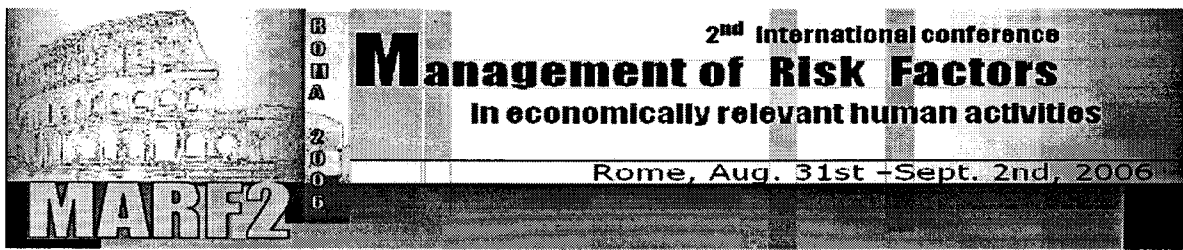


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## Preference to Oil Spot or Oil Futures for Risk-Seekers

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August, 2006

16/10/06

## **Abstract**

This paper develops the stochastic dominance (SD) tests for risk seekers. We find both MV criterion and CAPM measures unable to draw any conclusive preference between the returns but our SD results show that spot dominates futures in the downside risk while futures dominate spot in the upside profit. It also shows that the risk-averse investors prefer investing in spot index while risk seekers are attracted to futures index to maximize their utility. In addition, our SD results enable us to conclude that there is no arbitrage opportunity between these two prices and fail to reject market efficiency and market rationality.

**KEYWORDS:** stochastic dominance, risk averter, risk seeker, futures market, spot market.

**JEL CODES:** C14, G12, G15

## 1. Introduction

The mean-variance (MV) model developed by Markowitz (1952) and the capital asset pricing model (CAPM) statistics developed by Sharpe (1964), Treynor (1965) and Jensen (1969) are commonly used to compare investment prospects. These methodologies are well known to be derived from the Von Neumann-Morgenstern quadratic utility function and the first two moments of a normal distribution. Thus, they are inappropriate if returns are not normally distributed or investors' utility does not take a quadratic form. (Fung and Hsieh, 1999)

To circumvent the limitations of the MV approach and CAPM statistics, stochastic dominance (SD) rules offer superior criteria on prospects investment decisions because SD incorporates information on the entire return distribution, rather than the first two moments as used by employing the MV and CAPM. It requires no precise assessment to the specific form of the investor's risk preference or utility function and has direct utility interpretations in terms of risk and skewness preference. In addition, the theory of SD makes no assumptions about the distribution of asset returns nor does it require any model of asset pricing benchmarks. The outcome from the SD analysis not only allows us to identify the preferences of prospects for both risk averters and risk seekers, but also examines the existence of arbitrage opportunity between the investment prospects. When that opportunity presents itself, the investors can increase their utilities as well as wealth to make huge profits by setting up zero dollar portfolios to exploit this opportunity. In addition, the SD results could also be used to test for market efficiency and market rationality.

Owing to its superiority in comparing uncertain outcomes of different prospects, the SD theory associated with both risk averters and risk seekers has been well documented. The advantages presented by SD have motivated prior studies which use SD techniques to analyze many financial

puzzles. Unfortunately, these earlier studies were unable to establish the statistical significance of SD until recent advances in SD techniques. To date, the SD tests for risk averters have been well developed; for example, see McFadden (1989), Klecan et al. (1991), Kaur et al. (1994), Anderson (1996, 2004), Davidson and Duclos (DD, 2000), Barrett and Donald (BD, 2003), and Linton et al. (LMW, 2005). However, SD tests for risk seekers remain undeveloped.

To bridge the gap in the SD tests, this paper develops the SD tests for risk seekers. So far, the SD test developed by DD has been examined to be one of the most powerful approaches and yet less conservative in size. BD develop a Kolmogorov-Smirnov-type test, which is also found to be another good choice, while the strength from the LMW test enables us to analyse non-iid observations. Thus, we extend the SD tests for risk averters developed by DD, BD and LMW to the corresponding SD tests for risk seekers. Furthermore, we illustrate our approach in this paper by applying the SD tests to examine the behavior of both risk averters and risk seekers with regards to oil futures and spot. Most researches on analyzing spot and futures prices together with their associated returns concentrated only on the efficient market hypothesis (EMH). SD tests enable us not only to test for EMH in the futures and spot markets, but also reveal the existence of arbitrage opportunities and identify the preferences of risk averters and risk seekers in these markets.

Our results show that both the MV criterion and CAPM measures cannot draw any preference between the spot and futures prices. However, our SD results show that spot dominates futures in the downside risk while futures dominate spot in the upside profit. It also shows that the risk-averse investors prefer spot index while risk seekers are attracted to future index to maximize their utility for the entire period, especially apparent in the post-Asian Financial Crisis sub-period. In addition, our SD

results enable us to conclude no arbitrage opportunity between these two markets and fail to reject market efficiency and market rationality.

## 2. Literature Review

### 2.1 Mean-Variance criterion and CAPM statistics

In modern finance, the MV criterion and CAPM statistics are frequently used for constructing efficient portfolio and evaluation of investment performance. For any two investment prospects with the variables of profit or return  $Y_i$  and  $Y_j$  of means  $\mu_i$  and  $\mu_j$  and standard deviations  $\sigma_i$  and  $\sigma_j$  respectively,  $Y_j$  is said to dominate  $Y_i$  by the MV rule if  $\mu_j \geq \mu_i$  and  $\sigma_j \leq \sigma_i$  (Markowitz, 1952; Tobin, 1958). CAPM statistics includes the beta, Sharpe ratio, Treynor's index and Jensen (alpha) index to measure performance. Readers may refer to Sharpe (1964), Treynor (1965) and Jensen (1969) for details on the definitions of these indices and statistics.

However, Feldstein (1969), Hanoch and Levy (1969), Hakansson (1972) and others comment the MV criterion to be applicable only when the decision maker maximizes expected utility and that either the decision maker's utility function is quadratic or the probability distribution of return is normal. On the other hand, Rothschild and Stiglitz (1970) show that the preferences of an expected utility maximizing agent over different distributions of wealth cannot always be consistently stated as preferences over mean and variance alone. This can only be done if restrictive assumptions are made about either the Bernoulli utility function of the agent or the specific class of distributions from which the agent must choose. Arrow (1971) concludes that the quadratic form of utility function is not consistent with observed behavior and implies increasing absolute risk aversion. Baron (1977) points out that even if the return for each alternative has a normal distribution, the MV framework cannot be

used to rank alternatives consistently with the NM axioms unless a quadratic NM utility function is specified. In addition, Fung and Hsieh (1999) suggest that when returns are not normally distributed, the first two moments are not sufficient to give an accurate probability. As MV criterion and CAPM statistics depend on normal returns distributions and quadratic utility functions, they are not appropriate if returns distributions are not normal or investors' utility functions are not quadratic.

## **2.2 Stochastic Dominance Theory and Stochastic Dominance Tests**

SD approach is superior to the MV approach and the CAPM statistics as it does not require any assumption on the nature of distributions or any model of expected returns but studies the distribution of returns directly. The advantage of SD analysis over the MV and CAPM parametric tests becomes apparent when the asset returns distribution is non-normal. Therefore, it can be used for any distribution including both normal distributions and non-normal distributions.

SD theory developed initially by Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970) is one of the most useful tools in investment decision-making under uncertainty to rank investment prospects. SD is important when comparing investment prospects, as different SD relationships correspond to different preferences of the risk-averse or risk-seeking investors. Early theoretical works linked SD theory to the selection rules for risk averters under different restrictions on the utility functions include Quirk and Saposnik (1962), Fishburn (1964, 1974), Hanoch and Levy (1969), Whitmore (1970), Rothschild and Stiglitz (1970, 1971), Hammond (1974), Tesfatsion (1976), Meyer (1977) and Vickson (1977). Recent works include Gotoh and Konno (2000), Levy and Levy (2004), Post (2003), Kuosmanen (2004), Vinod (2004), Fong et al. (2005), Post and Levy (2005) and Broll et al. (2006). The SD theory for risk seekers has also been well developed in the literature (see,

for example, Hammond, 1974; Meyer, 1977; Stoyan, 1983; Levy and Wiener, 1998; Wong and Li, 1999; Li and Wong, 1999; Anderson, 2004; Post and Levy 2005).

Although the SD methodology has been developed for more than four decades, powerful SD tests become available only recently. Many articles in the literature, for example, Levy and Sarnat (1970, 1972), Joy and Porter (1974) and Wingender and Groff (1989), discuss the use of SD rules empirically, but they do not discuss the testing procedure for SD. There are two broad classes of SD tests. One is the minimum/maximum statistic, while the other is based on distribution values computed on a set of grid points. McFadden (1989) first develops a SD test using the minimum/maximum statistic, followed by Klecan et al. (1991) and Kaur et al. (1994). Barrett and Donald (2003) develop a Kolmogorov-Smirnov-type test and Linton et al. (2005) extend their work to relax the iid assumption. On the other hand, Anderson (1996, 2004) and Davidson and Duclos (2000) are the SD tests that compare the underlying distributions at a finite number of grid points.

