SHAPE ANALYSIS OF STRUCTURES SUBJECTED TO CONSTRAINTS IN NATURAL VIBRATION FREQUENCIES AND MODE SHAPES

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ABSTRACT

This paper deals with problems in finding the shapes of structures which satisfy the constraints imposed on their natural vibration frequencies and mode shapes. An algorithm which makes use of generalized inverse matrix is adopted. The purpose of this paper is to investigate and verify the applicability of shape finding analysis method using generalized inverse matrix. Governing equations are first presented. This is then followed by description of solution algorithm based on the use of generalized inverse. The equations to be solved are derived from the condition of existence of solutions. An iterative solution algorithm which makes use of Newton-Raphson method is explained. Results of analysis carried out on three examples are then presented and discussed. From the results of analysis, it can be concluded that the applicability of shape finding analysis method using generalized inverse matrix is satisfactory. Problems to be overcome such as shortening of computing time and effect of initial assumed shape on final solution need to be investigated further.

Keywords: *Shape Analysis, Constraints, Natural Vibration Frequencies, Mode Shapes, Generalized Inverse*

INTRODUCTION

Background

One of the critical factor in the design of long-span structures is lightness. As a consequence of being a light structural system, dynamic characteristics such as natural vibration frequencies and mode shapes become critical factor to be considered. Dynamic excitations that affect structures could originate for instance from movement of traffic that passes in the vicinity of a structure or from wind loading as shown in Figure.1. Such excitation might cause excessive vibration affecting the serviceability of structures or in worst case causing structural damage. Hence, it is imperative to have a clear understanding of structural response under dynamic influence.

In order that the structure to be designed possesses the desired dynamic characteristics in terms of its natural vibration frequencies and mode shapes so that damaging dynamic response could be avoided, we need to make proper design decision on types of structures, shapes, topology, materials to be used, condition of supports etc. Load carrying capacity of shell and spatial structures which are common choice of structural system adopted in large-span lightweight structures depends significantly on their shapes. Hence proper selection of shape is vital in ensuring structural soundness of the structures.

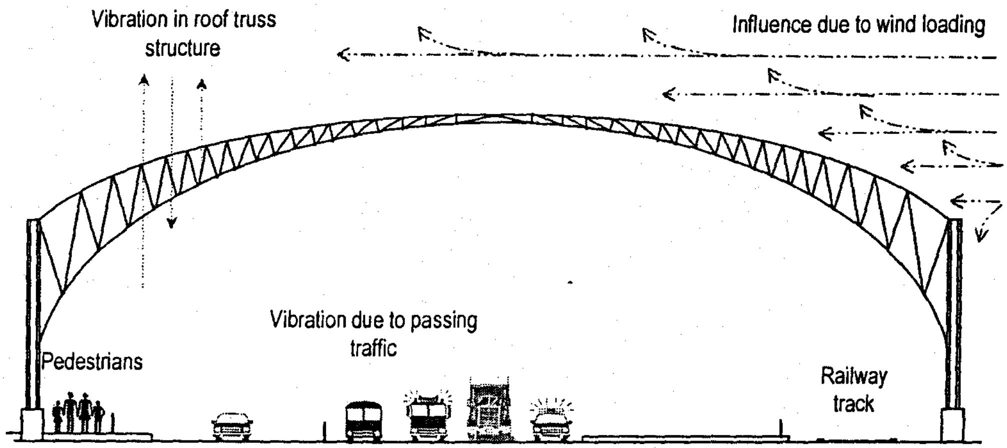


Figure. 1 : Factors causing vibration in a lightweight truss structure

Shape analysis

Analysis carried out to determine the shape of a structure is called shape analysis. It is an inverse problem in which a solution in the form of shape which satisfies the set of constraints imposed is sought. In this research study, shape analysis subjected to constraints in natural vibration frequencies and mode shapes is investigated.

Research on solution algorithm for shape analysis has been carried out by Tanami and Hangai(1990, 1991, 1992). Contraints on eigenvalues and their associated eigenmodes have been imposed. A direct approach with Newton-Raphson iteration has been proposed to solve the nonlinear governing equations. The solution strategy proposed makes use of generalized inverse. The applicability of the proposed algorithm has been verified using simple 2D problem.

