

---

UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2015/2016 Academic Session

June 2016

**EEE 377 – DIGITAL COMMUNICATIONS**  
**[PERHUBUNGAN DIGIT]**

Duration 3 hours

*[Masa : 3 jam]*

---

Please check that this examination paper consists of **TWENTY ONE (21)** pages and Appendices **SIX (6)** pages of printed material before you begin the examination. This examination paper consist of two versions, The English version and Malay version. The English version from page **TWO (2)** to page **ELEVEN (11)** and Malay version from page **TWELVE (12)** to page **TWENTY ONE (21)**.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH SATU (21)** muka surat dan Lampiran **ENAM (6)** muka surat bercetak sebelum anda memulakan peperiksaan ini. Kertas peperiksaan ini mengandungi dua versi, versi Bahasa Inggeris dan Bahasa Melayu. Versi Bahasa Inggeris daripada muka surat **DUA (2)** sehingga muka surat **SEBELAS (11)** dan versi Bahasa Melayu daripada muka surat **DUA BELAS (12)** sehingga muka surat **DUA PULUH SATU (21)**.*

**Instructions:** This question paper consists of **SIX (6)** questions. Answer **FIVE (5)** questions. All questions carry the same marks.

***[Arahan:** Kertas soalan ini mengandungi **ENAM (6)** soalan. Jawab **LIMA (5)** soalan. Semua soalan membawa jumlah markah yang sama]*

Begin your answer to each question on a new page.

*[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru]*

**“In the event of any discrepancies, the English version shall be used”.**

***[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai]***

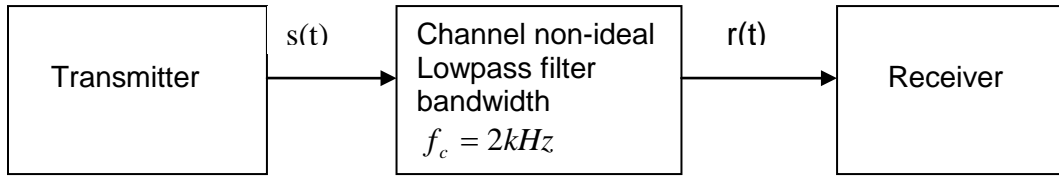
**ENGLISH VERSION**

1. (a) A Fourier series trigonometric representation of signal  $s(t)$  is given as follows;

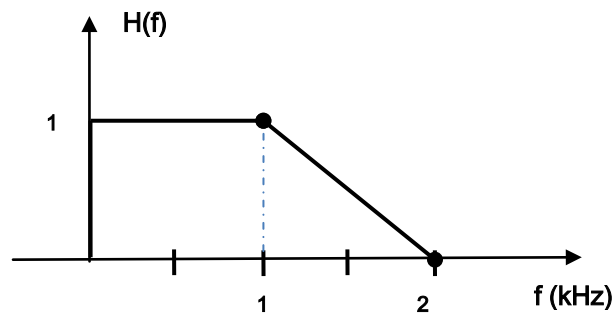
$$\begin{aligned} s(t) = & 0.6 + 0.490 \cos(400\pi t) + 0.796 \sin(400\pi t) - 0.058 \cos(800\pi t) + 0.398 \sin(800\pi t) \\ & + 0.039 \cos(1200\pi t) + 0.265 \sin(1200\pi t) - 0.122 \cos(1600\pi t) + 0.199 \sin(1600\pi t) \\ & + 0.082 \cos(2400\pi t) + 0.133 \sin(2400\pi t) - 0.017 \cos(2800\pi t) + 0.114 \sin(2800\pi t) \\ & + 0.014 \cos(3200\pi t) + 0.099 \sin(3200\pi t) - 0.054 \cos(3600\pi t) + 0.088 \sin(3600\pi t) \end{aligned}$$

Based on the above equation;

- (i) Find the period of  $s(t)$ . (10 marks)
- (ii) Find the expression of one-sided form for the first 10 harmonic components of  $s(t)$ . (20 marks)
- (iii) Draw the single-sided magnitude spectrum of the first 10 harmonic components of  $s(t)$ . (20 marks)
- (iv) Draw the single-sided average normalized power spectrum of the first 10 harmonic components of  $s(t)$ . (20 marks)
- (b) A communication system is designed as shown in Figure 1(a) attempts to transmit a periodic signal  $s(t)$  above via a channel that acts as a non-ideal low pass filter with cutoff frequency is 2kHz. The transfer function (frequency response) of the filter is as shown in Figure 1(b).



**Figure 1(a):** Communication system with non-ideal lowpass filter



**Figure 1(b):** Frequency response of the low pass filter

- (i) Examine your answer in part (a)(iv), then calculate the average normalized power of  $s(t)$  that can pass through the channel. (25 marks)
- (ii) Suggest on how to increase accuracy of the system. (5 marks)

2. (a) A PAM system using the sinc-shaped pulse transmits binary stream via a baseband channel.