So far, except for Anderson (2004), the SD tests developed in all of the above studies are for risk averters. Anderson (2004) develops a test for polarization to study economic inequality, economic growth, union/gender wage effects and assorted mating problems. This test could be extended to become SD tests for risk seeker.

### **2.3 Spot and Futures Prices of Petroleum**

The literature on the relationships between spot and futures prices of petroleum products has examined issues such as market efficiency and price discovery. Bopp and Sitzer (1987) find that futures prices have a significant positive contribution to describe past price changes even when crude oil prices, inventory levels, weather, and other important variables are accounted for. Serletis and

Banack (1990) use daily data for the spot and two-month futures crude oil prices, and for prices of gasoline and heating oil traded on the New York Stock Exchange (NYMEX), to test for market efficiency. They find evidence to support the market efficiency hypothesis. Crowder and Hamid (1993) use cointegration analysis to test the simple efficiency hypothesis and the arbitrage condition for crude oil futures. Their results support the simple efficiency hypothesis that the expected returns from futures speculation in the oil futures market are zero.

In the price discovery literature, Quan (1992) examines the price discovery process for the crude oil market using monthly data, and find that the futures price does not play an important role in this process. Using daily data from NYMEX closing futures prices, Schwartz and Szakmary (1994) find that futures prices strongly dominated in the price discovery relative to the deliverable spots in all three petroleum markets. Gulen (1999) applies cointegration tests in a series of oil markets with pairwise comparisons on post-1990 data, and concludes that oil markets have grown more unified during the period 1994-1996 as compared to the period 1991-1994. Silvapulle and Moosa (1999) examine the daily spot and futures prices of WTI crude using both linear and non-linear causality testing. They find that linear causality testing revealed that futures prices lead spot prices, whereas non-linear causality testing revealed a bidirectional effect. Lin and Tamvakis (2001) investigate information transmission between the NYMEX and London's International Petroleum Exchange. They find that NYMEX is a true leader in the crude oil market. Hammoudeh et al. (2003) also investigate information transmission among NYMEX WTI crude prices, NYMEX gasoline prices, NYMEX heating oil prices, and among international gasoline spot markets, including the Rotterdam and Singapore markets and conclude that the NYMEX gasoline market is the true leader.



Empirical studies indicate that commodity prices can be extremely volatile at times and sudden changes in volatility are quite common in commodity markets. For example, using an iterative cumulative sum-of-squares approach, Wilson et al. (1996) document sudden changes in the unconditional variance in daily returns on one-month through six month oil futures and relate these changes to exogenous shocks such as unusual weather, political conflicts and changes in OPEC oil policies. Fong and See (2002) conclude that regime switching models provide a useful framework in studying factors behind the evolution of volatility and short-term volatility forecasts. Moreover, Fong and See (2003) show that the regime switching model outperforms the standard GARCH model on all commonly used evaluation criteria for short-term volatility forecasts.

### 3. Theory

Let  $F$  and  $G$  be the cumulative distribution functions (CDF), and  $f$  and  $g$  be the corresponding probability density functions (PDF) of two investments,  $X$  and  $Y$ , respectively, with common support of  $[a, b]$ , where  $a < b$ . Define

$$H_0^A = H_0^D = h, \quad H_j^A(x) = \int_a^x H_{j-1}^A(t) dt \quad \text{and} \quad H_j^D(x) = \int_x^b H_{j-1}^D(t) dt \quad (1)$$

for  $h = f, g$ ,  $H = F, G$ , and  $j = 1, 2, 3$ .

One could modify the work from Davidson and Duclos (2000) to obtain the following:

$$H_j^A(x) = \frac{1}{(j-1)!} \int_a^x (x-t)^{j-1} h(t) dt \quad \text{and} \quad H_j^D(x) = \frac{1}{(j-1)!} \int_x^b (t-x)^{j-1} h(t) dt$$

for  $H = F, G$  and  $j = 1, 2, 3$ .

#### 3.1 SD for Risk Averters

The most commonly used SD rules corresponded with three broadly defined utility functions are

first-, second- and third-order Ascending SD (ASD)<sup>1</sup> for risk averters, denoted as FASD, SASD and TASD respectively. We first define the ASD rules as follows (see, for example, Quirk and Saposnik, 1962; Fishburn, 1964; Hanoch and Levy, 1969):

**Definition 1:**

1.  $X$  dominates  $Y$  by FASD, denoted by  $X \succ_1 Y$  or  $F \succ_1 G$ , if and only if  $F_1^A(x) \leq G_1^A(x)$  for all possible returns  $x$ , and strictly inequality for at least one value of  $x$ .
2.  $X$  dominates  $Y$  by SASD, denoted by  $X \succ_2 Y$  or  $F \succ_2 G$ , if and only if  $F_2^A(x) \leq G_2^A(x)$  for all possible returns  $x$ , and strictly inequality for at least one value of  $x$ .
3.  $X$  dominates  $Y$  by TASD, denoted by  $X \succ_3 Y$  or  $F \succ_3 G$ , if and only if  $\mu_F \geq \mu_G$  and  $F_3^A(x) \leq G_3^A(x)$  for all possible returns  $x$ , and strictly inequality for at least one value of  $x$ .

The following theorem states that investigating ASD relationship among different investments is equivalent to examining the choice of investments by utility maximization under SD theory (see, for example, Quirk and Saposnik, 1962; Fishburn, 1964; Hanoch and Levy, 1969; Jarrow, 1986).

**Theorem 1:**

1. All non-satiated investors with utility functions  $U'(x) \geq 0$  will prefer  $X$  to  $Y$  and will increase their wealth and utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ_1 Y$ .
2. All non-satiated and risk-averse investors with utility functions  $U'(x) \geq 0$  and  $U''(x) \leq 0$  will prefer  $X$  to  $Y$  and will increase their utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ_2 Y$ .

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<sup>1</sup> We call it Ascending SD as its integrals count from the worst return ascending to the best return.

3. All non-satiated and risk-averse investors with decreasing absolute risk aversion utility functions such that  $U'(x) \geq 0$ ,  $U''(x) \leq 0$  and  $U'''(x) \geq 0$  (prefer positive skewness) will prefer  $X$  to  $Y$  and will increase their utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ_3 Y$ .

The existence of SD implies that the investor's expected utility is always higher with the dominant asset than with the dominated asset and, consequently, the dominated asset would never be chosen by the investor. We note that a hierarchical relationship exists in SD: first-order SD implies second-order SD, which, in turn, implies third-order SD. However, the reverse is not true: a finding of second-order SD does not imply the existence of first-order SD. Likewise; a finding of third-order SD does imply neither the existence of second-order SD nor first-order SD. Thus, only the lowest dominance order of SD is reported in practice.

Finally, we note that under certain regularity conditions<sup>2</sup>, investment  $X$  stochastically dominates investment  $Y$  at first-order, if and only if there is an arbitrage opportunity between  $X$  and  $Y$ , such that the investor will increase his wealth as well as his utility if his investment is shifted from  $Y$  to  $X$  (see, Jarrow, 1986). In addition, if no first-order SD is found between  $X$  and  $Y$ , market efficiency and market rationality could not be rejected. Hence, SD approach provides a tool for revealing arbitrage opportunity among investment prospects as well as to examine market efficiency and rationality.

### 3.2 SD for Risk Seekers

The SD theory for risk seekers has also been well established in the literature (see, for example, Hammond, 1974; Meyer, 1977; Stoyan, 1983; Levy and Wiener, 1998; Wong and Li, 1999; Anderson,

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<sup>2</sup> See Jarrow (1986) for the conditions.

2004). Distinct from SD for risk averters which counts from the worst to the best return, it counts from the best return descending to the worst return. Hence, SD for risk seekers can be called Descending SD (DSD). We have the following definition for DSD (see, for example, Hammond, 1974; Meyer, 1977; Levy and Wiener, 1998; Wong and Li, 1999; Anderson, 2004).

**Definition 2:**

1.  $X$  dominates  $Y$  by first-order Descending SD (FDSD), denoted  $X \succ^1 Y$  or  $F \succ^1 G$ , if and only if  $F_1^D(x) \geq G_1^D(x)$ ;
2.  $X$  dominates  $Y$  by second-order Descending SD (SDSD), denoted  $X \succ^2 Y$  or  $F \succ^2 G$ , if and only if  $F_2^D(x) \geq G_2^D(x)$ ; and
3.  $X$  dominates  $Y$  by third-order Descending SD (TDSD), denoted  $X \succ^3 Y$  or  $F \succ^3 G$ , if and only if  $\mu_F \geq \mu_G$  and  $F_3^D(x) \geq G_3^D(x)$ ,  
where  $F_j^D(x)$  and  $G_j^D(x)$  are defined in (1) for  $j = 1, 2, 3$ .