In this research study, the applicability of the solution strategy by Tanami and Hangai (1990, 1991, 1992) has been further investigated using more examples including a simple 3D truss structure. A different approach has been adopted in the augmentation of constraint equation in order to form the governing equations. Moreover, indirect approach for solving the governing equations has been used. Design parameter used in shape analysis is the coordinates of joints/nodes of the structure to be analysed.

Objective

This research study is carried out with the objective of investigating the applicability of solution algorithm for shape analysis subjected to constraints in natural frequencies and mode shapes by the use of generalized inverse. Although type of structure treated in this research study has been restricted to truss structure, it can also be applied to other types of structures such as shells and frames.

BASIC EQUATIONS

Basic equations used in this research study are briefly described here. More details could be found in (Ngu, 2001). The two sets of basic equations involved are :

$$(\mathbf{K}(x)\text{-}\lambda_i\mathbf{M}(x))\{\phi\}_i = \{0\} \tag{2.1}$$

$$\begin{Bmatrix} \bar{\phi}_i^1 \\ \bar{\phi}_i^2 \\ \vdots \\ \bar{\phi}_i^j \\ \vdots \\ \bar{\phi}_i^{nm} \end{Bmatrix} = \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_j \\ \vdots \\ a_{nm} \end{Bmatrix} \Rightarrow \begin{matrix} \bar{\phi}_i^1 = a_1 \\ \bar{\phi}_i^2 = a_2 \\ \vdots \\ \bar{\phi}_i^j = a_j \\ \vdots \\ \bar{\phi}_i^{nm} = a_{nm} \end{matrix} \tag{2.2}$$

Eq.(2.1) is the equation for eigenvalue problem where $\boldsymbol{K}$: tangent stiffness matrix , $\boldsymbol{M}$: mass matrix, $\boldsymbol{x}$: vector representing the shape of structure, λ_i, ϕ_i : square of natural angular frequencies and corresponding mode shapes respectively. Eq.(2.2) is the constraints equations on natural mode shapes where m : no. of simultaneous linear equations and n : no. of unknown variables x_i . The overhead bar in Eq.(2.2) denotes value which is prescribed. Eqs.(2.1) and (2.2) could be rewritten in the following forms respectively :

$$\begin{bmatrix} 1 & & -\frac{a_1}{a_{max}} & & \\ & 1 \cdots & -\frac{a_2}{a_{max}} & & \mathbf{0} \\ & & \vdots & & \\ & & -\frac{a_j}{a_{max}} & \cdots 1 & \\ & \mathbf{0} & \vdots & & \ddots \\ & & -\frac{a_{nm}}{a_{max}} & & & 1 \end{bmatrix} \begin{Bmatrix} \bar{\phi}_i^1 \\ \bar{\phi}_i^2 \\ \vdots \\ \bar{\phi}_i^j \\ \vdots \\ \bar{\phi}_i^{nm} \end{Bmatrix} = {}^{(nm-1)\times nm}\left|\mathbf{C}_i(\bar{\phi}_i)\right.^{nm\times 1}\{\bar{\phi}_i\} = \{0\} \tag{2.3}$$

$$(\mathbf{K}(x)\text{-}\bar{\lambda}_i\mathbf{M}(x))\{\bar{\phi}\}_i = {}^{ndof\times ndof}\left|\mathbf{E}_i(x,\bar{\lambda}_i)\right.^{nm\times 1}\{\bar{\phi}\}_i = \{0\} \tag{2.4}$$

where $ndof$: total degrees of freedom of the problem. The set of governing equations is obtained by combining Eqs.(2.3) and (2.4) and it is given as follows :

$$\left[\begin{array}{ccccc} \mathbf{E}_1(x,\bar{\lambda}_1) & & & & \\ & \mathbf{E}_2(x,\bar{\lambda}_2) & & \mathbf{0} & \\ & & \ddots & & \\ & & \mathbf{E}_i(x,\bar{\lambda}_i) & & \\ & \mathbf{0} & & \ddots & \\ & & & & \mathbf{E}_{nf}(x,\bar{\lambda}_{nf}) \\ \hline \mathbf{C}_1(\bar{\phi}_1) & & & & \\ & \mathbf{C}_2(\bar{\phi}_2) & & \mathbf{0} & \\ & & \ddots & & \\ & & \mathbf{C}_i(\bar{\phi}_i) & & \\ & \mathbf{0} & & \ddots & \\ & & & \mathbf{C}_{nf}(\bar{\phi}_{nf}) & \end{array}\right] \left\{\begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \vdots \\ \bar{\phi}_i \\ \vdots \\ \bar{\phi}_{nf} \end{bmatrix}\right\} = {}^{m\times n}\left[\mathbf{A}(x,\bar{\lambda},\bar{\phi})\right]^{n\times 1}\{\bar{\phi}\} = {}^{m\times 1}\{\mathbf{b}\} \tag{2.5}$$

