Optical Bistability from Ferroelectric Fabry-Perot Interferometer

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We study the nonlinear optical bistability and multistability of a nonlinear ferroelectric (FE) Fabry-Perot (FP) interferometer assumed coated with partial dielectric mirrors on its interfaces. The FE material is considered to have an intensity-dependent refractive index where the third order nonlinear susceptibility $\chi^{(3)}_{ilmn}$ acts like Kerr coefficient. The nonlinear response of the FE medium is modeled by the Landau-Khalatnikov (LK) dynamical equation. Within a single frequency approximation, the nonlinear wave equation is written in terms of polarization P rather than the electric field E as the dependent variable. The resulting nonlinear polarization equation is numerically integrated across the FP film using the fourth order Runge-Kutta method. The behavior of the resulting polarization and transmission as a function of the field input intensity is investigated.

Keywords: Nonlinear optics; optical Bistability; Fabry-Perot; Ferroelectric; Intensity-Dependent refractive index; Landau-Khalatnikov equation

INTRODUCTION

Because bistable devices are useful in the field of optical communications, the optical bistable response of a Fabry-Perot (FB) interferometer containing a nonlinear material with Kerr nonlinearity has attracted a great deal of interest [1-3]. The standard analysis in studying this problem in nonlinear optics is to expand the optical polarization P as a Taylor series in the field E. However, Goldstone and Garmire [4] pointed out that the standard analysis is not suitable to describe this type of intrinsic optical bistability. Recently, K-H Chew et al [5] considered a dielectric nonlinear FP interferometer following the approach used in Ref [4]. They developed an alternative analysis based on the Duffing oscillator which is found to be more suitable to frequency ranges where the nonlinear response of materials like the FE materials is strong and resonant.

Recently, Rajan et al [6] have derived expressions of the tensor elements for various second- and third-order nonlinear optical effects including optical Kerr bistability for bulk FE having various symmetries. They have shown that many of these elements have large linear and nonlinear optical coefficients even in the visible and near-infrared frequency regions. They have found that it is the combination of the temperature divergence and the resonant frequency, which is typically in the THz region, dependence that underlies their large values. For these reasons we are interested in investigating theoretically into the possibility of generating optical bistability effect from FE materials. Moreover, because of the development of high-power THz sources [7,8] it may now be possible to demonstrate experimentally bistability phenomenon in these materials.

In this work we apply the Duffing model analysis to a Fabry-Perot (FP) interferometer containing a FE material coated with partially-reflecting mirrors. Since the material is FE, the Landau-Khalatnikov (LK) dynamical equation and the Landau-Devonshire free energy are used in the formulation [5]. A nonlinear polarization equation is derived and integrated across the etalon thickness. The resultant nonlinear polarization and transmission are plotted as functions of the input intensity of the field.

MATHEMATICAL FORMULATION

We consider a FP interferometer system consisting of a FE etalon (medium 2) of thickness L as shown in Fig. 1, which includes partially-reflecting mirrors at its interfaces. The mirrors can be modeled by a thin metallic layer of thickness δ_M, permittivity ε_M and conductivity σ_M [9]. For simplicity, a far-infrared (FIR) radiation with a single frequency ω is assumed incident normally on the etalon. The fields in medium 1 and 3 (considered vacuum) may be written as $E_1 = E_0\left[\exp(-ik_1z) + r\exp(ik_1z)\right]$ and $E_3 = tE_0\exp\left[-ik_3\left(z+L\right)\right]$ respectively. The reflection $R = |r|^2$ and transmission $T = |t|^2$ are evaluated following the standard analysis in linear optics [10].

Suppose the FE medium is in the paraelectric phase having a cubic symmetry. The Landau-Devonshire free energy for the bulk FE is [11]

$$F_i(P) = \frac{\alpha}{2\varepsilon_0}P_i^2 + \frac{\beta_1}{4\varepsilon_0^{\,2}}P_i^4 - E.P \tag{1}$$