Similar to ASD, investigating the DSD relationship among different investments is equivalent to examining the choice of investments by utility maximization under SD theory for risk seekers (see, for example, Stoyan, 1983; Levy and Wiener, 1998; Li and Wong, 1999; Anderson, 2004), as stated in the following theorem:

**Theorem 2:**

1. All non-satiated investors with utility functions  $U'(x) \geq 0$  will prefer  $X$  to  $Y$  and will increase their wealth and utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ^1 Y$ .

2. All non-satiated and risk-seeking investors with utility functions  $U'(x) \geq 0$  and  $U''(x) \geq 0$  will prefer  $X$  to  $Y$  and will increase their utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ^2 Y$ .
3. All non-satiated and risk-seeking investors with utility functions  $U'(x) \geq 0$ ,  $U''(x) \geq 0$  and  $U'''(x) \geq 0$  (prefer positive skewness) will prefer  $X$  to  $Y$  and will increase their utilities by shifting their investments from  $Y$  to  $X$  if and only if  $X \succ^3 Y$ .

From Theorems 1 and 2, the integral  $H_j^A$  integrating from the worst return ascending to the best return is known to be related to ASD and the integral  $H_j^D$  integrating from the best return descending to the worst return is related to DSD. Thus, we call the integral  $H_j^A$  in (1) to be FASD, SASD and TASD integrals or the  $j^{\text{th}}$  order ascending cumulative distribution function (ACDF) or simply the  $j^{\text{th}}$  order cumulative distribution function (CDF) and  $H_j^D$  in (1) to be FDSD, SDSD and TSDS integrals respectively or the  $j^{\text{th}}$  order DCDF (descending cumulative distribution function) for  $j = 1, 2$  and  $3$  and for  $H = F$  and  $G$

### 3.3 Davidson and Duclos (DD) Test

Let  $\{(f_i, s_i)\}$  be pairs of observations drawn from the random variables  $F$  and  $S$ , with distribution functions  $F(x)$  and  $G(x)$ , respectively<sup>3</sup>. The integrals  $F_j^A(x)$  and  $G_j^A(x)$  for  $F$  and  $G$  are defined in (1) for  $j = 1, 2, 3$ . For a grid of pre-selected points  $x_1, x_2, \dots, x_k$ , the  $j^{\text{th}}$  order Ascending DD test statistic,  $T_j^A(x)$  ( $j = 1, 2$  and  $3$ ), is:

$$T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}}, \text{ where} \quad (2)$$

$$\hat{V}_j^A(x) = \hat{V}_{F,j}^A(x) + \hat{V}_{S,j}^A(x) - 2\hat{V}_{FS,j}^A(x),$$

<sup>3</sup> As we will illustrate our test by analysing futures index and spot index.  $f$  here denotes the returns in futures index, while  $s$  denotes returns in spot index.

$$\hat{H}_j^A(x) = \frac{1}{N(j-1)!} \sum_{i=1}^N (x-h_i)_+^{j-1},$$

$$\hat{V}_{H,j}^A(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^N (x-h_i)_+^{2(j-1)} - \hat{H}_j^A(x)^2 \right], H = F, G; h = f, s;$$

$$\hat{V}_{FS,j}^A(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^N (x-f_i)_+^{j-1} (x-s_i)_+^{j-1} - \hat{F}_j^A(x) \hat{G}_j^A(x) \right].$$

It is empirically impossible to test the null hypothesis for the full support of the distributions.

Thus, Bishop et al. (1992) propose to test the null hypothesis for a pre-designated finite number of values  $x$ . Specifically, the following hypotheses are tested:

$$\begin{aligned} H_0 &: F_j^A(x_i) = G_j^A(x_i), \text{ for all } x_i, i = 1, 2, \dots, k; \\ H_A &: F_j^A(x_i) \neq G_j^A(x_i) \text{ for some } x_i; \\ H_{A1} &: F_j^A(x_i) \leq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) < G_j^A(x_i) \text{ for some } x_i; \\ H_{A2} &: F_j^A(x_i) \geq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) > G_j^A(x_i) \text{ for some } x_i. \end{aligned} \tag{3}$$

We note that in the above hypotheses,  $H_A$  is set to be exclusive of both  $H_{A1}$  and  $H_{A2}$ ; this means that if the test accepts  $H_{A1}$  or  $H_{A2}$ , it will not be classified as  $H_A$ . Under the null hypothesis, DD show that  $T_j^A(x)$  is asymptotically distributed as the Studentized Maximum Modulus (SMM) distribution (Richmond, 1982) to account for joint test size. To implement the DD test, the test statistic,  $T_j^A(x)$ , at each grid point,  $x$ , is computed and the null hypothesis,  $H_0$ , is rejected if there is a grid point  $x$  such that the test statistic,  $T_j^A(x)$ , is significant. The SMM distribution with  $k$  and infinite degrees of freedom, denoted by  $M_{\infty, \alpha}^k$ , is used to control the probability of Type I error, for  $j = 1, 2, 3$  the following decision rules are adopted based on the  $1-\alpha$  percentile of  $M_{\infty, \alpha}^k$  tabulated by Stoline and Ury (1979):

$$\begin{aligned} &\text{if } |T_j^A(x_i)| < M_{\infty, \alpha}^k \text{ for } i = 1, \dots, k, \text{ accept } H_0; \\ &\text{if } T_j^A(x_i) < M_{\infty, \alpha}^k \text{ for all } i \text{ and } -T_j^A(x_i) > M_{\infty, \alpha}^k \text{ for some } i, \text{ accept } H_{A1}; \\ &\text{if } -T_j^A(x_i) < M_{\infty, \alpha}^k \text{ for all } i \text{ and } T_j^A(x_i) > M_{\infty, \alpha}^k \text{ for some } i, \text{ accept } H_{A2}; \text{ and} \\ &\text{if } T_j^A(x_i) > M_{\infty, \alpha}^k \text{ for some } i \text{ and } -T_j^A(x_i) > M_{\infty, \alpha}^k \text{ for some } i, \text{ accept } H_A. \end{aligned} \tag{4}$$

Accepting either  $H_0$  or  $H_A$  implies non-existence of any SD relationship between  $F$  and  $G$ , non-existence of any arbitrage opportunity between these two markets, and neither of these markets is preferred to the other. If  $H_{A1}$  ( $H_{A2}$ ) of order one is accepted,  $F$  ( $G$ ) stochastically dominates  $G$  ( $F$ ) at first-order. In this situation and under certain regularity conditions<sup>4</sup>, arbitrage opportunity exists and any non-satiated investor will be better off if they switch from the dominated market to the dominant one. On the other hand, if  $H_{A1}$  or  $H_{A2}$  is accepted for orders two or three, a particular market stochastically dominates the other at second- or third-order. In this situation, arbitrage opportunity does not exist, and switching from one market to another will only increase risk averters' expected utilities, but not wealth (Jarrow, 1986; Falk and Levy, 1989). As discussed later, these results could also be used not to reject market efficiency and market rationality.

The DD test compares the distributions at a finite number of grid points. Various studies examine the choice of grid points. For example, Tse and Zhang (2004) show that an appropriate choice of  $k$  for reasonably large samples ranges from 6 to 15. Too few grids will miss information of the distributions between any two consecutive grids (Barrett and Donald, 2003), and too many grids will violate the independence assumption required by the SMM distribution (Richmond, 1982). In order to make the comparisons comprehensive without violating the independence assumption, we follow Fong et al. (2005) to make 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison, and show the statistical inference based on the SMM distribution for  $k=10$  and infinite degrees of freedom<sup>5</sup>. This allows the consistency of both the magnitude and sign of the DD statistics between any two consecutive major partitions to be examined.

To test SD for risk seekers, we modify the DD test for risk averters to be the Descending DD test

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<sup>4</sup> Refer to Jarrow (1986) for the conditions.

