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Unsteady Separated Mixed Convection Stagnation-Point Flow: The Case of Assisting Flow

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ABSTRACT

The problem of unsteady separated stagnation-point flow in mixed convection is studied. Only the case of assisting flow is considered here. By employing similarity transformation, the governing partial differential equations are reduced to a system of ordinary differential equations which are then solved numerically using bvp4c Matlab program. Comparison with results for the steady-state case from the open literature shows good agreement; therefore the bvp4c is able to give accurate results. The effects of governing parameters on the flow and heat transfer characteristics are presented.

Keywords: Mixed Convection, Unsteady Separated Stagnation-Point Flow, Assisting Flow.

1. Introduction

The study of unsteady boundary layer flow has gained considerable attention due to the fact that many boundary layers, which occur in practice, are unsteady (Riley, 1990). One of the important features of unsteady flow is the phenomenon of separation. Boundary layer separation occurs when the portion of the boundary layer that close to the wall reverses in flow direction. The unsteadiness in the flow field increases the dimension of the problem by one and thus increases the complexity of the problem. Stewartson (1960), Stuart (1964, 1971), Riley (1990), Telionis (1981), Wang (1989) and Ingham (1984) have concisely reviewed the main ideas and important contributions on the topic of unsteady flow and heat transfer.

Using the method of Lie group transformations, Ma and Hui (1990) derived all possible group-invariant similarity solutions for the unsteady two-dimensional laminar boundary layer equations. These results are shown to include all the existing solutions as special cases. A detailed analysis had been given to several classes of solutions which are also solutions to the Navier-Stokes equations and which exhibit flow separation, therefore the solutions remain valid in the boundary layer region even when flow separation or reversal occurs.

The aim of the present paper is to extend the problem studied by Ma and Hui (1990) to the case of unsteady mixed convection flow past a vertical heated surface. It is worth mentioning to this end that an analysis of the unsteady convective problems is important since these have wide applications in nuclear reactor technology, floated gyrocompasses and liquid heat sink cooling of re-entry vehicles (Slaouti *et al.*, 1998).

2. Basic Equations

Consider the unsteady two-dimensional stagnation-point flow of a viscous and incompressible fluid over a vertical flat plate in mixed convection.

2.1 Governing Equations

Following Ma and Hui (1990), it is assumed that the free stream velocity is $u_e(x,t) = a(x/t)$, where a is a positive constant known as acceleration parameter, t is the time and x is the axis measured along the plate in the vertical direction. It is also assumed that the surface temperature is $T_w(x,t) = T_\infty + T_0(x/t^2)$, where T_∞ is the constant temperature of the ambient fluid and T_0 is a characteristic temperature with $T_0 > 0$ for a heated plate (assisting flow) and $T_0 < 0$ for a cooled plate (opposing flow), respectively. Under these assumptions the basic boundary layer equations of this problem can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where y is the coordinate measured in the direction normal to the plate, u and v are the velocity components along the x– and y– axes, g is the acceleration due to gravity, α is the thermal diffusivity, β and ν is the kinematic viscosity of the fluid. The initial and boundary conditions of these equations are

$$\begin{aligned} &t \le 0:\ u = v = 0, \qquad T = T_\infty \quad \text{for any } x, y \\ &t > 0:\ u = v = 0,\ T = T_w(x,t) = T_\infty + T_0(x/t^2) \quad \text{at } y = 0 \\ &\qquad u = u_e(x,t) = a(x/t),\ T = T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

2.2 ~~Non~~similar Transformation

It is assumed that Eqs. (1) to (3) subject to the boundary conditions (4) admit the similarity solution

$$u = a(x/t)f'(\eta), \qquad v = -a\sqrt{\nu/t}f(\eta),$$

$$\theta(\eta) = (T - T_{\infty})/(T_w - T_\infty), \qquad \eta = y\sqrt{\nu/t} \tag{5}$$

Substituting (5) into Eqs. (2) and (3), we get the following ordinary differential equations

$$f''' + a(ff'' - f'^2 + 1) + \left(\frac{\eta}{2}f'' + f' - 1\right) + \lambda\theta = 0 \tag{6}$$

$$\frac{1}{Pr}\theta'' + a(f\theta' - f'\theta) + \left(\frac{\eta}{2}\theta' + 2\theta\right) = 0 \tag{7}$$

subject to the boundary conditions

$$f(0) = 0, \qquad f'(0) = 0, \qquad \theta(0) = 1$$
$$f' \to 1, \qquad \theta \to 0 \qquad \text{as } \eta \to \infty \tag{8}$$

where primes denote differentiation with respect to η. Here Pr = ν/α is the Prandtl number and λ is the constant mixed convection parameter, which is defined as

$$\lambda = \frac{Gr_x}{Re_x^2} \tag{9}$$

Where $Gr_x = g\beta(T_w - T_\infty)x^3/\nu^2$ is the local Grashof number and $Re_x = u_e x/\nu$ is the local Reynolds number. Here, $\lambda > 0$ corresponds to assisting flow while $\lambda < 0$ is associates with opposing flow, respectively.