where $\mathbf{C}_i$, $\mathbf{E}_i$: matrices on the left-hand-side of Eq.(2.3) and (2.4) respectively and $\{\bar{\phi}_i\}$: i^{th} prescribed mode shape. Matrix A in Eq.(2.5) which is of size $m \times n$ is in general a rectangular matrix. It is a function of λ_i, $\bar{\phi}_i$ and x. Generalized inverse [2, 4,5] is used in the solution of the above governing equations. 2 node truss element has been used in the analysis (Bath & K. J, 1982; Chandrupatla *et al* 1991).

SOLUTION ALGORITHM

Solution to Eq.(2.5) could be expressed by using Moore-Penrose generalized inverse as follows :

$$\{\bar{\phi}\} = \mathrm{A}^{+}\{\mathrm{b}\} + \left[\mathrm{I}_n - \mathrm{A}^{+}\mathrm{A}\right]\alpha \tag{3.1}$$

where $\mathbf{A}^+$: Moore-Penrose generalized inverse of $\mathbf{A}$, $\mathbf{I}_n$: identity matrix of size $n \times n$ and α : a vector containing arbitrary constants. For problems considered in this research study, $\mathbf{A}^+\mathbf{A}=\mathbf{I}_n$. Hence solution to Eq.(2.5) becomes as follows :

$$\{\bar{\phi}\} = \mathbf{A}^{+}(x,\bar{\lambda},\bar{\phi})\{\mathbf{b}\} \tag{3.2}$$

Newton-Raphson iteration method is used to solve the nonlinear equation Eq.(3.2). Linearization of Eq.(3.2) will yield the following incremental equations :

$$\left(\left[\mathbf{A}^{+}(x)\right]_{,x} dx\right)\{\mathbf{b}\} = \bar{f} - \mathbf{A}^{+}(x)\{\mathbf{b}\} = \{\bar{\phi}\} - \{\phi\}_N \tag{3.3}$$

where $\bar{f} = \{\bar{\phi}\}$ and subscript N denotes iteration step. Solution to Eq.(3.3) is given by the following equation :

$$^{np\times 1}\{dx\}_N = {}^{n\times np}\left[\mathbf{J}(x)\right]_N^{+}\ {}^{n\times 1}\{\Delta\phi\}_N \tag{3.4}$$

where dx : correction to the required shape, $\mathbf{J}(x) = [\mathbf{A}^{+}(x)]_{,x}\{\mathbf{b}\}$, $\{\Delta\phi\}_N = \{\bar{\phi}\} - \{\phi\}_N$ and $\{x\}_{N+1} = \{x\}_N + \{dx\}_N$. Differentiation of matrix $\mathbf{A}^{+}$ with respect to x which appears in Eq.(3.5) could be computed by using the following equation :

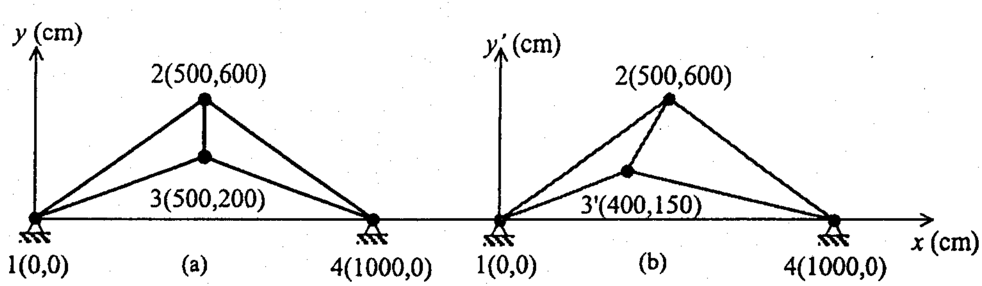
$$\frac{d\mathbf{A}^{+}}{dx} = -\mathbf{A}^{+}\frac{d\mathbf{A}}{dx}\mathbf{A}^{+} + \left(\mathbf{A}^{+}\frac{d\mathbf{A}}{dx}\mathbf{H}\right)^{\mathrm{T}}\mathbf{A}^{+} + \mathbf{A}^{+}\left(\mathbf{G}\frac{d\mathbf{A}}{dx}\mathbf{A}^{+}\right)^{\mathrm{T}} \tag{3.5}$$