<sup>5</sup> Refer to Lean et al. (2006) for explanation.

statistic,  $T_j^D(x)$ , such that:

$$T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}}, \text{ where} \quad (5)$$

$$\hat{V}_j^D(x) = \hat{V}_{F,j}^D(x) + \hat{V}_{S,j}^D(x) - 2\hat{V}_{FS,j}^D(x),$$

$$\hat{H}_j^D(x) = \frac{1}{N(s-1)!} \sum_{i=1}^N (h_i - x)_+^{j-1},$$

$$\hat{V}_{H,j}^D(x) = \frac{1}{N} \left[ \frac{1}{N((s-1)!)^2} \sum_{i=1}^N (h_i - x)_+^{2(j-1)} - \hat{H}_j^D(x)^2 \right],$$

$$\hat{V}_{FS,j}^D(x) = \frac{1}{N} \left[ \frac{1}{N((s-1)!)^2} \sum_{i=1}^N (f_i - x)_+^{j-1} (s_i - x)_+^{j-1} - \hat{F}_j^D(x) \hat{G}_j^D(x) \right],$$

where the integrals  $F_j^D(x)$  and  $G_j^D(x)$  are defined in (1) for  $j=1,2,3$ . The hypotheses for risk seekers can be obtained from modifying (4) such that:

$$H_0 : F_j^D(x_i) = G_j^D(x_i), \text{ for all } x_i, i=1,2,\dots,k;$$

$$H_D : F_j^D(x_i) \neq G_j^D(x_i) \text{ for some } x_i;$$

$$H_{D1} : F_j^D(x_i) \geq G_j^D(x_i) \text{ for all } x_i, F_j^D(x_i) > G_j^D(x_i) \text{ for some } x_i;$$

$$H_{D2} : F_j^D(x_i) \leq G_j^D(x_i) \text{ for all } x_i, F_j^D(x_i) < G_j^D(x_i) \text{ for some } x_i;$$

and the following decision rules are adopted for risk seekers such that:

$$\text{If } |T_j^D(x_i)| > M_{\infty,\alpha}^k \text{ for } i=1,\dots,k, \text{ accept } H_0;$$

$$\text{if } -T_j^D(x_i) < M_{\infty,\alpha}^k \text{ for all } i \text{ and } T_j^D(x_i) > M_{\infty,\alpha}^k \text{ for some } i, \text{ accept } H_{D1};$$

$$\text{if } T_j^D(x_i) < M_{\infty,\alpha}^k \text{ for all } i \text{ and } -T_j^D(x_i) > M_{\infty,\alpha}^k \text{ for some } i, \text{ accept } H_{D2}; \text{ and}$$

$$\text{if } T_j^D(x_i) < M_{\infty,\alpha}^k \text{ for some } i \text{ and } -T_j^D(x_i) > M_{\infty,\alpha}^k \text{ for some } i, \text{ accept } H_D.$$

As in the case of the test for risk averter, accepting either  $H_0$  or  $H_D$  implies non-existence of any SD relationship between  $F$  and  $G$ , non-existence of any arbitrage opportunity between these two markets, and neither of these markets is preferred to the other. If  $H_{D1}$  ( $H_{D2}$ ) of order one is accepted, asset  $F$  ( $G$ ) stochastically dominates  $F$  ( $G$ ) at first-order. In this situation, an arbitrage opportunity exists and any non-satiated investor will be better off if s/he switches her/his investment from the dominated market to the dominant one. On the other hand, if  $H_{D1}$  or  $H_{D2}$  is accepted at



order two or three, a particular market stochastically dominates the other at second- or third-order. In this situation, an arbitrage opportunity does not exist and switching from one market to another will only increase the risk seekers' expected utility, but not wealth.

### 3.4 Barrett and Donald (BD) Test

Barrett and Donald (2003) develop a Kolmogorov-Smirnov-type test for SD of any pre-specified order. The test applies to two independent samples of possibly unequal sample sizes. Hereafter, this test will be referred to as the BD test, which evaluates the following two sets of null and alternative hypotheses:

$$H_{A0} : F_j^A(x_i) \leq G_j^A(x_i) \text{ for all } x; \text{ and}$$

$$H_{A1} : F_j^A(x_i) > G_j^A(x_i) \text{ for some } x.$$

$$H'_{A0} : G_j^A(x_i) \leq F_j^A(x_i) \text{ for all } x; \text{ and}$$

$$H'_{A1} : G_j^A(x_i) > F_j^A(x_i) \text{ for some } x.$$

where  $F_j^A(x)$  and  $G_j^A(x)$  are defined in (1) for  $j = 1, 2$  and  $3$ . The null hypothesis that futures index dominates (but does not strictly dominate) spot index is stated in  $H_{A0}$  and the null hypothesis that spot index dominates (but does not strictly dominate) futures index is stated in  $H'_{A0}$ . The BD test statistic for risk averters in brevity is:

$$\hat{K}_j^A = \left( \frac{N^2}{2N} \right)^{1/2} \sup_x [\hat{F}_j^A(x) - \hat{G}_j^A(x)],$$

where  $\hat{H}_j^A(x)$  is defined in (2) for  $H = F, S$ .

To compute the critical value for BD statistic, we rely on the following theorem:

**Theorem 3:** (Barrett and Donald, 2003)

*Define the random variable,  $K_j^{F,G} = \sup_x [\sqrt{\lambda} F_j^A(x, B_F \circ F) - \sqrt{1-\lambda} G_j^A(x, B_G \circ G)]$ , where*

*$\lambda \in (0,1)$  and  $B_F \circ F$  and  $B_G \circ G$  are independent Brownian Bridge processes. The limiting*

distributions of the test statistics under the null hypothesis are characterized as

$$\sqrt{N}(\hat{F}_j^A - F) \Rightarrow B_F \circ F \quad \text{and} \quad \sqrt{N}(\hat{G}_j^A - G) \Rightarrow B_G \circ G \quad . \quad \text{If } H_0 \text{ is true,}$$

$$\lim_{N \rightarrow \infty} P(\text{reject } H_0) \leq P(K_j^{F,G} > c_j) \equiv \alpha(c_j). \quad \text{If } H_0 \text{ is false, } \lim_{N \rightarrow \infty} P(\text{reject } H_0) = 1.$$

If  $H_{A_0}$  is rejected, we conclude that the futures index does not dominate spot index by risk averters. Similarly, if  $H'_{A_0}$  is rejected, we conclude that the spot index does not dominate the futures index by risk averters.

Theorem 3 shows how to compute the critical values corresponding to any desired Type I error, such as  $\alpha(c_j) = 0.05$ . It also indicates that if the null hypothesis is true, the Type I error rate will not exceed  $\alpha(c_j)$  asymptotically. For  $j \geq 2$ , it is analytically intractable to derive the critical values of the test statistic because the limiting distribution of  $K_j^{F,G}$  depends on the underlying CDFs. The  $p$ -values can be simulated by using arbitrarily fine grids for calculating the suprema of  $F_j^A(x)$  and  $G_j^A(x)$ <sup>6</sup>.

In this paper, we extend Barrett and Donald's Kolmogorov-Smirnov-type test for risk seekers for testing the following two sets of null and alternative hypotheses:

$$H_{D_0} : F_j^D(x_i) \leq G_j^D(x_i) \text{ for all } x; \text{ and}$$

$$H_{D_1} : F_j^D(x_i) > G_j^D(x_i) \text{ for some } x.$$

$$H'_{D_0} : G_j^D(x_i) \leq F_j^D(x_i) \text{ for all } x; \text{ and}$$

$$H'_{D_1} : G_j^D(x_i) > F_j^D(x_i) \text{ for some } x.$$

The BD test statistic for risk seekers is given by:

$$\hat{K}_j^D = \left( \frac{N^2}{2N} \right)^{1/2} \sup_x \left[ \hat{F}_j^D(x) - \hat{G}_j^D(x) \right], \text{ where } \hat{H}_j^D(x) \text{ is defined in (1) for } H = F, S.$$

If  $H_{D_0}$  is rejected, the spot index is concluded not to dominate the futures index by risk seekers. On

the other hand, if  $H'_{D_0}$  is rejected, the futures index is concluded not to dominate the spot index by

<sup>6</sup> Refer to Barrett and Donald (2003) for the details.

risk seekers.

### 3.5 Linton, Maasoumi and Whang (LMW) Test

Linton et al. (2005) propose a procedure for estimating the critical values of the extended Kolmogorov-Smirnov tests of SD. Their method is based on sub-sampling and the resulting tests are consistent and powerful against some  $N^{-1/2}$  local alternatives. This method allows for general dependence amongst the prospects, and for non-iid observations. The hypotheses set up are the same as BD test and test statistic for risk averters is:

$$T_j^A = \min \sup_x \sqrt{N \left[ \hat{F}_j^A(x) - \hat{G}_j^A(x) \right]} \quad \text{where } \hat{H}_j^A(x) \text{ is defined in (1) for } H = F, G.$$

They follow DD (2000) for the computation of  $\hat{H}_j^A(x)$  and compute approximations to the suprema in  $\hat{H}_j^A(x)$  based on taking maxima over some smaller grid of points,  $x_1, x_2, \dots, x_k$ .

In this paper, we extend the LMW test for risk seekers with the corresponding hypotheses as follows:

$$H_{D0} : F_j^D(x_i) \leq G_j^D(x_i) \text{ for all } x; \text{ and}$$

$$H_{D1} : F_j^D(x_i) > G_j^D(x_i) \text{ for some } x.$$

$$H'_{D0} : G_j^D(x_i) \leq F_j^D(x_i) \text{ for all } x; \text{ and}$$

$$H'_{D1} : G_j^D(x_i) > F_j^D(x_i) \text{ for some } x.$$

The test statistic is  $T_j^D = \min \sup_x \sqrt{N \left[ \hat{F}_j^D(x) - \hat{G}_j^D(x) \right]}$  where  $\hat{H}_j^D(x)$  is defined in (1) for  $H = F, G$ .

