UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Session

June 2016

EUM 114/3 – KALKULUS KEJURUTERAAN LANJUTAN
[ADVANCED ENGINEERING CALCULUS]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN (7)** pages of printed material before you begin the examination. English version from page **TWO (2)** to page **FOUR (4)** and Malay version from page **FIVE (5)** to page **SEVEN (7)**.
*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat bercetak sebelum anda memulakan peperiksaan ini. Versi Bahasa Inggeris daripada muka surat **DUA (2)** sehingga muka surat **EMPAT (4)** dan versi Bahasa Melayu daripada muka surat **LIMA (5)** sehingga muka surat **TUJUH (7)**.*

Instructions: This question paper consists **FIVE (5)** questions. Answer **ALL** questions. All questions carry the same marks.
*[**Arahan:** Kertas soalan ini mengandungi **LIMA (5)** soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama]*

Answer to any question must start on a new page
[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baharu].

"In the event of any discrepancies, the English version shall be used".
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

ENGLISH VERSION

1. (a) Evaluate the line integral $\int_C (y^2 + 3x^2)dx + 2xydy$ where C is the arc on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ from point (2, 0) to point (0, -1)

(40 marks)

(b) Use Green's theorem to evaluate the integral $\oint_C Pdx + Qdy$ where $P(x, y) = x - y$ and $Q(x, y) = y$, C is the boundary of the region between the x-axis and the graph of $y = sinx$ for $0 \leq x \leq \pi$.

(30 marks)

(c) Evaluate the integral $\iint_R (x + y)dA$ where R is the region bounded by $xy = 4$ and $x + y = 5.$

(30 marks)

2. (a) An experiment was conducted to analyze temperature distribution $T(x)$ at a distance x, measured from one side along a plate having length L. The result is given by the following function

$T(x) = Kx(L - x)$, $0 \leq x \leq L$ where K = constant

Show that the Fourier sine series expansion representing $T(x)$ is

$$\frac{8KL^2}{\pi^3}\left\{\sin\left(\frac{\pi x}{L}\right) + \frac{1}{27}\sin\left(\frac{3\pi x}{L}\right) + \frac{1}{125}\sin\left(\frac{5\pi x}{L}\right) + \cdots\right\}$$

(55 marks)

(b) Consider a function $f(x) = e^x$, $-\pi < x < \pi$.

Find its Fourier series.Given $f(x + 2\pi) = f(x)$

(45 marks)

3. (a) A solid rod of 1 unit length is perfectly insulated at one of its ends (at x = 0), with the other end being subjected to 1 unit temperature. The initial temperature profile is given by $u_o(x,0) = x^2$ for $0 \le x \le 1$, where x is measured from one end of the rod to the other end. The temperature variation in the rod, $u(x,t)$, satisfies the heat equation:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Using numerical method with Δt = 0.01 and Δx = 0.2, obtain the approximate values of $u(x,t)$ at time t = 0.02, giving your answer at 3 decimal points. Given k = 1.

Use the following explicit formula to solve the above heat-conduction equation numerically.

$$u(x_i, t_{j+1}) = \lambda u(x_{i-1}, t_j) + (1 - 2\lambda)u(x_i, t_j) + \lambda u(x_{i+1}, t_j)$$

Where $\lambda = k\frac{\Delta t}{(\Delta x)^2}$

Hints:

The boundary condition at one of the end is $\frac{\partial u}{\partial x}(0, t) = 0$ (where no heat flow at (x = 0). This also means that $u(x_0, t) = u(x_1, t)$, since $\frac{\partial u(0,t)}{\partial x} \approx \frac{u(x_0,t) - u(x_1,t)}{\Delta x} = 0$

(45 marks)

(b) An elastic string is stretched and fixed at two points 2 m apart. The centre of the string is displaced vertically 0.1 m upward from its rest position, and then released with initial zero velocity. Find the solution for this problem by obtaining the displacement $u(x,t)$at any time t with respect to any location x based on its rest position. Use wave equation below with c = 1 to obtain $u(x,t)$.

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

(55 marks)

4. (a) Solve the following equations by Doolittle's method.

$$5x_1 + 4x_2 + x_3 = 6.8$$
$$10x_1 + 9x_2 + 4x_3 = 17.6$$
$$10x_1 + 13x_2 + 15x_3 = 38.4$$

(50 marks)

(b) Determine the inverse of the following matrix using elementary row operations

$$C = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 5 & 0 & -1 \end{bmatrix}$$

(50 marks)

5. (a) Given the vector field $\boldsymbol{F} = x^2\boldsymbol{i} + 2xy\boldsymbol{j} + z^2\boldsymbol{k}$ and the closed curve is a square with vertices at $(0, 0, 3), (1, 0, 3), (1, 1, 3), \text{and } (0, 1, 3)$, verify Stoke's Theorem.

(50 marks)

(b) Use the Gauss-Seidel iterative technique to find approximate solutions to

$$\begin{array}{rrrrcr} 10x_1 & -x_2 & +2x_3 & & = & 6 \\ -x_1 & +11x_2 & -x_3 & +3x_4 & = & 25 \\ 2x_1 & -x_2 & +10x_3 & -x_4 & = & -11 \\ & 3x_2 & -x_3 & +8x_4 & = & 15 \end{array}$$

Starting with $\boldsymbol{X}^{(0)} = [0 \quad 0 \quad 0 \quad 0]^T$ and iterating until

$$\frac{\|x^{(k)} - x^{(k-1)}\|_\infty}{\|x^{(k)}\|_\infty} < 10^{-3}$$

Note: Use rounding to 4 decimal places in all calculations.