where $\mathbf{G} = \mathbf{I}_m - \mathbf{A}\mathbf{A}^{+}$ and $\mathbf{H} = \mathbf{I}_n - \mathbf{A}^{+}\mathbf{A}$. Differentiation of matrix $\mathbf{A}$ and $\mathbf{A}^{+}$ with respect to x are obtained numerically using Stirling method[6]. Singular value decomposition method is used in the determination of $\mathbf{A}^{+}$[4]. Scalar dot product of normalized vector $\{\lambda_i\}$and $\{\phi_i\}$ at the end of iteration N with normalized target $\{\bar{\lambda}\}$ and $\{\bar{\phi}\}_i$ are used as convergence criteria for Newton-Raphson iterative calculation process. The solution x is considered as converged to the required shape when the above dot product approaching the the value of 1.0.

NUMERICAL EXAMPLES

The proposed solution algorithm has been tested on three simple truss structures in order to check its applicability. An initial shape is first assumed. Constraints corresponding to natural vibration frequencies and mode shapes of target shape have been imposed. Analysis is then carried out iteratively until target shape is obtained.

A simple five-member 2D truss



$E = 2.10 \times 10^7$ N/cm^2 ; $A = 1.12 \times 10^1$ cm^2 ; $W/L = 8.792 \times 10^{-2}$ kg/cm

— : initial shape — : target shape

Figure 2 A simple five-member 2D truss : (a) target shape (b) initial assumed shape

Figure 2 shows the analysis data, initial assumed and target shape. Results of iteration are as shown in Figure 3.

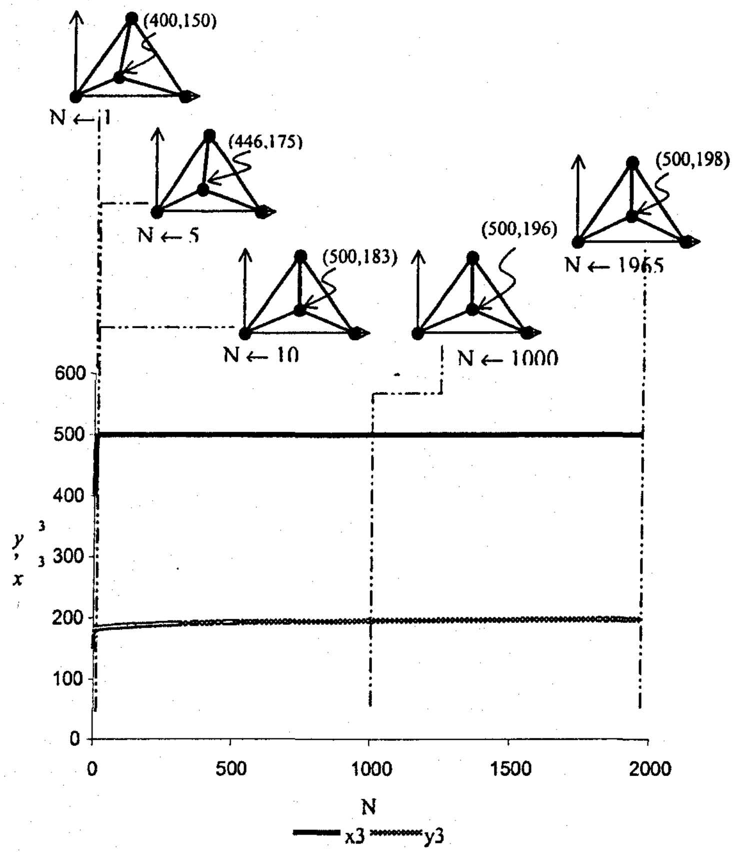
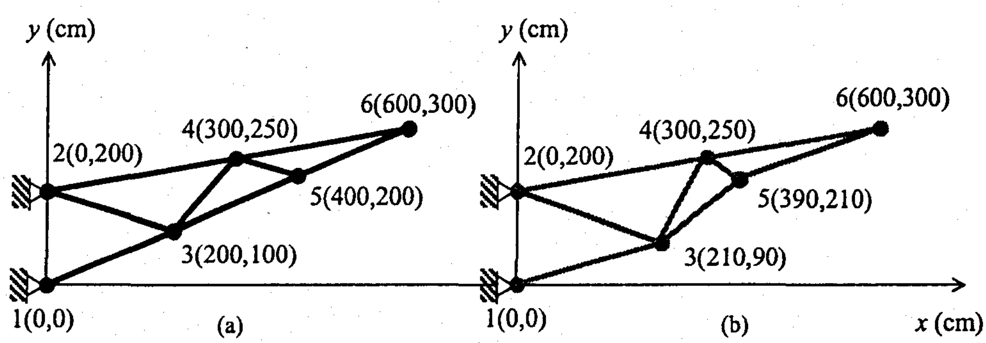