Since the DD, BD and LMW tests have both advantages and disadvantages, we will use all these tests to assess the statistical significance of dominance. Evident from the similar results produced, these tests provide a greater degree of confidence in the empirical results.<sup>7</sup>

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<sup>7</sup> We find similar results from using DD, BD and LMW tests, so we only report the DD results. The BD and LMW results are available upon request.

### 3.6 Market Rationality and Market Efficiency

In this section, we will discuss the relationship between SD and market rationality/efficiency. Our focus here is to show how market rationality/efficiency can be tested by using the SD rules without identifying a risk index or a specific model. In the examination of market data, SD employs the criterion of whether the investors' wealth increases when they switch their portfolio choice. If futures FASD stocks, there exists an arbitrage opportunity and all non-satiated investors will increase their wealth and utility by switching from stocks to futures. Such a situation contradicts the notion of market rationality and market efficiency. Thus, we expect first-order SD results could lead us to reject that market is rational and efficient.<sup>8</sup> We note that it should be possible in a rational market that two assets or portfolios where one FASD another for a short period of time. However, this phenomenon should not last for an extended period of time because market forces mitigate adjustments to a state whereby no FASD will result if the market is efficient and investor is rational.

The next situation to consider is whether some investors can increase their expected utility by switching their portfolio choice provided others have no utility loss. Are investors rational and the market efficient, especially if all individuals have increasing utility with decreasing marginal utility functions? Given a market with only this type of investors, if futures dominate stocks, then all investors would buy futures and short stocks. This will continue driving up the price of futures relative to stocks, until the price of futures relative to stocks cause the marginal investor to be indifferent between futures and stocks. But the higher price for futures (and lower price for stocks) implies lower expected return for futures (and higher expected return for stocks). This has the effect of altering the equilibrium distributions of returns until there is no SASD for the two assets. This means that in a rational and

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<sup>8</sup> See Bawa (1978), Jarrow (1986), Falk and Levy (1989), Bernard and Seyhun (1997) and Larsen and Resnick (1999) for more discussion of this condition.

efficient market where all individuals have increasing utility but decreasing marginal utility functions, we should not observe SASD relationship between any two distinct assets/portfolios continue to exist for a considerable length of time. A similar argument can be made for the T ASD criterion which assumes that all investors' utility functions exhibit non-satiation, risk aversion, and decreasing absolute risk aversion. Given a market with only these type of investors, we should not observe T ASD relationship between any two distinct assets/portfolios continue to exist for a considerable length of time.

From the above argument, the existence of SASD (T ASD) could infer that markets are inefficient and irrational. However, this argument requires the assumption that all investors in the markets possess similar increasing utility with decreasing marginal utility functions (i.e. decreasing absolute risk aversion). Is this assumption realistic? Many studies disagree with it. Friedman and Savage (1948) claim that the strictly concave functions may not be able to explain the behavior why investors buy insurance or lottery tickets while Fishburn and Kochenberger (1979) document the prevalence of risk seeking in choices between negative prospects. On the other hand, Hammond (1974), Meyer (1977), Stoyan (1983), Levy and Wiener (1998), Anderson (2004) and Post and Levy (2005) suggest that both risk averters and risk seekers co-exist in the market.

In this paper, we follow the study from Post and Levy (2005) and others, to study the situation where markets consist of both risk averters and risk seekers. Hence, the existence of SDS D or T DSD should not lead to the rejection of market efficiency and market rationality. Our argument is very simple. As shown in our findings in the next section, risk averters prefer stocks to futures and risk seekers prefer futures to stocks. Both risk averters and risk seekers could choose their preferred prospects or markets to

invest and the prices could still reach equilibrium without prices adjustment as the markets only have risk averters.

Jegadeesh and Titman (1993) find a striking seasonality in momentum profits.. If the markets contain risk averters or risk seekers only, the momentum profits should be diminished and eventually disappeared after its discovery as more investors would exploit this effect but in reality they are not. The only reasonable explanation is that the markets contain more than one type of investors, for example, risk averters and risk seekers. As found by Fong et al. (2005) and Wong et al. (2006) the winners portfolio dominates at the right-hand side of the distribution of returns while the losers portfolio dominates at the left-hand side of the distribution. If our approach is used, their results will imply that risk averters prefer losers while risk seekers prefer the winners and hence it is not surprised that Jegadeesh and Titman (2001) find that the momentum profits still exists after its discovery in 1993.

Conceptually, market rationality within the SD framework is not different from the conventional understanding in some rational asset pricing models, such as the CAPM. The only difference is that the latter approach defines an abnormal return as excess return adjusted to some risk measure, while SD market rationality tests employ the whole distribution of returns. In particular, both SD and the asset pricing model based on residual analyses are consistent with the concept of expected utility maximization. Nonetheless, the SD approach with less restriction on investors' preferences and returns distributions can help us to understand better the relationship between stocks and futures markets.

#### **4. Illustration**

To demonstrate the superiority of our approach, we examine the performance of Brent Crude oil

spot and futures by using their daily closing prices for the period January 1, 1989 to December 31, 2004. The price data are collected from Datastream and the daily log returns,  $R_{i,t}$ , for the oil spot and futures indices is defined to be  $R_{i,t} = \ln (P_{i,t} / P_{i,t-1})$ <sup>9</sup>, where  $P_{i,t}$  is the daily index at day  $t$  for index  $i$ , with  $i = S$  (Spot) and  $F$  (Futures) respectively. We further study the effect of the Asian Financial Crisis on oil prices by examining two sub-periods: the first sub-period is the pre Asian Financial Crisis (pre-AFC) period and the second sub-period is the period after the Asian Financial Crisis (post AFC), using July 1, 1997 as the cut-off point.

#### 4.1 MV and CAPM analysis

For comparative purposes, we adopt the MV criterion and use several statistics derived from the CAPM literature, including beta, Sharpe Ratio, Treynor's Index and Jensen's Alpha (referred to as CAPM statistics) to study the performance of the two indices. For computing the CAPM statistics, we use the 3-month U.S. T-bill rate and the Morgan Stanley Capital International index returns (MSCI) to proxy the risk-free rate and the global market index, respectively.

[Table 1 here]

Table 1 provides the descriptive statistics for the daily returns of oil spot prices and oil futures prices for the entire sample period. Their mean daily returns are about 0.02%, which are not significantly different from zero. The mean return of futures is insignificantly higher than that of spot while, as expected, the standard deviation of spot is insignificantly lower than that of futures. As both means and standard deviations are insignificantly different for the two indices, the MV criterion is unable to indicate any preference between these two indices.

For the CAPM measures, the beta (absolute value) of spot is smaller than that of futures; with

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<sup>9</sup> The definition is commonly used; see for example, Chen et al. (2002) and Pok and Poshakwale (2004).

both being negative and less than one. Both indices have similar Sharpe ratios, Treynor and Jensen indices, and none is significantly different from zero. Thus, the information drawn from the CAPM statistics cannot lead to any preference between the spot and futures prices. In addition, when returns are not normally distributed, the MV rule and CAPM may lead to paradoxical results (see Hanoch and Levy, 1969; Falk and Levy, 1989; Fung and Hsieh, 1999). The highly significant Kolmogorov-Smirnov (K-S) and Jarque-Bera (J-B) statistics shown in Table 1 indicate that both returns are non-normal.<sup>10</sup> Moreover, both daily returns are negatively skewed. As expected, futures have much higher kurtosis than spot, with both being higher than that of normality. Both significant skewness and kurtosis indicate non-normality in the returns distributions and thus lead to the conclusion that the normality requirement in the traditional MV and CAPM measures is violated.

#### 4.2 SD analysis for Risk Averters

[Figure 1 here]

To circumvent the limitations of the MV approach and CAPM statistics, we employ the SD methodology to the issue. We first depict the CDF of the returns distributions for both oil spot prices and oil futures prices and the first three orders of the Ascending DD statistics for risk averters in Figure 1. If futures dominate spot in the sense of FASD, then the CDF of futures returns should lie below that of the spot for the entire range. However, the plots in Figure 1 show that spot lies below futures in the downside risk while futures lies below spot on the upside profit. This indicates that there is no FASD between the two returns and that spot dominates futures on the downside risk while futures dominate spot on the upside profit range. To verify this finding formally, we employ the first three orders of the Ascending DD statistics,  $T_j^A$  ( $j=1,2,3$ ), for the two series with the results

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<sup>10</sup> The results of other normality tests, such as Shapiro-Wilk lead to the same conclusion. The results are available on request.



reported in Table 2 and depicted in Figure 1.