(i) If accuracy requirement dictate that 100% of the signal's average power spectral density must be within the bandwidth channel of 50kHz. Determine the maximum transmission speed of the system.

(20 marks)

(ii) If accuracy requirement dictate that 90% of the signal's average power spectral density must be within the bandwidth channel. Determine the bandwidth of the channel, if the transmission speed is 50,000 bits/sec.

(20 marks)

(b) Figure 2(a) below show the sample and hold receiver of the PAM baseband transmission receiver. The received signal is sampled at the center of bit period

$t_0 = (i-1)T_b + \frac{T_b}{2}$  and the sample is compared to a 0V threshold for decision of

the demodulated binary data. The received signal is corrupted by noise in the channel which has uniform probability density function as shown in Figure 2(b).

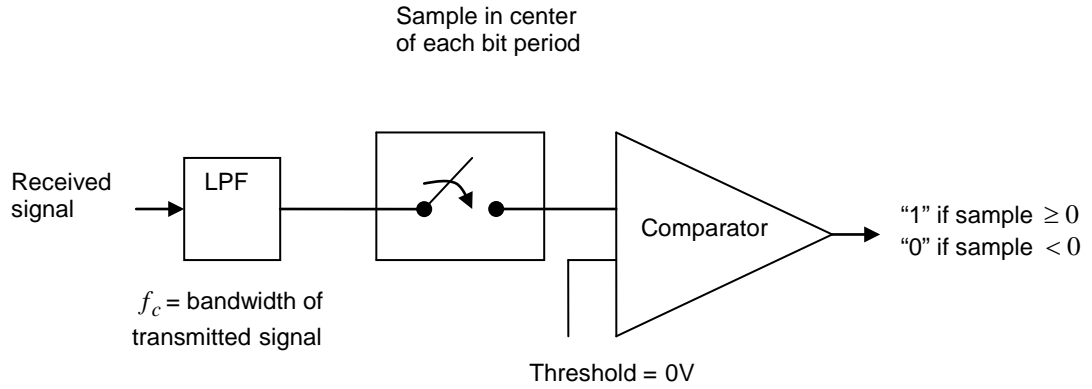


Figure 2(a): Sample and hold receiver for PAM baseband transmission

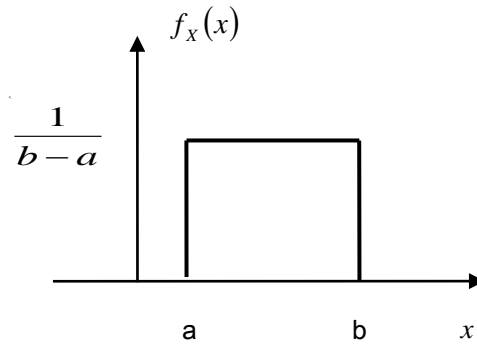


Figure 2(b): pdf of uniform distribution of noise

$$f_X(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for } x > b \end{cases}$$

where  $a$  and  $b$  are always positive number.

Based on the Figure 2(a) above;

- (i) Explain the use of lowpass filter in the receiver design. (10 marks)
- (ii) Explain the nature of the signal after the lowpass filter. (10 marks)
- (iii) Starting from the fact that the probability of bit error;

$$P_b = \frac{1}{2} P\{n_o(t_o) < -\gamma A\} + \frac{1}{2} P\{n_o(t_o) \geq \gamma A\}$$

where the  $\gamma A$  is the term to indicated the attenuated received signal.

Show the probability of bit error  $P_b$  ;

$$P_b = \frac{1}{2} \int_{\gamma A}^{\infty} \left( \frac{1}{b-a} \right) dx \quad \text{for } a \leq \gamma A \leq b$$

(30 marks)

[Hint: Use Appendix for appropriate useful relations]

- (iv) A communication system transmits the PAM signal with 4V pulse and the channel is assumed to have the uniform distribution noise as in Figure 2(b). By the time the signal arrive at the receiver, the voltage is only 20% of the transmitted signal. Calculate the probability of bit error  $P_b$  where  $a$  and  $b$  are 0.2 and 1.2 respectively.

(10 marks)

3. (a) Bandpass modulation is the technique that enables data to be transmitted through a bandpass channel band limited by  $f_1$  and  $f_2$ , using a carrier signal such as  $\cos(2\pi f_c t)$ . A Phase Shift Keying (PSK) modulation technique is used to modulate a binary sequence that should be transmitted with bit rate  $r_b = 50,000 \text{ bits/sec}$ . Given that bandpass frequencies are  $f_1 = 100 \text{ kHz}$  and  $f_2 = 300 \text{ kHz}$  and the carrier frequency is  $f_c = 200 \text{ kHz}$ .

