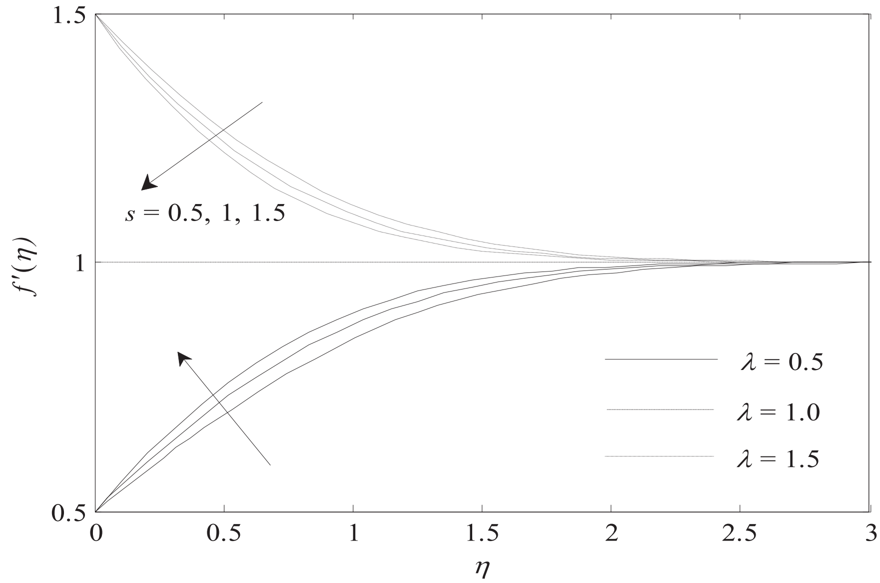

FIGURE 3. Variations of $f'(\eta)$ against η for several values of stretching parameter λ and suction parameter s when $a = 0.5$: attached flow solutions

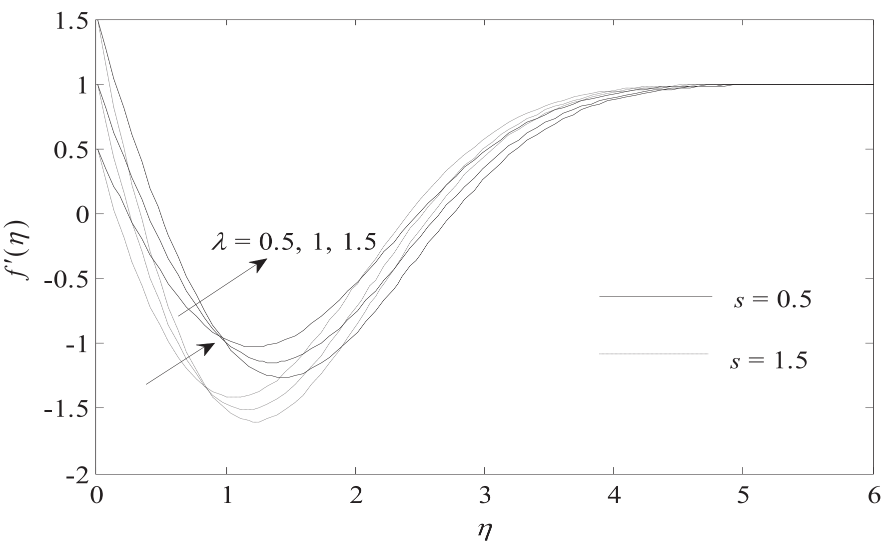

FIGURE 4. Variations of $f'(\eta)$ against η for several values of stretching parameter λ and suction parameter s when $a = 0.5$: reverse flow solutions.

CONCLUSION

The problem of unsteady separated stagnation-point flow over a permeable stretching sheet was studied theoretically. A similarity transformation was employed to reduce the governing partial differential equations into a system of ordinary differential equations, before being solved numerically. It is found that dual solutions exist in stretching case, one exhibits reattached flow solution while the other one is reverse flow solution. As expected, the effect of suction is to decrease the velocity of fluid, hence make the flow controllable.

ACKNOWLEDGMENTS

The first and second authors wish to thank the Department of Higher Education, Ministry of Education Malaysia for the financial support received under the Fundamental Research Grant Scheme (Project Code: 203/PJJAUH/6711293). The support provided by the Universiti Sains Malaysia is also acknowledged.

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