DD state that the null hypothesis can be rejected if any of the test statistics  $T_j^A$  is significant with the wrong sign. In order to minimize the Type II error of dominance and to accommodate the effect of almost SD (Leshno and Levy, 2002), we use a conservative 5% cut-off point for the proportion of test statistics for statistical inference. Using a 5% cut-off point, if futures dominate spot, we should find at least 5% of  $T_j^A$  to be significantly negative and no portion of  $T_j^A$  is significantly positive. The reverse holds if the spot dominates futures.

[Table 2 here]

The values of  $T_1^A$  depicted in Figure 1 move from positive to negative along the distribution of returns, together with the percentage of significant values reported in Table 2, implying that spot dominates futures in the downside risk with 4% significance, while futures dominate spot in the upside profit, with 5% significance. Thus, the hypotheses that futures stochastically dominate spot or vice-versa at first-order are rejected, implying no arbitrage opportunity between these two series. We can, however, state that spot dominates futures marginally in the downside risk returns, while futures dominate spot marginally in the upside profit.

The SD criterion enables us to compare utility interpretations in terms of investors' risk aversion and decreasing absolute risk aversion (DARA) respectively, by studying the higher order SD relationships. The Ascending DD statistics  $T_2^A$  and  $T_3^A$  depicted in Figure 1 are positive in the entire range of the return distribution, with 6% of  $T_2^A$  (5% of  $T_3^A$ ) being significantly positive and no  $T_2^A$  ( $T_3^A$ ) being significantly negative; implying that spot marginally SASD (TASD) dominates futures, and hence the risk averse investors prefer investing in spot to maximize their expected utility, but not wealth.

### 4.3 Will Risk Seekers Have Different Preferences?

So far, if we apply the existing ASD tests on the issue, we could only draw conclusion on the preference of the risk-averse investors in the markets but not of risk seekers. Nonetheless, the result also shows that futures dominate spot for the upside profit. However, applying ASD test alone could not yield any inference based on this information. Thus, an extension of the SD test for risk seekers is necessary, as discussed in the earlier sections. Subsequent discussions illustrate the applicability of DSD test for risk seekers in this section

It is well known that investors could be risk-seeking (see, for example, Markowitz, 1952; Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Levy and Levy, 2004; Post and Levy, 2005). In order to examine the risk seeking behavior, DSD theory for risk seeking is developed. In this section, we put the theory into practice by extending the DD test for risk seekers, namely Descending DD statistics,  $T_j^D$  ( $j = 1, 2$  and  $3$ ), of the first three orders for risk seekers with the correspondence statistics as discussed in the previous section.

[Figure 2 here]

Figure 2 shows the descending cumulative density function (DCDF) for the daily returns of both oil spot prices and oil futures prices, and the values of the  $T_j^D$  ( $j = 1, 2$  and  $3$ ) over the entire distribution range for the whole sample period. The cross of the two DCDFs suggests that there is no FSD between futures and spot returns. The DCDF of the futures lies above that of spot for the upside profit while the DCDF of the spot lies above that of futures for the downside risk, indicating that futures are preferred to spot for upside profit while spot is preferred to futures for downside risk.

[Table 3 here]

To test this phenomenon formally, we plot the Descending DD statistics,  $T_j^D$ , of the first three orders in Figure 2, and report the percentages of their significant positive and negative portions in Table 3. Figure 2 shows that  $T_1^D$  is positive in the upside profit range and negative in the downside risk range while Table 3 shows that 5% (4%) of the positive (negative) values of  $T_1^D$  is significant; indicating that there is no FSDS relationship between the two series for the entire period.

As there is no FSDS, we examine the  $T_j^D$  for the second and third orders. Both  $T_2^D$  and  $T_3^D$  depicted in Figure 2 are positive for the entire range; implying that risk seeking investors could prefer futures to spot. To verify this statement statistically, we use the results in Table 3 that 7% (6%) of  $T_2^D$  ( $T_3^D$ ) are significantly positive while no  $T_2^D$  ( $T_3^D$ ) is significantly negative. This leads us to conclude statistically that futures SDDS and TDDS spot and consequently, risk-seeking investors prefer futures to spot to maximize their utilities, not their wealth.

In addition, neither FASD nor FSDS lead us not to reject market efficiency or market rationality. The preference of risk-averse and risk-seeking investors towards spot and futures do not infer market inefficiency unless the oil market has only one type of investors. Our results are consistent with Fong et al. (2005) and Wong et al. (2006) who study the momentum profits in stocks markets.

#### **4.4 The Impact of Asian Financial Crisis on Oil**

It is of interest to examine the effects of the Asian Financial Crisis (AFC) on the behavior of investors and the performance of the markets in the sub-periods. In order to achieve these, we employ the MV criterion, CAPM statistics and DD test to the return series in the pre-AFC and post-AFC periods.

[Table 4 here]

Table 4 provides descriptive statistics of daily returns of oil spot prices and oil futures prices for the pre-AFC and the post-AFC sub-periods. Both the mean returns for spot and futures dramatically increase four times in the post-AFC sub-period but, the difference between the means for spot and futures in the post-AFC is not significant. Although the mean return for spot is slightly more than futures with the standard deviation of spot being lower than futures, both statistics are insignificant. The MV criterion is thus unable to indicate any preferences between the two indices.

In addition, the values of beta for spot in both the pre-AFC and post-AFC sub-periods are not significantly different from zero. Nonetheless, the beta for futures is significantly negative in the pre-AFC sub-period but becomes insignificant in the post-AFC sub-period. This indicates that the spot market does not move along with the global market while the futures market moves in a different direction from the global market in the pre-AFC sub-period. In addition, the other CAPM statistics are unable to conclude any significant difference between the two indices. As these CAPM statistics cannot be used to draw the preference between spot and futures in both sub-periods, we rely on the SD analysis for this possibility.

On the other hand, we find all values of  $T_j^A$  and  $T_j^D$  ( $j = 1, 2$  and  $3$ ) are not significantly positive nor significantly negative at the 5% level for the first three orders in the pre-AFC sub-period for both risk averters and risk seekers. Therefore, there is neither arbitrage opportunity nor any preference between these two indices for both risk-averse and risk-seeking investors in the pre-AFC sub-period. However, Table 2 displays that 15% (13%) of  $T_2^A$  ( $T_3^A$ ) are significantly positive and none of the  $T_2^A$  ( $T_3^A$ ) is significantly negative while Table 3 displays that 15% (17%) of  $T_2^D$  ( $T_3^D$ ) are significantly positive and none of the  $T_2^D$  ( $T_3^D$ ) is significantly negative at the 5% level in the

post-AFC sub-period. Hence, we conclude that the risk-averse investors prefer spot index but risk seekers are attracted to future index to maximize their utility, but not their wealth.

## 5. Conclusions

In order to circumvent the limitations of MV and CAPM, we have extended the application of SD rules and techniques of analyses to offer a more robust decision tool for investment decisions. The SD tests enable us to reveal the existence of arbitrage opportunities, identify the preferences for both risk averters and risk seekers over different investment prospects and examine market rationality and market efficiency. This paper further develops the SD tests of DD, BD and LMW for risk seekers and apply the SD tests to examine the behavior of both risk averters and risk seekers with regards to oil spot and futures markets and the performance of these markets.

Our results show conclusively that oil spot dominates oil futures on the downside risk while futures dominate spot on the upside profit range. We conclude that there is neither arbitrage opportunity nor preference being prevalent between these two indices for both risk-averse and risk-seeking investors in the pre-AFC sub-period. However, the risk-averse investors and DARA investors prefer spot index while risk seekers are attracted to future index in order to maximize their utility in the post-AFC sub-period.

Finally, we note that some authors propose to use higher order (higher than three) SD in empirical application. For example, Vinod (2004) recommends employing the 4<sup>th</sup> order SD to make the choice among investment prospects with illustration in his analysis of 1281 mutual funds realistic. We also note that the most commonly-used orders in SD for empirical analyses, regardless whether they are simple or complicated, are the first three and one could easily extend the theory developed in this paper to any order. We thus stop at third order in this paper.

Table 1: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for 1989 – 2004 (Whole Period)

Variable	Oil Spot Prices	Oil Futures Prices
Mean (%)	0.02282 (0.76)	0.02296 (0.65)
Std Dev	0.01941	0.02274
Skewness	-0.96197	-1.83211
Kurtosis	13.2812	33.6070
Jarque-Bera (J-B)	19027.27*	165258.40*
Kolmogorov-Smirnov (K-S)	0.0719*	0.0851*
Beta	-0.0182	-0.2253
Sharpe Ratio	0.0061	0.0050
Treynor Index	-0.0065	-0.0005
Jensen Index	0.0001	0.0001
F Statistics	0.7281	
N	4174	4174

Numbers in parentheses are t statistics. \* significant at 1% level, \*\* significant at 5% level, \*\*\* significant at 10%. F Statistics is for testing the equality of variances.