Figure 3: Convergence of initial shape towards the target shape for five-member truss

A 2D cantilever truss

Figure 4 shows the analysis data, initial assumed and target shapes. Results of iteration are as shown in Figure 5.



$E = 2.10 \times 10^7$ N/cm^2
$A = 1.12 \times 10^1$ cm^2
$W/L = 8.792 \times 10^{-2}$ kg/cm

Figure 4 A 2D cantilever truss : (a) target shape ; (b) initial assumed shape

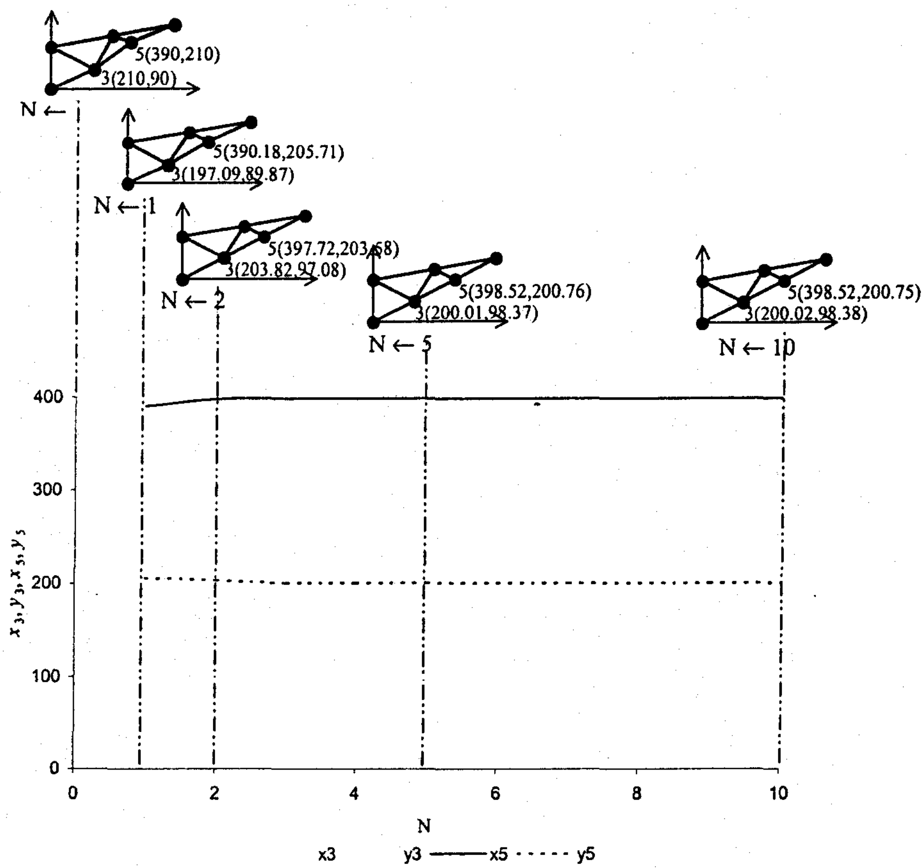


Figure 5: Convergence of initial shape to target shape for 2D cantilever truss

A simple 3D truss

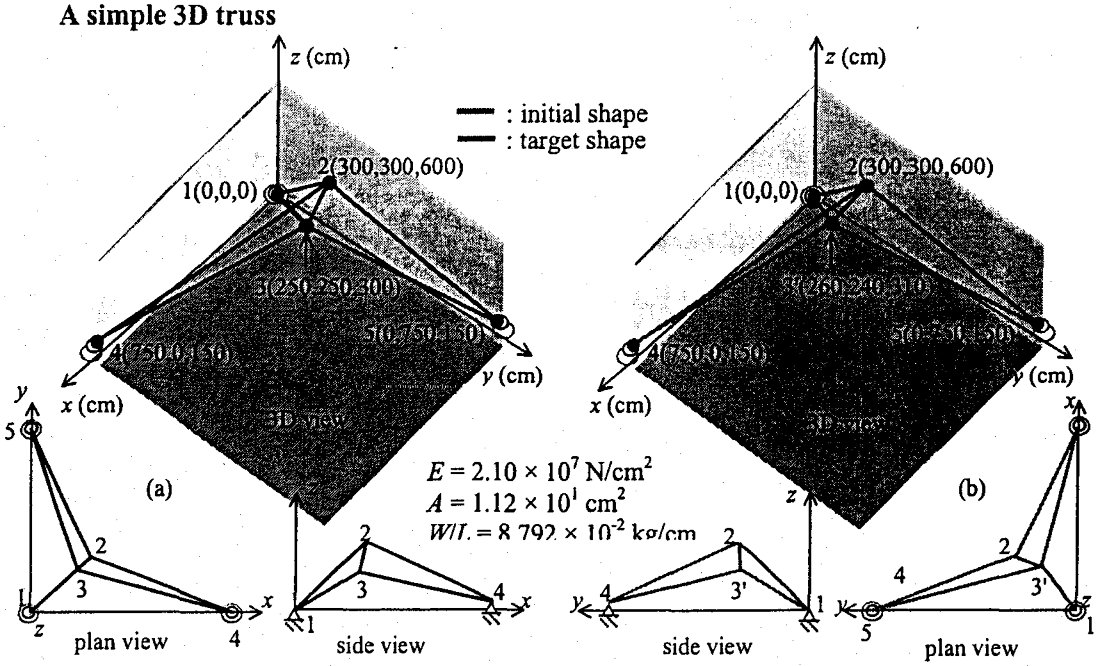


Figure 6 A simple 3D truss : (a) target shape ; (b) initial assumed shape

The third example analysed is a simple 3D truss structure. Analysis data, initial and target shapes are as shown in Figure 6. Result of analysis showing the convergence of initial assumed shape to target shape is shown in Figure 7.