(i) Draw the modulated signal  $S_{PSK}(t)$  with timing sequence of binary 10101 and indicate the number of cycles per bit period.

(20 marks)

(ii) Give the mathematical expression of  $S_{PSK}(t)$  signal in terms of baseband and carrier signal and draw the corresponding baseband signal.

(10 marks)

(iii) Draw the corresponding average normalized power spectral density  $G_{PSK}(f)$  and indicate for 90% and 95% of the in-band power.

(10 marks)

(b) Suppose you are going to design a PSK bandpass communication system with the following requirements;

- accuracy is 90%
- bandpass channel with  $f_1 = 100 \text{ kHz}$  and  $f_2 = 300 \text{ kHz}$
- communication is supposed to fully utilize the bandwidth of its bandpass channel.
- carrier frequency is  $f_c = 200 \text{ kHz}$ .
- Sequence =

...8/-

- (i) Draw a diagram of the communications system (with a transmitter and receiver) and indicate the following terms on the diagram the followings; Channel (spectrum of the bandpass channel), source binary data, transmitted signal  $S_{PSK}(t)$  (just this symbol), transmission rate (require to be calculated).

(20 marks)

- (ii) Suppose that the accuracy is changed to 95%, calculate the transmission speed and comment on the different values obtained as compared in part (b)(i).

(10 marks)

- (c) A PSK communication system transmits 100,000 bits/sec using signal with a peak amplitude of 0.1V. By the time the signal arrives at the receiver, its voltage is only 40% of the transmitted voltage. The noise at the input to the receiver is Gaussian noise with average normalized power spectral density as shown in Figure 3 where the constant  $\frac{N_o}{2} = 1.6 \times 10^{-9} \frac{\text{volts}^2}{\text{Hz}}$ . Assuming the system is perfectly synchronized;

- (i) Discuss whether the receiver is symmetry or asymmetry. Show your appropriate steps for the conclusion.

(10 marks)

- (ii) Calculate the probability of bit error of the corresponding receiver.

(20 marks)



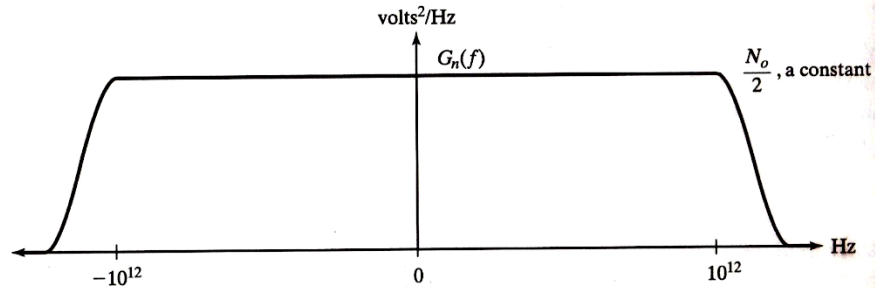


Figure 3 : Average normalized power spectrum density of noise at the input of the receiver

4. (a) It is stated that the minimum Hamming Distance is the same as the minimum Hamming Weight. Do you agree with this statement, and if so, discuss the reason behind it. (20 marks)
- (b) State your understanding of Generator Matrix and describe its implementation in coding. Use (7,4) Hamming Code as an example. (20 marks)
- (c) A differential pulse code modulation (DPCM) yields 4-bit words as output, which are to be protected against random single errors.
- (i) Determine the number of parity bits needed if block coding is employed. (10 marks)

- (ii) Determine the generator matrix and parity matrix. (40 marks)
- (iii) Build the parity check matrix, H (10 marks)

5. (a) A generator matrix for generating linear codes is as follows:

- (i) Determine the message length for this code generator, and the code length. (10 marks)
- (ii) Determine the codewords for the possible messages. (30 marks)
- (iii) Determine the minimum distance. (15 marks)

(b) From generator matrix in Q5 (a), determine the parity check matrix. If a received code is 1000111, find the corrected message. (45 marks)

6. (a) An instantaneously adaptive delta modulator adapts its step size exponentially, i.e., if the predictor is unable to track the input the step size is doubled. Thus, the steps generated have the values given by  $s_n = 2^n$ , where  $n = 1, 2, 3, \dots, 10$ .

Determine the entropy of the step generator if the probability of a step,  $s_n$  is  $P(s_n) = 1/s_n$  for  $n = 1, 2, \dots, 9$  and  $P(s_{10}) = P(s_9)$ .

(40 marks)

(b) A digital data stream is encoded with (15, 11) block Hamming code with the capability to correct all single errors. The coded data is PSK modulated and transmitted through a AWGN channel.

(i) Determine the generator matrix by using the parity check equations.