Table 2: Results of DD Test for Risk Averters

Sample	FASD		SASD		TASD	
	$\% T_1^A > 0$	$\% T_1^A < 0$	$\% T_2^A > 0$	$\% T_2^A < 0$	$\% T_3^A > 0$	$\% T_3^A < 0$
Whole Period	4	5	6	0	5	0
Pre-AFC	0	0	0	0	0	0
Post-AFC	9	11	15	0	13	0

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution.

Table 3: Results of DD Test for Risk Seekers

Sample	FDSD		SDSD		TDSD	
	$\% T_1^D > 0$	$\% T_1^D < 0$	$\% T_2^D > 0$	$\% T_2^D < 0$	$\% T_3^D > 0$	$\% T_3^D < 0$
Whole Period	5	4	7	0	6	0
Pre-AFC	0	0	0	0	0	0
Post-AFC	11	9	15	0	17	0

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% significance level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution.

Table 4: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for Pre- and Asian Financial Crisis Sub-Period (cut at 1/7/1997)

Variable	Pre-AFC		Post-AFC	
	Oil Spot Prices	Oil Futures Prices	Oil Spot Prices	Oil Futures Prices
Mean (%)	0.008086 (0.19)	0.008652 (0.18)	0.0395 (0.92)	0.03914 (0.75)
Std Dev	0.01980	0.02231	0.01895	0.02323
Skewness	-1.4906	-3.4757	-0.2766	-0.1839
Kurtosis	20.7164	65.9635	3.1724	2.3039
J-B	29801.42*	370508.48*	27.3834	50.5721
K-S	0.1006*	0.1215*	0.0525*	0.0449*
Beta	-0.1008	-0.7427	0.0109	-0.0451
Sharpe Ratio	-0.0046	-0.0042	0.0163	0.0131
Treynor Index	0.0009	0.0001	0.0284	-0.0068
Jensen Index	$-7.83 \times 10^{-5}$	$2.35 \times 10^{-5}$	0.0003	0.0003
F Statistics	0.7876		0.6658	
N	2216	2216	1958	1958

Numbers in parentheses are t statistics. \* significant at 5% level, \*\* significant at 1% level, \*\*\* significant at 10%. F Statistics is for testing the equality of variances.

Figure 1: Distribution of Returns and DD Statistics for Risk Averters - Whole Period

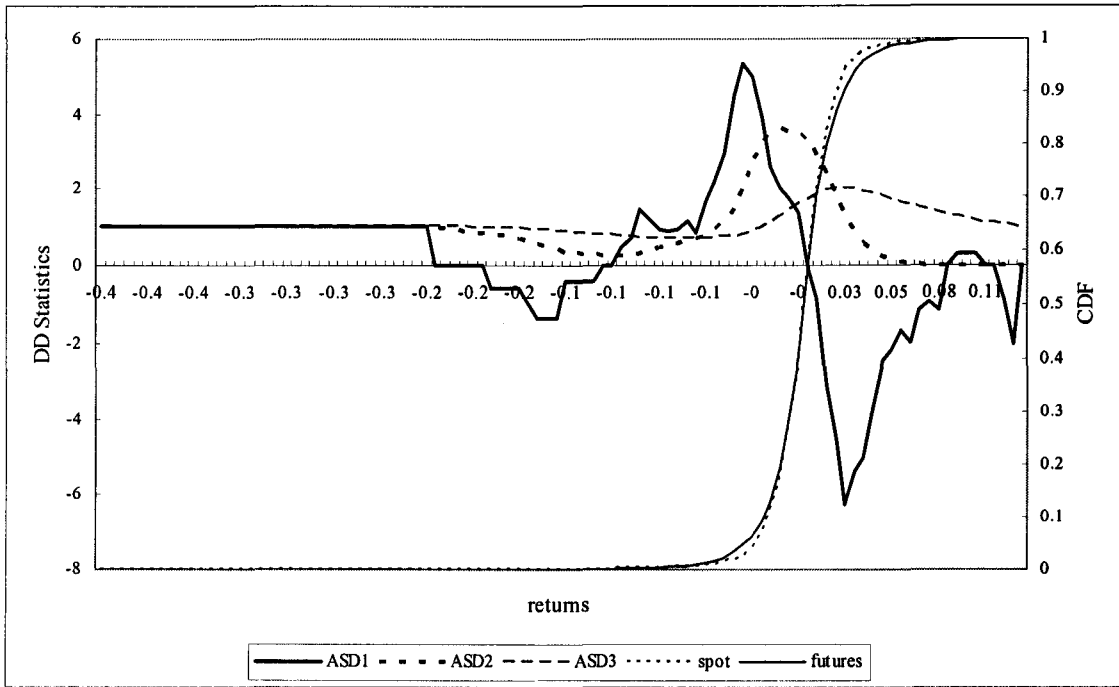


Figure 2: Descending Distribution of Returns and DD Statistics for Risk Seekers - Whole Period

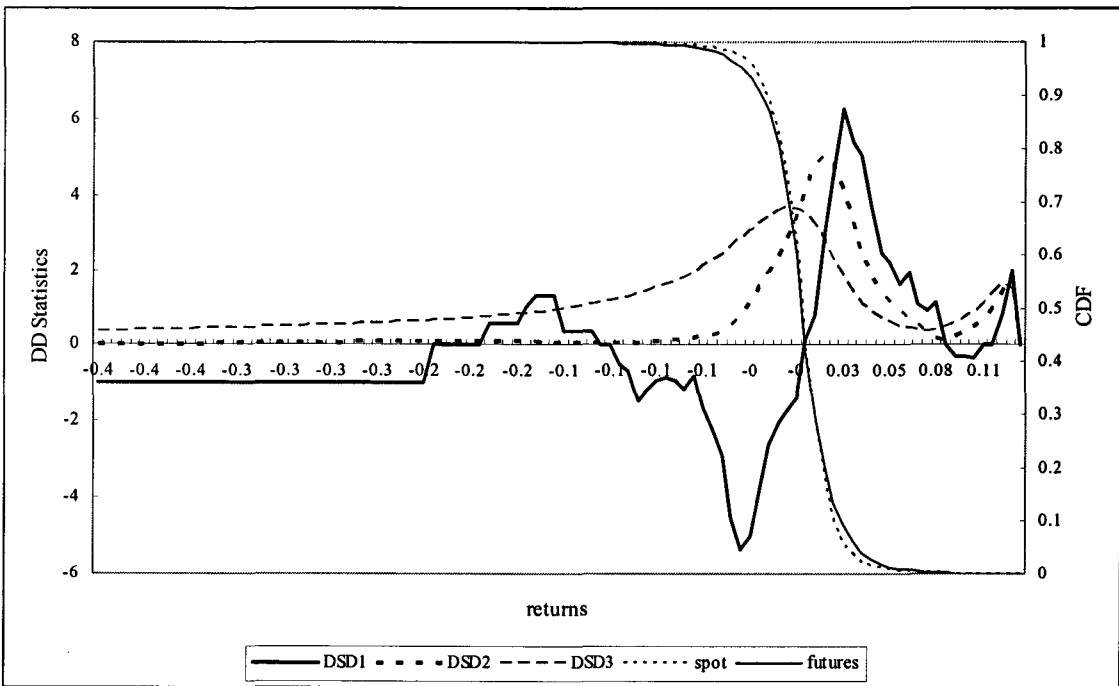




Figure 3: Distribution of Returns and DD Statistics for Risk Averters – Pre-AFC

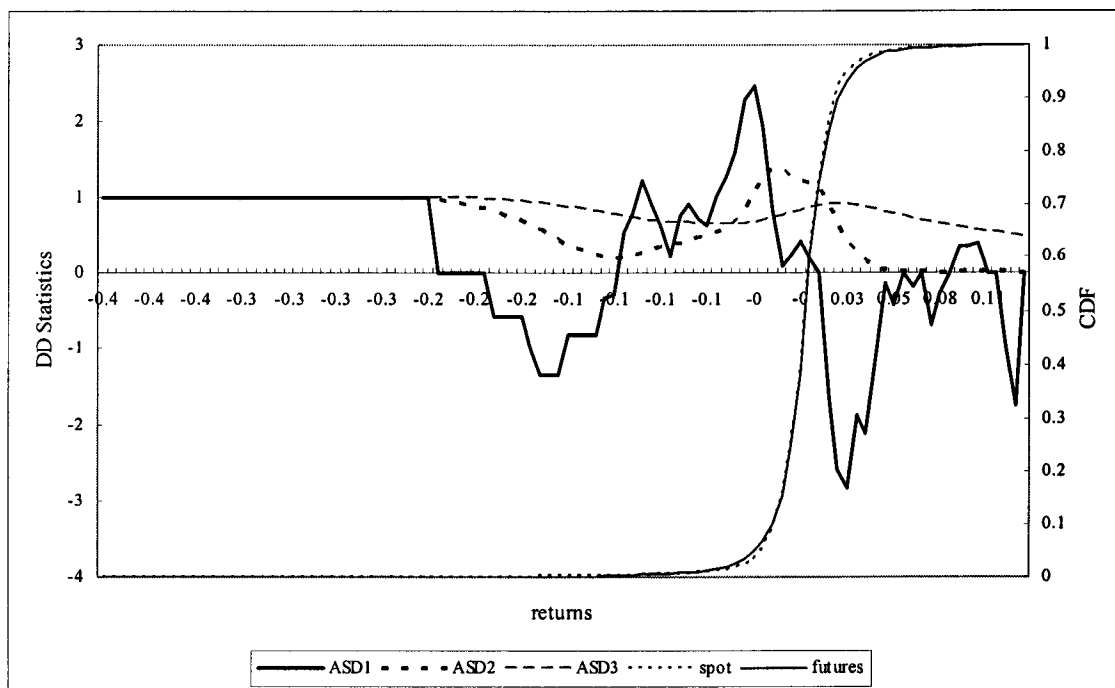


Figure 4: Descending Distribution of Returns and DD Statistics for Risk Seekers - Pre-AFC