(50 marks)

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VERSI BAHASA MELAYU

1. (a) *Cari nilai kamiran garis* $\int_C (y^2 + 3x^2)dx + 2xydy$ *di mana C adalah arka pada elips* $\frac{x^2}{4} + \frac{y^2}{1} = 1$ *dari titik (2, 0) hingga titik (0, -1).*

(40 markah)

(b) *Gunakan teorem Green untuk menilai kamiran* $\oint_C Pdx + Qdy$ *di mana P (x, y) = x-y dan Q(x, y) = y, C adalah sempadan di rantau antara paksi-x dan graf y = sinx untuk* $0 \leq x \leq \pi$ *.*

(30 markah)

(c) *Nilaikan kamiran* $\iint_R (x+y)dA$ *di mana R ialah rantau yang dibatasi oleh xy = 4 dan x + y = 5.*

(30 markah)

2. (a) *Di dalam satu ujikaji untuk menentukan taburan suhu* $T(x)$ *pada jarak* x *yang diukur pada satu hujung plat yang panjangnya* L *. Keputusannya diberi oleh fungsi*

$T(x) = Kx(L-x)$, $\quad 0 \leq x \leq L \quad$ *dengan* K *= pemalar*

Tunjukkan siri Fourier sinus bagi $T(x)$ *ialah*

$$\frac{8KL^2}{\pi^3}\left\{sin\left(\frac{\pi x}{L}\right) + \frac{1}{27}sin\left(\frac{3\pi x}{L}\right) + \frac{1}{125}sin\left(\frac{5\pi x}{L}\right) + \cdots\right\}$$

(55 markah)

(b) *Pertimbangkan suatu fungsi* $f(x) = e^x, \quad -\pi < x < \pi.$

Dapatkan siri Fouriernya. Diberi $f(x + 2\pi) = f(x)$

(45 markah)

3. (a) *Satu rod yang panjangnya 1 unit telah ditebat pada satu hujungnya (pada $x = 0$) dengan satu hujung lagi berada pada suhu 1 unit. Nilai suhu awal diberi oleh persamaan $u_0(x,0) = x^2$ untuk $0 \leq x \leq 1$, dengan x diukur dari satu hujung ke satu hujung rod. Perubahan suhu rod $u(x,t)$memenuhi persamaan haba*

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Dengan menggunakan kaedah berangka, di mana Δt = 0.01 dan Δx = 0.2, dapatkan anggaran $u(x,t)$ pada masa t = 0.02 dengan memberikan jawapan pada 3 titik perpuluhan. Diberi k = 1. Guna rumus berikut untuk menyelesaikan masalah persamaan haba secara berangka.

$$u(x_i, t_{j+1}) = \lambda u(x_{i-1}, t_j) + (1 - 2\lambda)u(x_i, t_j) + \lambda u(x_{i+1}, t_j)$$

Dengan $\lambda = k\frac{\Delta t}{(\Delta x)^2}$

Petunjuk : Syarat sempadan ialah $\frac{\partial u}{\partial x}(0,t) = 0$ *(di mana tiada aliran haba pada (x = 0). Ini juga bermakna bahawa* $u(x_0,t) = u(x_1,t)$*, memandangkan*

$$\frac{\partial u(0,t)}{\partial x} \approx \frac{u(x_0,t) - u(x_1,t)}{\Delta x} = 0$$

(45 markah)

(b) *Seutas tali elastik diregang dan ditetapkan di dua hujung sepanjang 2m. Titik tengah tali tersebut diangkat secara menegak 0.1m daripada titik rehat dan dilepaskan dengan pecutan sifar. Dapatkan penyelesaian bagi masalah nilai awal dengan mencari persamaan $u(x,t)$ pada sebarang masa t dan sepanjang lokasi x dari kedudukan rehat. Guna persamaan gelombang dengan nilai $c = 1$ untuk mendapatkan $u(x,t)$.*

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

(55 markah)

4. (a) *Selesaikan persamaan berikut dengan menggunakan kaedah Doolittle.*

$$5x_1 + 4x_2 + x_3 = 6.8$$
$$10x_1 + 9x_2 + 4x_3 = 17.6$$
$$10x_1 + 13x_2 + 15x_3 = 38.4$$

(50 markah)

(b) *Dapatkan songsangan matrik dengan kaedah operasi baris permulaan.*

$$C = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 5 & 0 & -1 \end{bmatrix}$$

(50 markah)

5. (a) *Diberi vektor* $\boldsymbol{F} = x^2\boldsymbol{i} + 2x\boldsymbol{j} + z^2\boldsymbol{k}$ *dan lengkung tertutup merupakan segi empat sama dengan paksi pada* $(0,0,3),(1,0,3),(1,1,3), dan\ (0,1,3)$, *tentusahkan Teorem Stoke's .*

(50 markah)

(b) *Dengan menggunakan kaedah Gauss- Seidel, selesaikan*

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6 \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25 \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11 \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$

Mulakan dengan nilai awal $\boldsymbol{X}^{(0)} = [0 \quad 0 \quad 0 \quad 0]^T$ *dan lelaran bagi*

$$\frac{\left\| x^{(k)} - x^{(k-1)} \right\|_\infty}{\left\| x^{(k)} \right\|_\infty} < 10^{-3}$$

Nota : Beri jawapan hingga 4 titik perpuluhan.

(50 markah)

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