(50 marks)

(ii) A message word 00001000011 is ready to be transmitted, determine its correct codeword at the output.

(10 marks)

**VERSI BAHASA MALAYSIA**

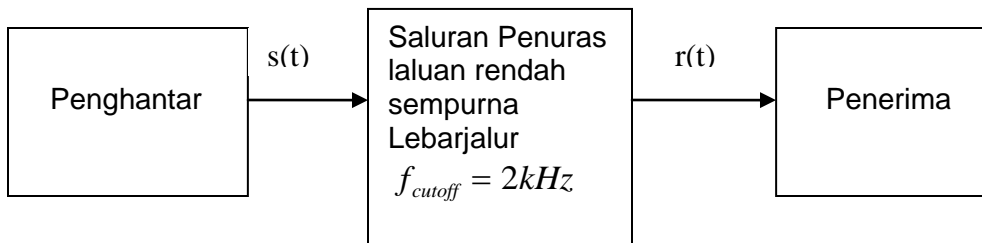
1. (a) Satu seri Fourier dalam bentuk trigonometri untuk isyarat  $s(t)$  adalah diberikan seperti berikut;

$$\begin{aligned} s(t) = & 0.6 + 0.490 \cos(400\pi t) + 0.796 \sin(400\pi t) - 0.058 \cos(800\pi t) + 0.398 \sin(800\pi t) \\ & + 0.039 \cos(1200\pi t) + 0.265 \sin(1200\pi t) - 0.122 \cos(1600\pi t) + 0.199 \sin(1600\pi t) \\ & + 0.082 \cos(2400\pi t) + 0.133 \sin(2400\pi t) - 0.017 \cos(2800\pi t) + 0.114 \sin(2800\pi t) \\ & + 0.014 \cos(3200\pi t) + 0.099 \sin(3200\pi t) - 0.054 \cos(3600\pi t) + 0.088 \sin(3600\pi t) \end{aligned}$$

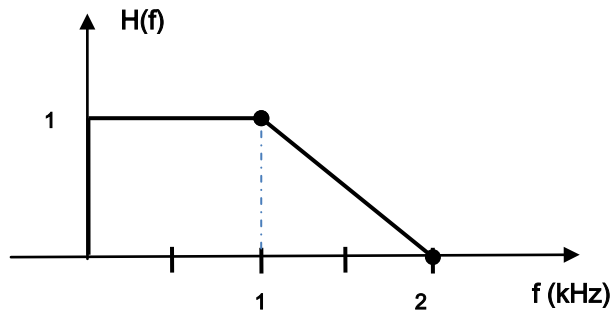
Bergantung kepada persamaan di atas;

- (i) Cari kala bagi  $s(t)$ . (10 markah)
- (ii) Cari persamaan sebelah untuk 10 komponen harmonik pertama bagi  $s(t)$ . (20 markah)
- (iii) Lakarkan magnitude spectrum sebelah bagi 10 harmonik pertama bagi isyarat  $s(t)$ . (20 markah)
- (iv) Lakarkan spektrum purata kuasa ternormal belah bagi 10 harmonik pertama bagi isyarat  $s(t)$ . (20 markah)

- (b) Sebuah sistem komunikasi direkapipta seperti dalam Rajah 1(a) cuba menghantar isyarat seperti Jadual 1 di atas melalui saluran yang berfungsi sebagai penapis laluan rendah yang tidak unggul yang mempunyai frekuensi potong adalah 2 kHz. Pemandahan fungsi sambutan frekuensi penuras adalah seperti yang ditunjukkan dalam Rajah 1(b).



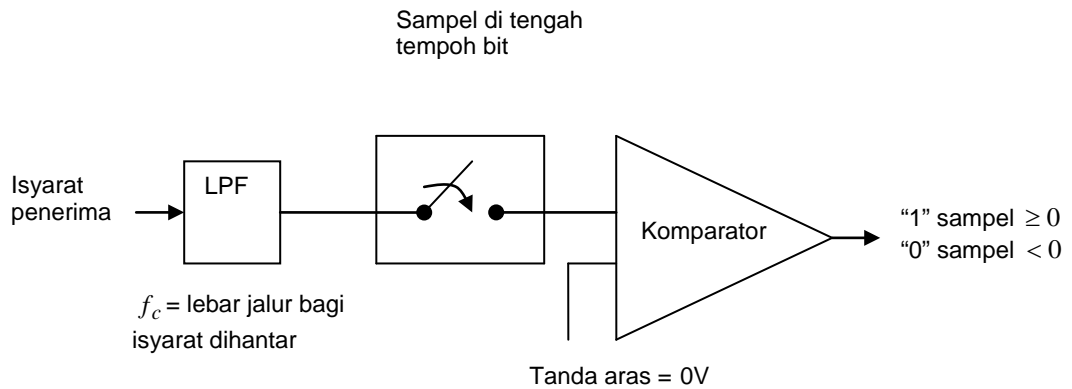
**Rajah 1(a):** Sistem komunikasi dengan penapis laluan rendah tidak unggul