2.3 Steady-State form of Equations

In order that we validate the present numerical code for solving Eqs. (6) and (7), we consider the steady-state form of these equations. In this respect, we need to consider the following similarity variables

$$u = axf'(\eta), \qquad v = -\sqrt{a\nu}f(\eta),$$

$$\theta(\eta) = (T - T_{\infty})/(T_w - T_\infty), \qquad \eta = y\sqrt{a/\nu} \tag{10}$$

so that the corresponding steady-state equations of this problem are

$$f''' + ff'' - f'^2 + 1 + \lambda\theta = 0 \tag{11}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta = 0 \tag{12}$$

subject to the same boundary conditions Eq. (8). Equations (11) and (12) subject to the boundary conditions (8) are exactly Eqs. (17) to (19) of the paper by Ramachandran *et al*. (1988), when *n* = 1.

3. Numerical Method

The ordinary differential equations (6) and (7), equations (11) and (12), together with the boundary conditions (8) have been solved numerically using the program bvp4c in Matlab. bvp4c is a finite difference method codes that implements the three-stage Lobatto IIIa formula, which is a collocation method with forth-order accuracy. In this approach, the differential equations are first reduced to a system of first-order equations by introducing new variables. The mesh selection and error control are based on the residual of the continuous solution. In this study we set the relative error tolerance to 10^{-7}. The examples of solving boundary value problems with bvp4c can be found in a book by Shampine *et al*. (2003).

4. Results and Discussion

For comparison purpose, the values of the reduced skin friction $f''(0)$ and the reduced heat flux $-\theta'(0)$ for steady case given by Equations (11), (12) and boundary conditions (8) are compared with those given in the paper by Ramachandran *et al*. (1988) and Lok *et al*. (2005). The comparison is shown in Table 1 and it is seen that the results are in excellent agreement. We are, therefore, confident that the present results are accurate.

Table 1 Comparison of values of $f''(0)$ and $-\theta'(0)$ for some values of Pr with published results when the flow is buoyancy assisting ($\lambda = 1$).

Pr	$f''(0)$		$-\theta'(0)$	
	Present method	Other methods	Present method	Other methods
0.7	1.70632	(1.7063) [1.706376]	0.76406	(0.7641) [0.764087]
7	1.51791	(1.5179) [1.517952]	1.72238	(1.7224) [1.722775]
20	1.44848	(1.4485) [1.448520]	2.45759	(2.4576) [2.458836]
60	1.39027	(1.3903) [1.390311]	3.55141	(3.5514) [3.555404]
100	1.36824	(1.3680) [1.368070]	4.21189	(4.2116) [4.218462]

Note: Present method – bvp4c function in Matlab

( ) – Results from Ramachandran et al. (1988) – Runge-Kutta with shooting

[ ] – Results from Lok *et al*. (2005) – Keller box method

Table 2 shows the initial values $f''(0)$ and $-\theta'(0)$ for some values of governing parameters. Multiple solutions are found for the case when $\lambda > 0$ and $a > 0$. The values of $f''(0)$ increases as λ and a increase. On the other hand, the values of $|-\theta'(0)|$ decreases when λ is increased; however the absolute value increases as a increases except for the third solution.

The velocity and temperature profiles for $a = 1$, Pr = 0.7 and some positive values of mixed convection parameter λ (assisting flow case) are presented in Figs. 1 and 2. Three different solutions are obtained and all profiles satisfy the boundary conditions (8) asymptotically. For the first solution, the velocity profiles overshoot when λ increases. Reverse flows are observed for the second and the third solutions. In Fig. 2, the temperature decreases as λ increases for the 1st and the 2nd solutions, however, opposite trend is observed for the 3rd solution.

Figures 3 and 4 shows the variation of velocity and temperature as a function of η for fixed Pr and λ, and some values of acceleration parameter a. It is found that the velocity boundary layer thickness increases as a increases but the thermal boundary layer thickness decreases as a increases. The temperature profiles for the 2nd solution have negative gradient near to the plate before they attain the far field boundary condition. The temperature for the 3rd solution are positive near to the plane but reduced to negative values when far from the plate and finally ended at zero value.

Table 2 Initial values of $f''(0)$ and $-\theta'(0)$ for Pr = 0.7 when λ = 0.5, 1, 1.5 and a = 1, 2, 3.