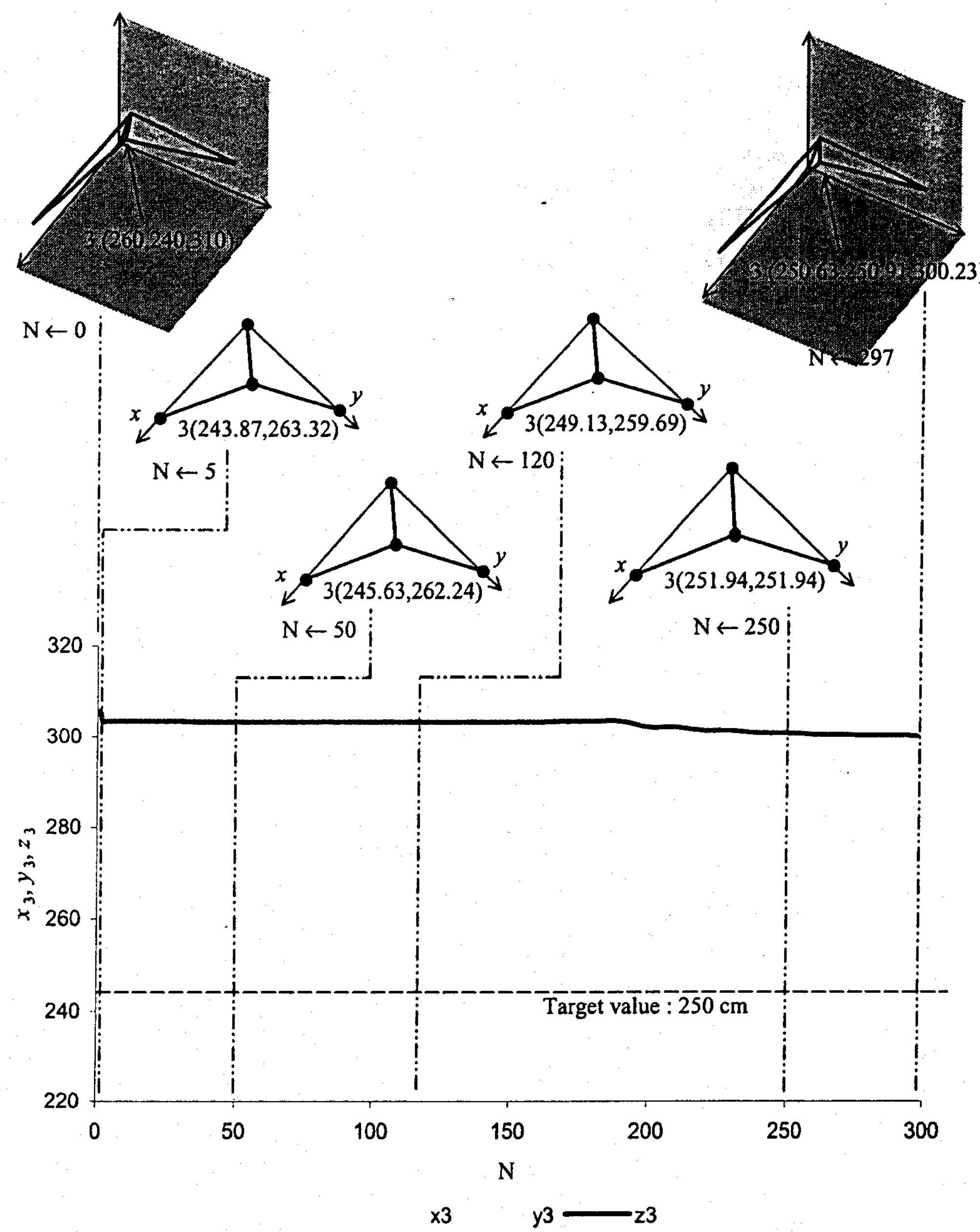


Figure 7: Convergence of intial shape to target shape for a simple 3D truss

DISCUSSION

From the results of analysis presented above, it can be seen that in all three examples, the shape satisfying the constraints imposed could be obtained. Convergence to the prescribed natural frequencies and mode shapes could also be clearly seen from the following plot of dot product versus iteration step for the third example.

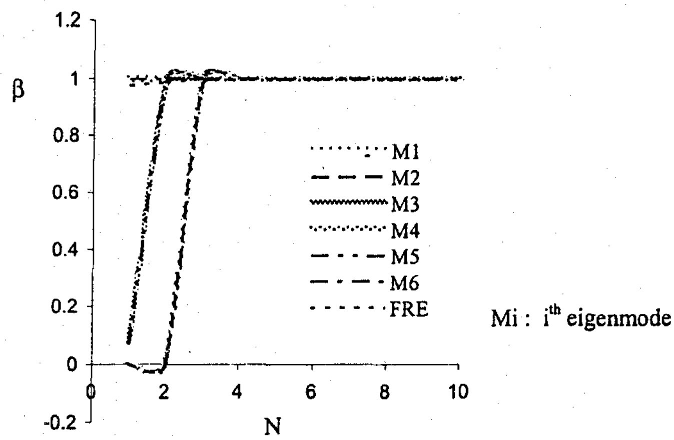


Figure 8: Variation of dot product β of calculated and prescribed eigenvectors with iteration step N for example on 3D truss structure

Based on the above, it can be said that the proposed solution algorithm is applicable and yields accurate results. Verification of its effectiveness by using problems with higher degrees of freedom is necessary.

CONCLUSIONS

Shape analysis of structures with constraints in natural vibration frequencies and mode shapes have been studied. The applicability of solution strategy with the use of generalized inverse has been investigated. Results obtained from numerical analysis carried out on two examples show that the solution strategy adopted performs well and yields satisfactory results. Verification of the effectiveness of the adopted solution strategy in terms of accuracy of solution, sensitivity of solution to initial data and analysis time required by using examples with larger degrees of freedom forms the topics for future investigation.

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