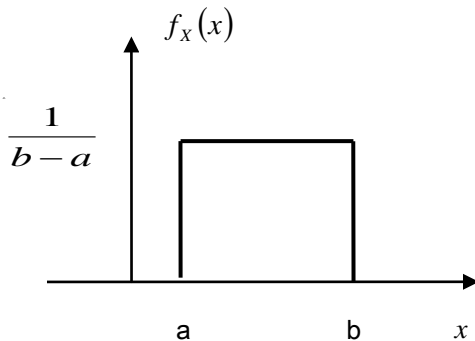
**Figure 1(b):** Sambutan frequensi penapis laluan rendah tidak unggul

- (i) Dengan memeriksa jawapan anda dalam bahagian a) iv, kirakan purata kuasa ternormal yang boleh melalui saluran.  
(25 markah)
  - (ii) Bincangkan bagaimana ketepatan dapat ditingkatkan.  
(5 markah)
  
- 2. (a) Sebuah Sistem PAM yang menggunakan denyut berbentuk sinc menghantar data binari melalui saluran jalur asas.
  - (i) Sekiranya ketepatan yang diperlukan adalah 100% untuk purata kuasa ketumpatan spektra bagi isyarat itu mesti berada dalam saluran lebar jalur adalah 50kHz. Tentukan kelajuan maksima penghantaran sistem tersebut.  
(20 markah)
  - (ii) Sekiranya ketepatan yang diperlukan adalah 90% untuk purata kuasa ketumpatan spektra bagi isyarat itu mesti berada dalam saluran lebar jalur. Cari lebar jalur untuk saluran, sekiranya kelajuan penghantaran adalah 50,000 bits/sec.  
(20 markah)

- (b) Rajah 2(a) berikut menunjukkan satu penerima sampel dan pegang untuk penghantaran jalur asas PAM. Isyarat yang diterima disampel pada pertengahan tempoh bit  $t_0 = (i - 1)T_b + \frac{T_b}{2}$  dan sampel dibandingkan dengan penanda aras 0V untuk membuat keputusan bagi proses penyahmodulatan perduaan data. Isyarat yang diterima dirosakkan oleh hingar dalam saluran yang mempunyai fungsi ketumpatan kebarangkalian seragam seperti dalam Rajah 2(b).



Rajah 2(a): Penerima sampel dan pegang untuk penghantaran PAM



Rajah 2(b): pdf bagi fungsi ketumpatan kebarangkalian seragam hingar

$$f_X(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for } x > b \end{cases}$$

Di mana  $a$  dan  $b$  sentiasa bernombor positif.

Berdasarkan Rajah 2(a) di atas;

- (i) Terangkan kegunaan penuras laluan rendah dalam rekabentuk penerima.

(10 markah)

- (ii) Terangkan keadaan isyarat selepas penuras laluan rendah.

(10 markah)

- (iii) Bermula daripada fakta bahawa kebarangkalian ralat bit;

$$P_b = \frac{1}{2} P\{n_o(t_o) < -\gamma A\} + \frac{1}{2} P\{n_o(t_o) \geq \gamma A\}$$

Di mana  $\gamma A$  mewakili isyarat penerima dikurangkan.

Tunjukkan kebarangkalian ralat bit  $P_b$ ;

$$P_b = \frac{1}{2} \int_{\gamma A}^{\infty} \left( \frac{1}{b-a} \right) dx \quad \text{for } a \leq \gamma A \leq b$$

(30 markah)

[Petunjuk: Gunakan lampiran untuk persamaan yang sesuai]



- (iv) Sebuah sistem komunikasi menghantar isyarat PAM dengan denyut 4V dan melalui saluran yang mempunyai fungsi ketumpatan kebarangkalian seragam seperti dalam Rajah 2(b). Bila isyarat tiba di penerima, voltannya adalah hanya 20% dari isyarat yang di hantar. Kirakan kebarangkalian ralat bit  $P_b$ , dimana parameter  $a$  dan  $b$  adalah 0.2 and 1.2 setiap satu.

(10 markah)

3. (a) Permodulatan laluanlulus adalah teknik yang membolehkan data yang dapat dihantar melalui jalur saluran laluanlulus yang dihadkan oleh \_\_\_\_\_, dengan menggunakan isyarat pembawa seperti \_\_\_\_\_. Permodulatan fasa beralih penaipan (PSK) digunakan untuk memodulatkan data binari dengan kadar penghantaran  $r_b = 50,000 \text{ bits/sec}$ . Diberikan bahawa frekuensi jalur lulus adalah  $f_1 = 100 \text{ kHz}$  and  $f_2 = 300 \text{ kHz}$  dan frekuensi pembawa adalah  $f_c = 200 \text{ kHz}$ .