		1st solution		2nd solution		3rd solution	
		$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$a = 1$	$\lambda = 0.5$	1.32339	0.18124	–0.18578	–2.42841	0.24511	–1.98905
	$\lambda = 1.0$	1.65769	0.007009	0.00882	–1.01328	0.89350	–1.42425
	$\lambda = 1.5$	1.95676	–0.01149	0.34422	–0.49737	1.25474	–0.98723
$\lambda = 1.0$	$a = 1$	1.65769	0.07009	0.00882	–1.01328	0.89350	–1.42425
	$a = 2$	1.95991	–0.50487	0.18509	–2.94284	1.47459	0.88670
	$a = 3$	2.29242	–0.83499	0.37493	–4.81654	1.83050	0.21133

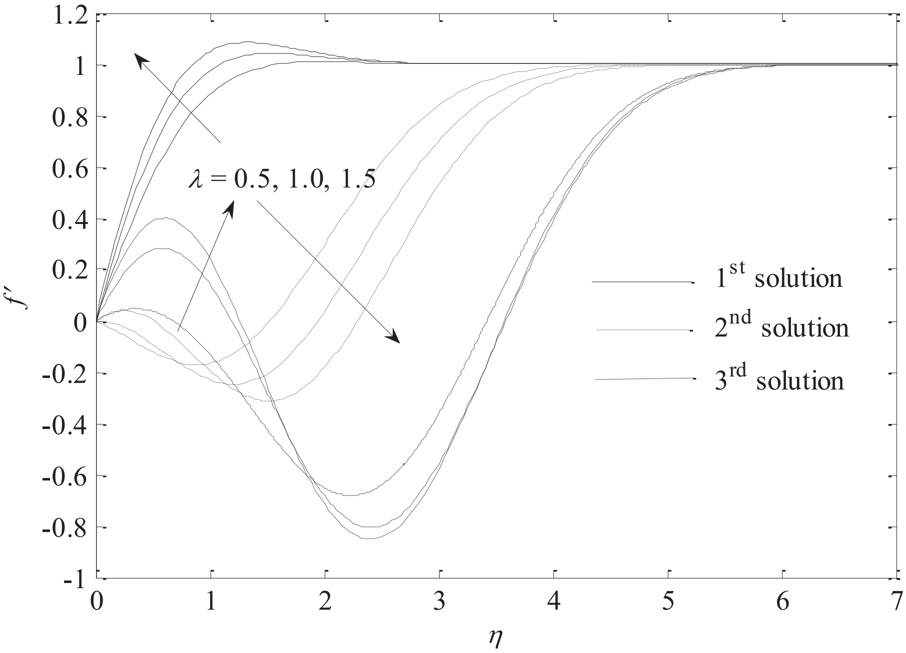

Figure 1 Velocity profiles for a = 1, Pr = 0.7 and different values of λ.

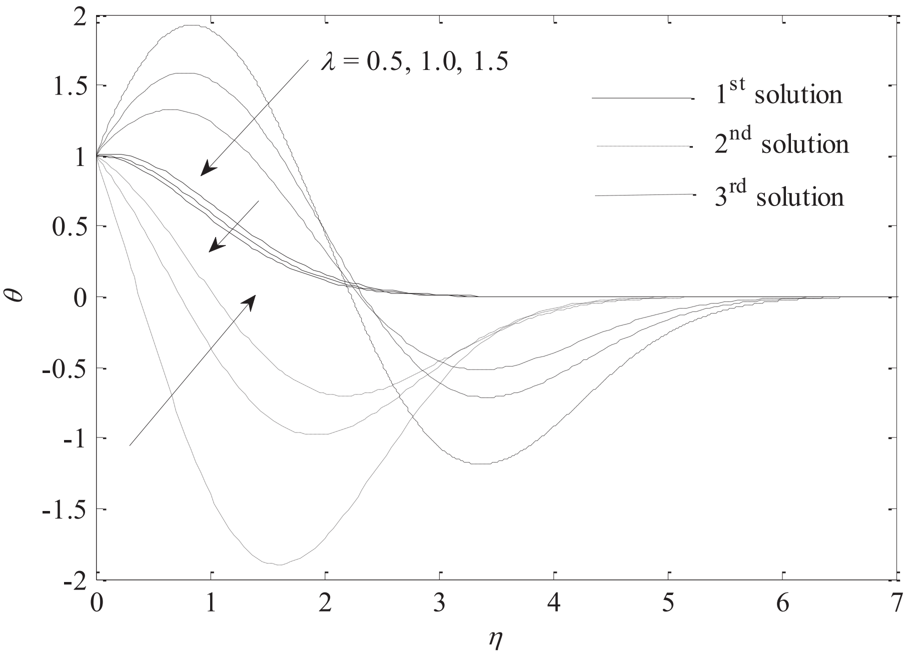

Figure 2 Temperature profiles for a = 1, Pr = 0.7 and different values of λ.