(1) is the simplest binding potential that leads to bistable behavior where $\alpha = a\left(T - T_C\right)$ with a being the inverse of the Curie constant and T_C is the Curie temperature, and β_1 is a material dependent parameter. The term $E.P$ accounts for the coupling of the *FIR* radiation to the driving field *E*. The response of a FE material exposed to a high-

intensity FIR radiation may be described by the LK dynamical equation of motion in terms of polarization, P,

$$M(d^2P_i/dt^2)+\gamma\left(dP_i/dt\right)=-\partial F/\partial P \qquad (1)$$

M and γ are inertial and damping parameters respectively. $\gamma\, dP_i/dt$ represents a linear loss. Using (1) and (2), it is possible to derive E in terms of P, which is

$$E_d=(3\sqrt{3}/2)\left[\left[-M_d f^2-i\gamma_d f+(T_d-1)\right]p+(3/4)|p|^2 p\right] \qquad (3)$$

in the dimensionless form, using the following scaling: $f=\omega/\omega_0$, $E_d=E/E_0$, $p=P/P_0$, $T_d=T/T_C$, $P_0^2=aT_c\varepsilon_0/\beta_1$ and $E_0^2=\left[4a^3T_C^3/27\varepsilon_0\beta_1\right]$. ω_0 is the material resonance frequency. In (3), $M_d=\left[M\omega_0^2\varepsilon_0/aT_C\right]$ and $\gamma_d=\left[\gamma\omega_0\varepsilon_0/aT_C\right]$. In the high frequency limit, the scaled form of the electromagnetic wave equation for waves propagating in the z-direction is

$$-\frac{d^2E_d}{du^2}=f^2\varepsilon_\infty E_d+\left[P_0/E_0\varepsilon_0\right]f^2 p \qquad (4)$$

Here ε_∞ is the high-frequency limit of the complex dielectric function $\varepsilon(\omega)$ and $u=\omega_0 z/c$ is the scaled thickness. Microscopically, the driving field in (4) is approximately considered to be equivalent to the net field inside the material [12]. (3) and (4) give the following ODE in polarization, which is

$$\begin{aligned}&\left[\psi_d+\frac{3}{2}|p|^2\right]\frac{d^2p}{du^2}+\frac{3}{4}p^2\frac{d^2p^*}{du^2}+3p\frac{dp}{du}\frac{dp^*}{du}+\\&\frac{3}{2}p^*\left(\frac{dp}{du}\right)^2=-f^2\varepsilon_\infty\left[\psi_d+\frac{3}{4}|p|^2+w\right]p\end{aligned} \qquad (5)$$

with $\psi_d=\left[-M_d f^2-if\gamma_d+(T_d-1)\right]$ and $w=\left(2\sqrt{3}/9\right)\left(P_0/E_0\varepsilon_0\varepsilon_\infty\right)$. d^2p^*/du^2 may be eliminated from (5); and the resulting form may be integrated numerically as an initial value problem to evaluate the desired polarization. Applications of the standard electromagnetic boundary conditions lead to

$$(1+r)E_d = \frac{3\sqrt{3}}{2}\left[\psi_d p_t + \frac{3}{4}|p_t|^2 p_t\right] \tag{6}$$

$$\left[(1-r)n_0 + i(1+r)\alpha_d\right] f E_d = i\frac{27}{4}\left[\psi_d \frac{dp_t}{du} + \frac{3}{4}\left(p_t^2 \frac{dp_t^*}{du} + 2|p_t|^2 \frac{dp_t}{du}\right)\right] \tag{7}$$

$$t E_d = \frac{3\sqrt{3}}{2}\left[\psi_d p_b + \frac{3}{4}|p_b|^2 p_b\right] \tag{8}$$

$$(n_0 + i\alpha_d) f t E_d = i\frac{27}{4}\left[\psi_d \frac{dp_b}{du} + \frac{3}{4}\left(p_b^2 \frac{dp_b^*}{du} + 2|p_b|^2 \frac{dp_b}{du}\right)\right] \tag{9}$$

where p_t and p_b each refers to the polarization at the top and bottom boundaries respectively and the term $\alpha_d = [\varepsilon_M \delta_M \omega / c]$ accounts for the mirror. (6) and (7) may be solved for E_d, while (8) and (9) for dp_b / du.

RESULTS AND DISCUSSIONS

Since there is no incoming wave in medium 3, it is more convenient to integrate the ODE in (5) from the boundary $u = -l$ to $u = 0$. Our numerical strategy is as follows: we assume an arbitrary value of $p_b = p_b^* = p_0$ at the boundary $u = -l$ and evaluate dp_b / du. We then integrate (6) as an initial value problem from $u = -l$ to $u = 0$. This gives p_t, dp_t / du and their complex conjugates. Subsequently, values of E_d, r and t are calculated. The procedure is repeated for a large number of arbitrary p_b values. The available experimental data of $BaTiO_3$ is used in our computation to obtain more realistic results [13]. The operating frequency $f = 1.3\omega_0$ is selected. We also use relatively small value of thickness $l = 4$ appropriate for ionic solid in the FIR.