- (i) Lakarkan isyarat  $S_{PSK}(t)$  dengan urutan masa binari \_\_\_\_\_ dan tunjukkan bilangan kitaran setiap tempoh bit.

(20 markah)

- (ii) Berikan persamaan matematik untuk isyarat  $S_{PSK}(t)$  dengan menggunakan isyarat jalur asas dan pembawa dan lukiskan isyarat jalur asas itu.

(10 markah)

(iii) Lakarkan purata ketumpatan kuasa ternormal spektra  $G_{PSK}(f)$  dan tunjukkan 90% and 95% kuasa dalam julat.

(10 markah)

(b) Anda akan merekabentuk satu sistem komunikasi PSK jalur lurus dengan keperluan berikut;

- Ketepatan adalah 90%
- Saluran jalur lurus dengan  $f_1 = 100 \text{ kHz}$  and  $f_2 = 300 \text{ kHz}$
- Komunikasi sepatutnya menggunakan lebar jalur sepenuhnya
- Frekuensi pembawa adalah  $f_c = 200 \text{ kHz}$ .
- Turutan = .

(i) Lakarkan rajah sistem komunikasi tersebut (dengan penghantar dan penerima) dan tunjukkan dalam rajah perkara berikut; saluran - spektra jalur lurus, binari data, isyarat yang dihantar  $S_{PSK}(t)$  (tunjukkan hanya symbol ini sahaja), kadar penghantaran (perlu dikira).

(20 markah)

(ii) Sekiranya ketepatan ditukar kepada 95%, kirakan kadar penghantaran dan beri komen tentang perbezaan nilai yang diperolehi.

(10 markah)

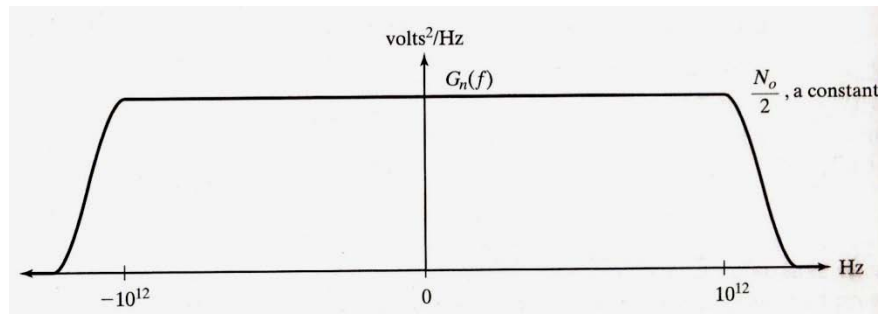
(c) Sebuah sistem komunikasi PSK system menghantar  $r_b = 100,000 \text{ bits/sec}$  menggunakan isyarat puncak yang amplitudnya adalah 0.1V. Bila isyarat ini sampai di penerima, voltannya adalah hanya 40% dari yang telah dihantar. Hingar pada kemasukan di penerima adalah hingar Gaussian dengan purata kuasa ternormal ketumpatan spektra adalah ditunjukkan dalam Rajah 3 di mana nilai tetap  $\frac{N_o}{2} = 1.6 \times 10^{-9} \frac{\text{volts}^2}{\text{Hz}}$ . Anggapkan sistem ini mempunyai kesempurnaan segerak;

(i) Bincangkan sama ada penerima ini adalah penerima simetri atau asimetri. Tunjukkan langkah yang membawa kepada kesimpulan.

(10 markah)

(ii) Kirakan kebarangkalian ralat bit untuk penerima ini.

(20 markah)



Rajah 3 : Purata kuasa ternormal ketumpatan spektra bagi hingar pada kemasukan penerima

4. (a) Diketahui bahawa jarak minimum Hamming adalah sama dengan berat minimum Hamming. Adakah anda setuju dengan kenyataan ini, dan jika ya bincangkan sebabnya.