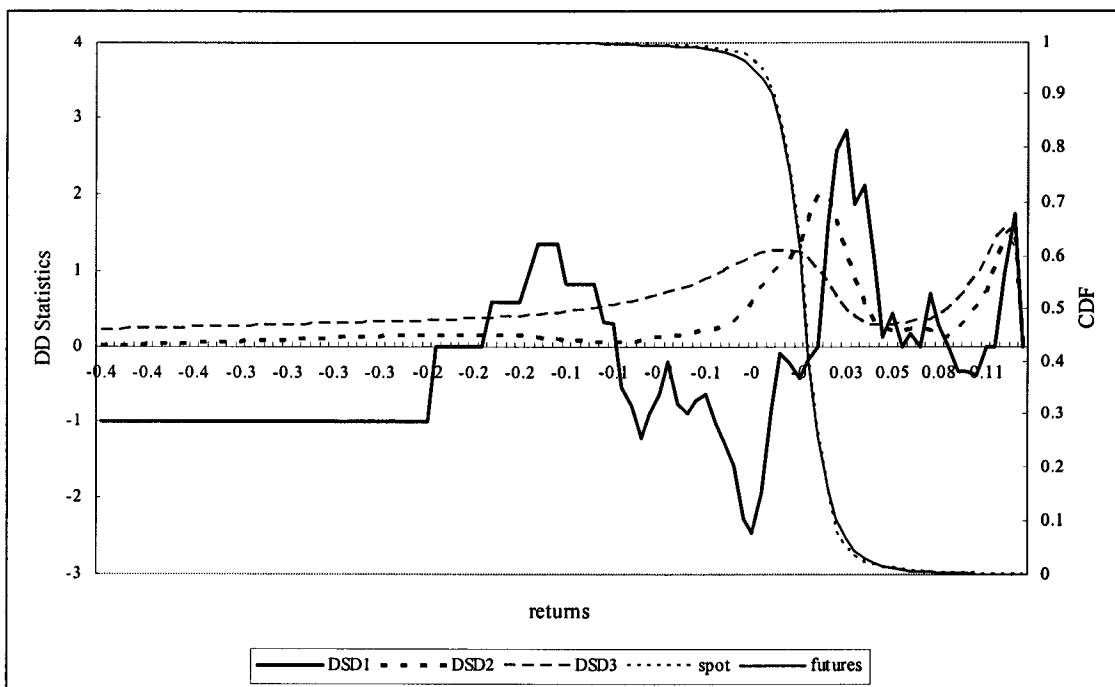


Figure 5: Distribution of Returns and DD Statistics for Risk Averters – Post AFC

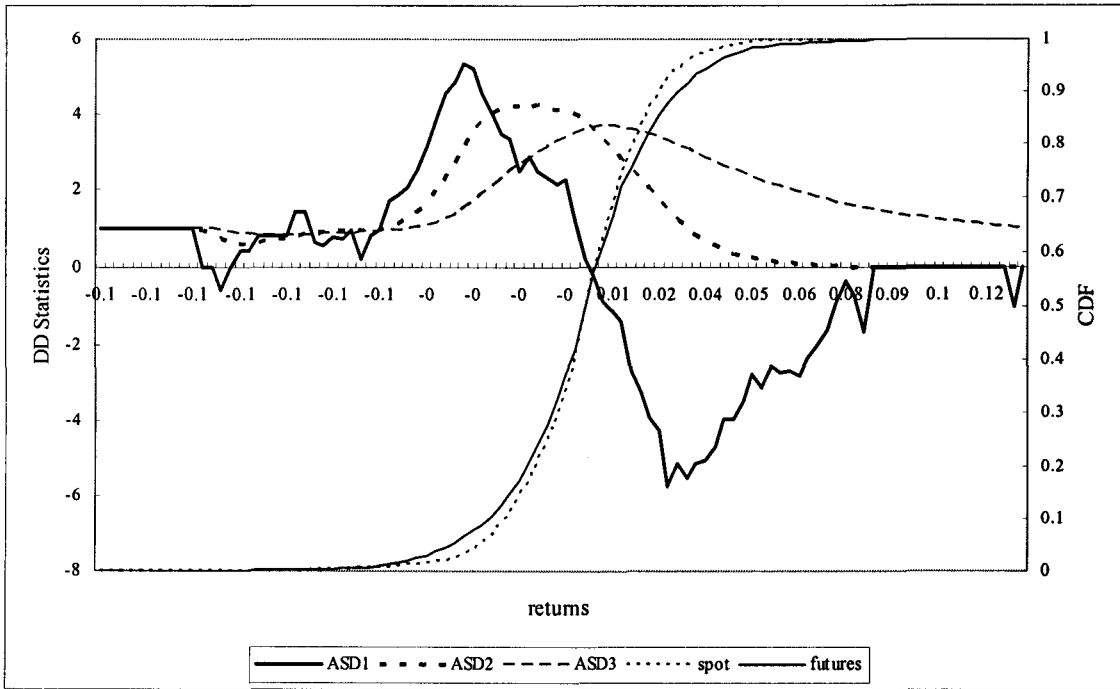
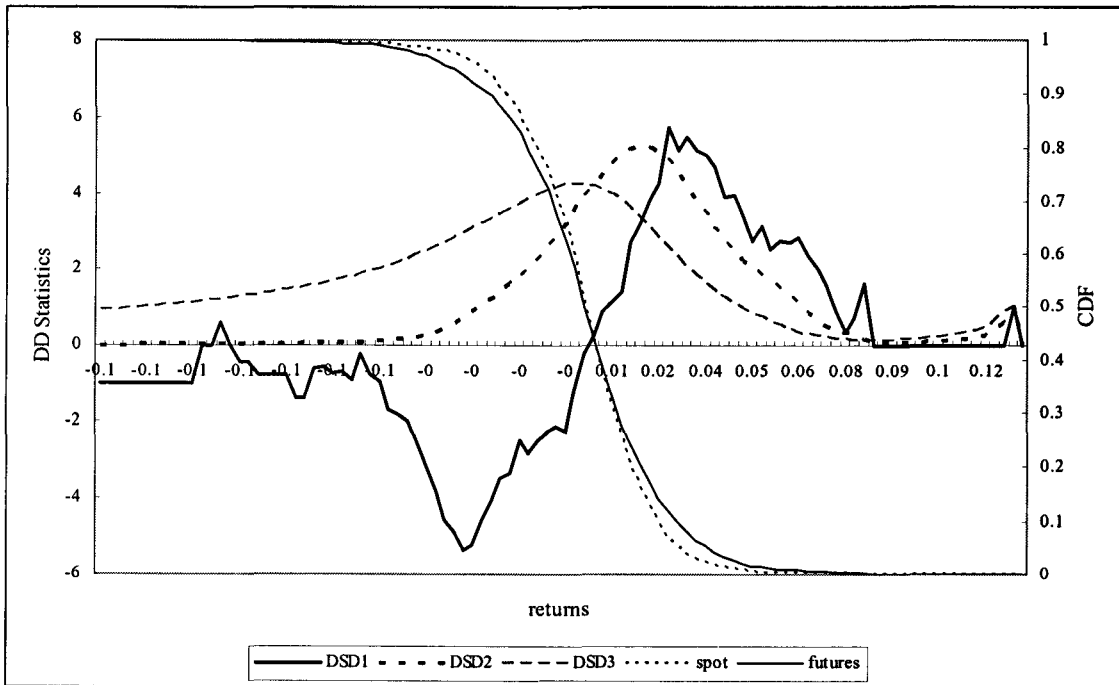


Figure 6: Descending Distribution of Returns and DD Statistics for Risk Seekers – Post AFC



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