Fig. (2) shows plot of $|p_b|$ as a function of $|E_d|$. It is observed that the system responds linearly for small values of $|E_d|$ and after reaching a certain threshold value, a bistable behavior is exhibited but for a very narrow range of $|E_d|$, i.e. ~ 870 to 900, which is clearly observed from the inset. Here, the threshold value is $|E_d| \approx 870$ which is $|E| \approx 2.95 \times 10^{10}\ Vm^{-1}$, a physically realizable value. It is also observed that as $|E_d|$ increases beyond 900, a multistability phenomenon sets in. Fig. (3) illustrates the transmittance T curve (Inset: the reflectance R curve), which has typical features found in bistability and multistability behaviour.

CONCLUSIONS

Our results show that it is possible to generate bistability and multistability phenomena from a FE Fabry-Perot etalon. We have demonstrated that the approach we adopted is more suitable for ferroelectrics particularly at frequency ranges where the nonlinear response of the material is strong and resonant. It may be possible to achieve a lower threshold value of input intensity at different operating frequencies, and by varying the system parameters like etalon thickness, mirror reflectivity etc., which we are currently investigating.

ACKNOWLEDGEMENT

One of the authors (JO) would like to thank the Academy of Science Malaysia for awarding the SAGA grant and for making this work possible.

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FIGURE CAPTIONS

Fig. 1: Geometry of the Fabry-Perot interferometer system.

Fig. 2: $|p_t|$ versus $|E_d|$ for $BaTiO_3$ at scaled frequency $f = 1.3$ with mirror parameter $\alpha = 0.2$ corresponding to mirror reflectance of $R_M = 0.827$, $\varepsilon_\infty = 2$, and $l = 4$.(Inset: magnification of the curve for small values of $|E_d|$).

Fig. 3: T versus $|E_d|$ for $BaTiO_3$ with other parameters remain unchanged. The vertical line drawn at $|E_d| = 1.5\text{x}10^3$ shows that T assumes seven stable points. (Inset: R versus $|E_d|$).

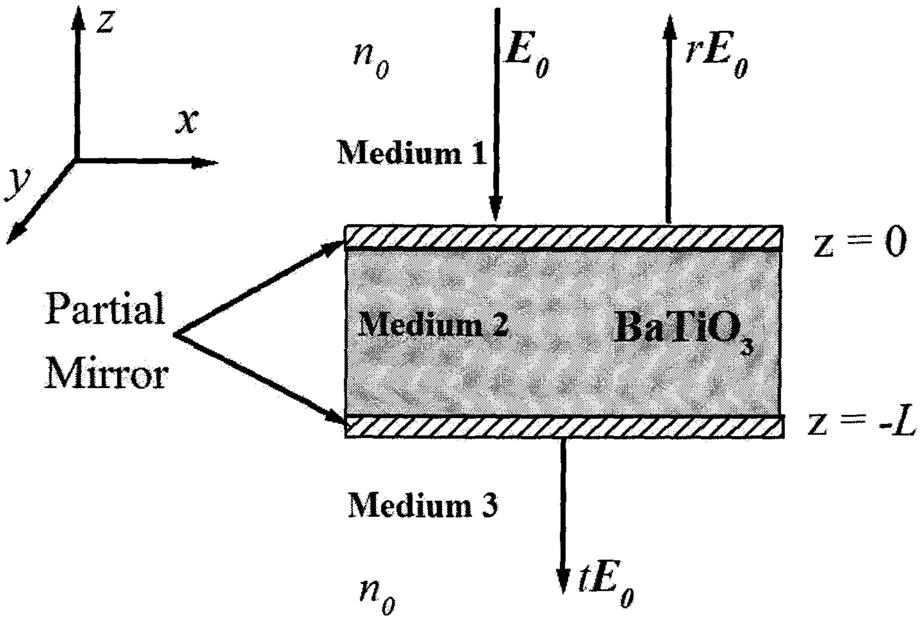

z
x
y
n_0
E_0
rE_0
Medium 1
z = 0
Partial
Mirror
Medium 2
$BaTiO_3$
z = -L
Medium 3
n_0
tE_0

Fig.(1)