(20 markah)

...20/-

- (b) Nyatakan pemahaman anda tentang Generator Matrix dan terangkan kaedah implementasinya di dalam kod. Gunakan (7,4) Kod Hamming sebagai contoh.  
(20 markah)
- (c) Satu pemodulatan kod denyut pembeza menghasilkan kata 4-bit sebagai keluaran, yang akan dilindungi daripada satu ralat rawak.
- (i) Tentukan bilangan bit kesetarafan yang diperlukan jika kod blok digunakan.  
(10 markah)
- (ii) Tentukan matrix penjana dan matrix kesetarafan.  
(40 markah)
- (iii) Bina matrix semakan kesetarafan, H.  
(10 markah)
5. (a) Satu penjana matrix bagi menjana kod linear adalah seperti berikut:
- (i) Tentukan panjang mesej bagi penjana kod ini dan panjang kod.  
(10 markah)
- (ii) Tentukan katakod bagi mesej yang berkaitan.  
(30 markah)
- (iii) Tentukan jarak minimum.  
(15 markah)

- (b) Daripada penjana matrix di Q5 (a), tentukan matriks semakan kesetarafan. Jika kod diterima adalah 1000111, carikan mesej yang betul.
- (45 markah)

6. (a) Satu pemodulat delta suai ketika mempunyai saiz langkah secara eksponen, i.e. jika peramal tidak mampu untuk mengesan kemasukan, maka saiz langkah akan digandakan. Oleh itu, langkah yang terhasil mempunyai nilai seperti  $s_n = 2^n$ , di mana  $n = 1, 2, 3, \dots, 10$ .

Tentukan entropi bagi penjana langkah ini jika kebarangkalian bagi langkah,  $s_n$  adalah  $P(s_n) = 1/s_n$  bagi  $n = 1, 2, \dots, 9$  dan  $P(s_{10}) = P(s_9)$ .

(40 markah)

- (b) Satu jurusan data digital dikod dengan (15, 11) kod blok Hamming yang mempunyai kebolehan membetulkan semua ralat tunggal. Data yang telah dikod akan dimodulasikan dengan PSK dan dihantar melalui satu laluan AWGN.

- (i) Tentukan matriks penjana dengan menggunakan persamaan semakan kesetarafan.

(50 markah)

- (ii) Satu kata mesej 00001000011 bersedia untuk dihantar, tentukan katakod yang betul di keluaran.

(10 markah)

**1. Fourier Series (Trigonometric form)**

$S(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t))$	
$f_0 = \frac{1}{T}$	$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} s(t) dt$
$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} s(t) \cos(2\pi n f_0 t) dt$	$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} s(t) \sin(2\pi n f_0 t) dt$

**2. Fourier Series (One-sided form)**

$S(t) = X_0 + \sum_{n=1}^{\infty} X_n \cos(2\pi n f_0 t + \phi_n)$		
$X_0 = a_0$	$X_n = \sqrt{a_n^2 + b_n^2}$	$\phi_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$

**3. Fourier Series (Double-sided form)**

$s(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$	$c_n = \frac{1}{T} \int_{t_0}^{t_0+T} s(t) e^{-j2\pi n f_0 t} dt$
--	---

**4. Conversion between 2-sided to 1-sides Fourier Series**

$n > 0$	$c_n = \frac{1}{2}(a_n - j b_n)$	$c_{-n} = \frac{1}{2}(a_n + j b_n)$	$c_{-n} = c_n^*$
	$ c_n  =  c_{-n}  = \frac{X_n}{2}$	$\angle c_n = \phi_n$	$\angle c_{-n} = -\phi_n$
$n = 0$	$c_0 = a_0 = X_0$		

**5. Fourier Transform and Inverse Fourier Transform**

$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t} df$	$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$
--	---

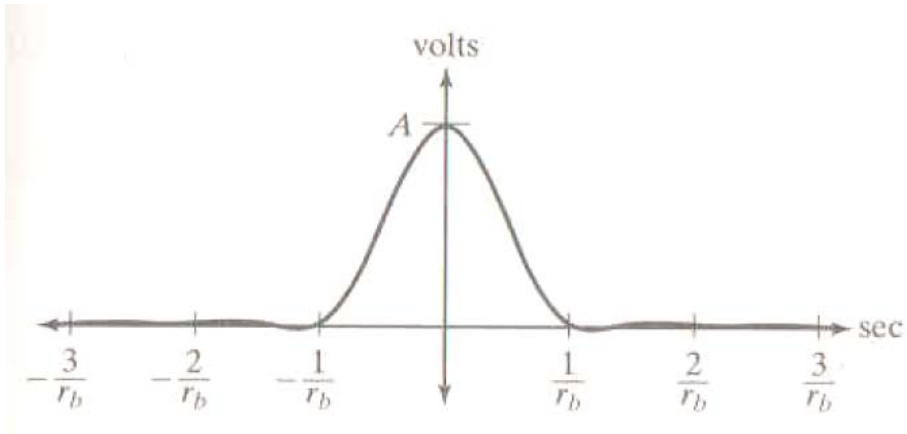
**6. Parseval 's Theorems**

Parseval's Power Theorem	$P_s = \frac{1}{T} \int_{t_0}^{t_0+T} s^2(t) dt = X_0^2 + \sum_{n=1}^{\infty} \frac{X_n^2}{2}$ <p style="text-align: center;"><i>time domain</i>                      <i>frequency domain</i></p>
Parseval's Energy Theorem	$E_s = \int_{-\infty}^{\infty} s^2(t) dt = \int_{-\infty}^{\infty}  S(f) ^2 df$ <p style="text-align: center;"><u><i>time domain</i></u>                      <u><i>frequency domain</i></u></p>