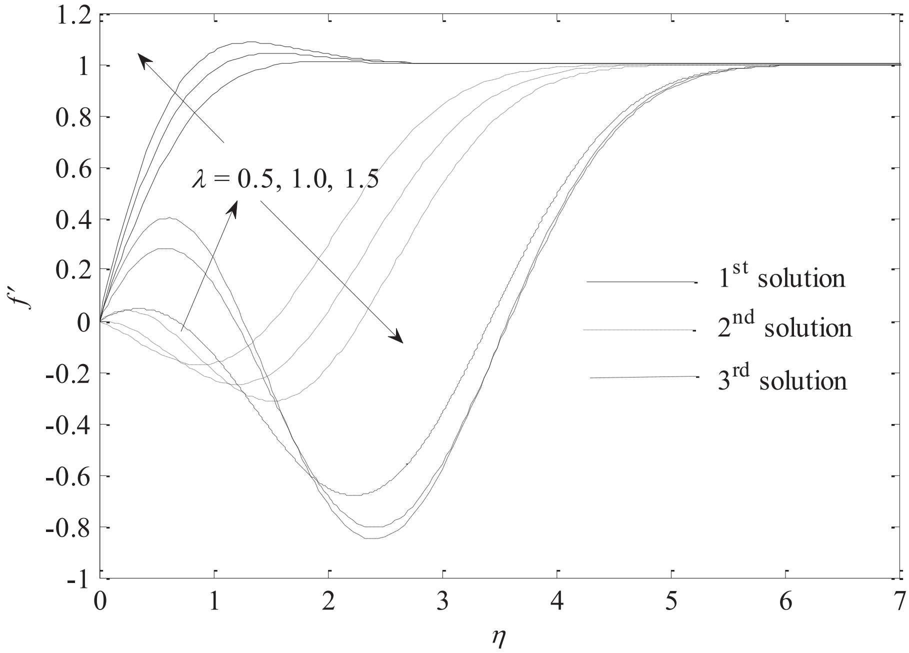

Figure 3 Velocity profiles for $\lambda = 1$, Pr = 0.7 and different values of *a*.

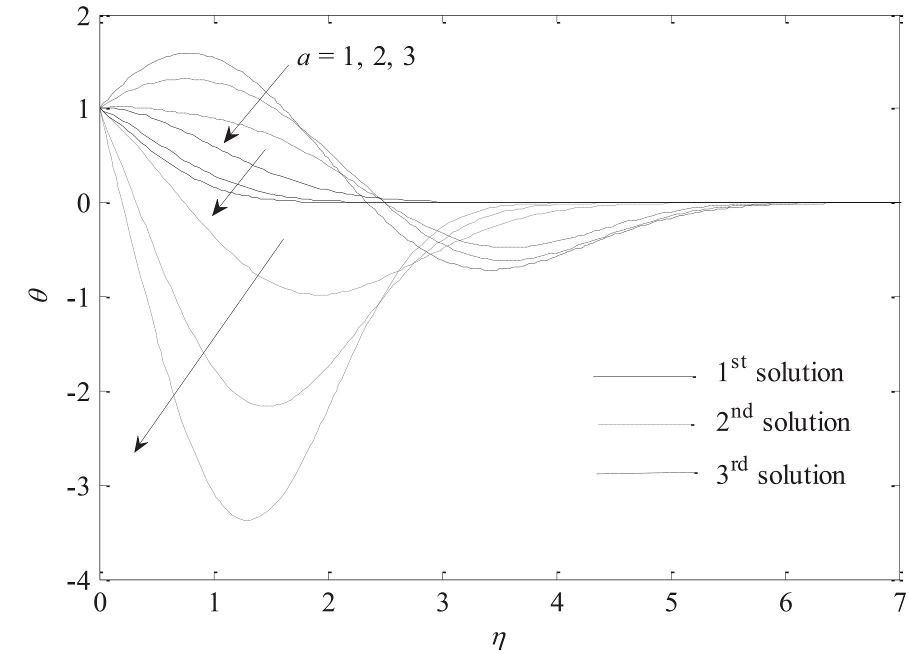

Figure 4 Temperature profiles for $\lambda = 1$, Pr = 0.7 and different values of *a*.

5. Conclusion

We have studied the problem of unsteady separated mixed convection near the stagnation-point over a vertical flat plate. Only the case of assisting flow ($\lambda > 0$) and positive acceleration parameter ($a > 0$) are considered here. According to Ma and Hui

(1990), the case $a > 1$ exhibits flow reattachment, therefore multiple solutions are observed in this study. Stability analysis is needed to investigate the nature of these three solutions.

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