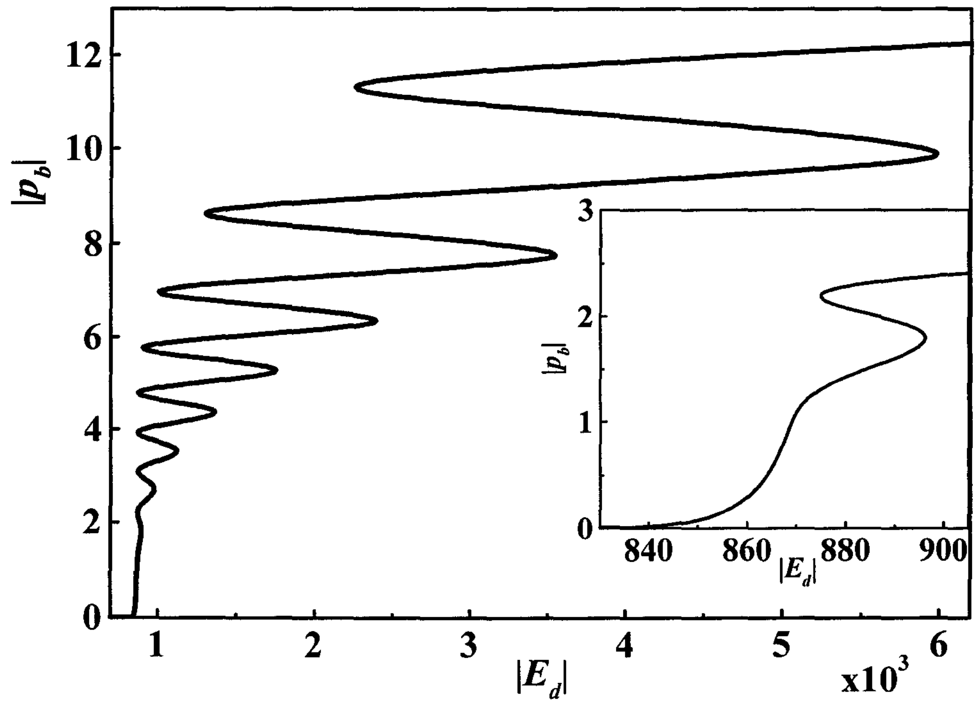
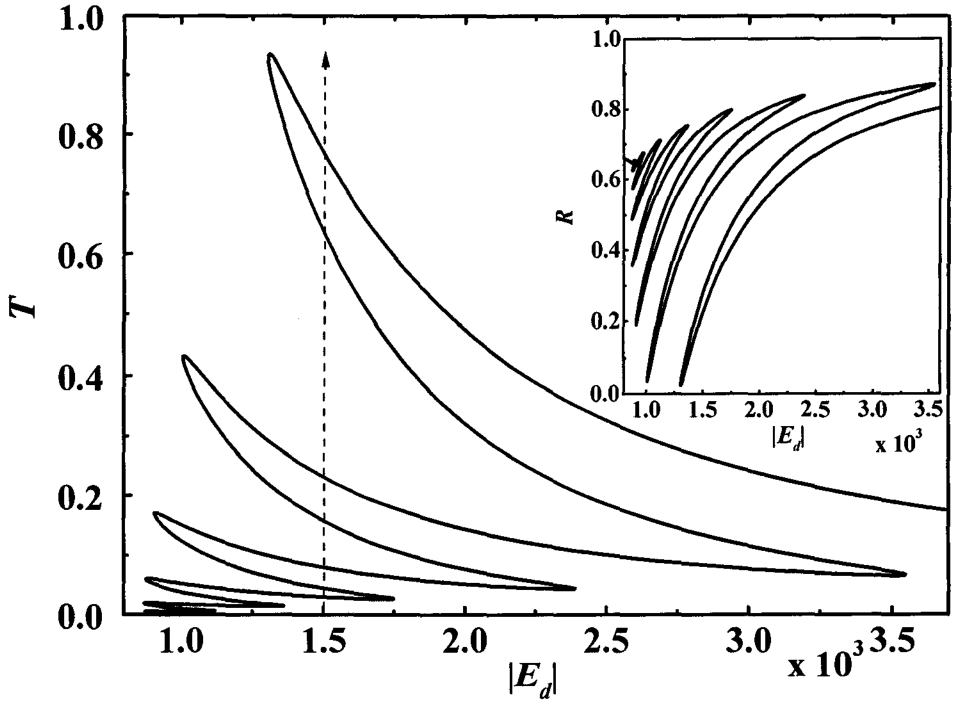


Fig.(2)



1.0
0.8
0.6
0.4
0.2
0.0
T
1.0
1.5
2.0
2.5
3.0
3.5
x 10^3
|E_d|
R
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.5
2.0
2.5
3.0
3.5
|E_d|
x 10^3

Fig.(3)