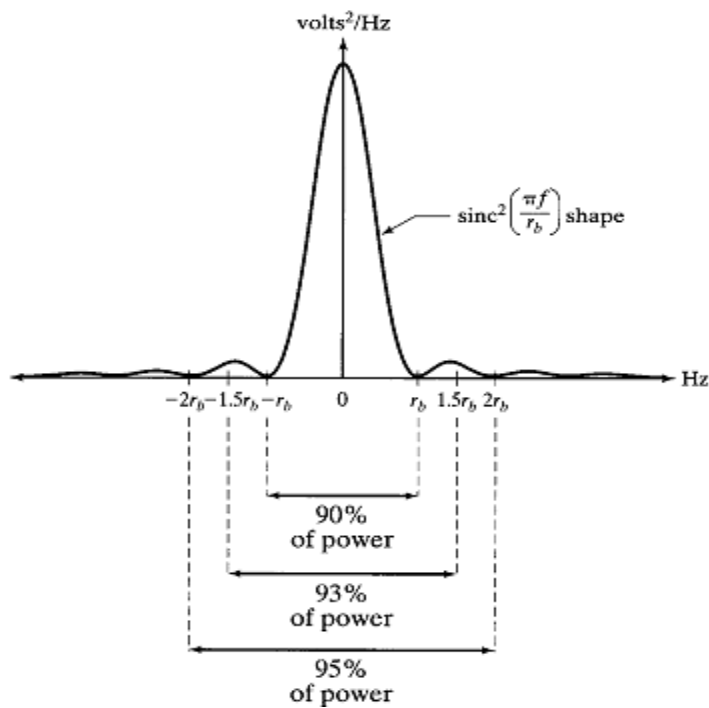
**7. Normalized energy spectral density**

$$\Psi(f) = |S(f)|^2 \left[ \frac{\text{volts}^2}{\text{Hz}^2} \right] \text{ or } \left[ \frac{\text{volts}^2 \text{ - sec}}{\text{Hz}} \right]$$

**8. Raised-cosine shape pulse with roll-off factor  $\alpha=1$**



9. Average normalized power spectral density ( $G(f)$ ) for PAM signal





**10. Stochastic Relations**

i. Probability Distribution Function

$$F_X(a) = P\{X \leq a\} = \int_{-\infty}^a f_X(x) dx$$

ii. Probability Density Function

$$f_X(x) = \frac{d}{dx} F_X(x)$$

iii. Gaussian probability Density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

iv. Other Relationships

$$P\{a < X \leq b\} = P\{X \leq b\} - P\{X \leq a\} = F_X(b) - F_X(a) = \int_a^b f_X(x) dx$$

$$P\{X > c\} = 1 - P\{X \leq c\} = 1 - F_X(c) = \int_c^{\infty} f_X(x) dx = \int_{-\infty}^c f_X(x) dx = \int_c^{\infty} f_X(x) dx$$

**11. Receiver design**

Sample and hold receiver  $(t_0 = (i-1)T_b + \frac{T_b}{2})$	$P_b = Q\left(\frac{\gamma A - \mu}{\sigma}\right)$		
Optimum receiver  $(t_0 = iT_b)$	Symmetric	$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$  $E_b = \gamma^2 \int_{-\infty}^{\infty}  S(f) ^2 df = \gamma^2 \int_{(i-1)T_b}^{iT_b} s^2(t) dt$	$P_b = Q\left(\sqrt{\frac{(\gamma A)^2 T_b}{N_0}}\right)$  Coherent receiver
	Asymmetric	$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$  $E_d = \gamma^2 \int_{(i-1)T_b}^{iT_b}  s_1(t) - s_2(t) ^2 dt$	$P_b = Q\left(\sqrt{\frac{(\gamma A)^2 T_b}{4N_0}}\right)$  Coherent receiver

The Q Function (Gaussian distribution with

$$Q(a) = \int_a^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$Q(a) \cong \frac{1}{a\sqrt{2\pi}} e^{-\frac{a^2}{2}} \text{ for } a \geq 3$$

<i>a</i>	Third Significant Digit									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

<i>Q(a)</i>	<i>a</i>
10 <sup>-4</sup>	3.73
5 × 10 <sup>-5</sup>	3.90
10 <sup>-5</sup>	4.27
5 × 10 <sup>-6</sup>	4.43

<i>Q(a)</i>	<i>a</i>
10 <sup>-6</sup>	4.76
10 <sup>-7</sup>	5.20
10 <sup>-8</sup>	5.61
10 <sup>-9</sup>	6.00

