UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

ESA 322/3 – Structural Dynamics
[Dinamik Struktur]

Duration : 3 hours
[Masa : 3 jam]

Please check that this paper contains **TWELVE (12)** printed pages, **THREE (3)** pages appendix and **SIX (6)** questions before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **DUA BELAS (13)** mukasurat bercetak,**TIGA (3)** mukasurat lampiran dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **SIX (6)** questions.

***[ARAHAN :** Jawab **ENAM (6)** soalan.]*

Answer to each question must begin from a new page.

[Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.]

Appendix/*Lampiran :*

1. Fundamental Equations in Vibration [1 page/*mukasurat*]
2. Vibration-Related Formulas [1 page/*mukasurat*]
3. Finite Element method [1 page/*mukasurat*]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. [a] Answer the following questions in words. Supplement your answer with diagrams if required.

(i) What are the three elementary parts of a vibrating system

(ii) What is viscous damping and how does it transpires?

(iii) Write short notes about Fourier transform.

(30 marks)

[b] Plot vibration time response that illustrates a single degree of freedom system undergoing oscillation for these cases;

(i) Resonance

(ii) Beating phenomenon

(iii) Underdamped

Responses must be plotted on separate axis and label clearly

(30 marks)

[c] **Figure 1** shows a free response of a single degree of freedom system undergoing decay oscillation due to the existence of viscous damping. Based on **Figure 1** and by using appropriate equations compute:

(i) Damped natural frequency of the system in rad/s.

(ii) Damping ratio of the system.

(iii) Undamped natural frequency of the system in rad/s.

(iv) Viscous damping coefficient of the system in N-s/m if mass m = 1.0kg.

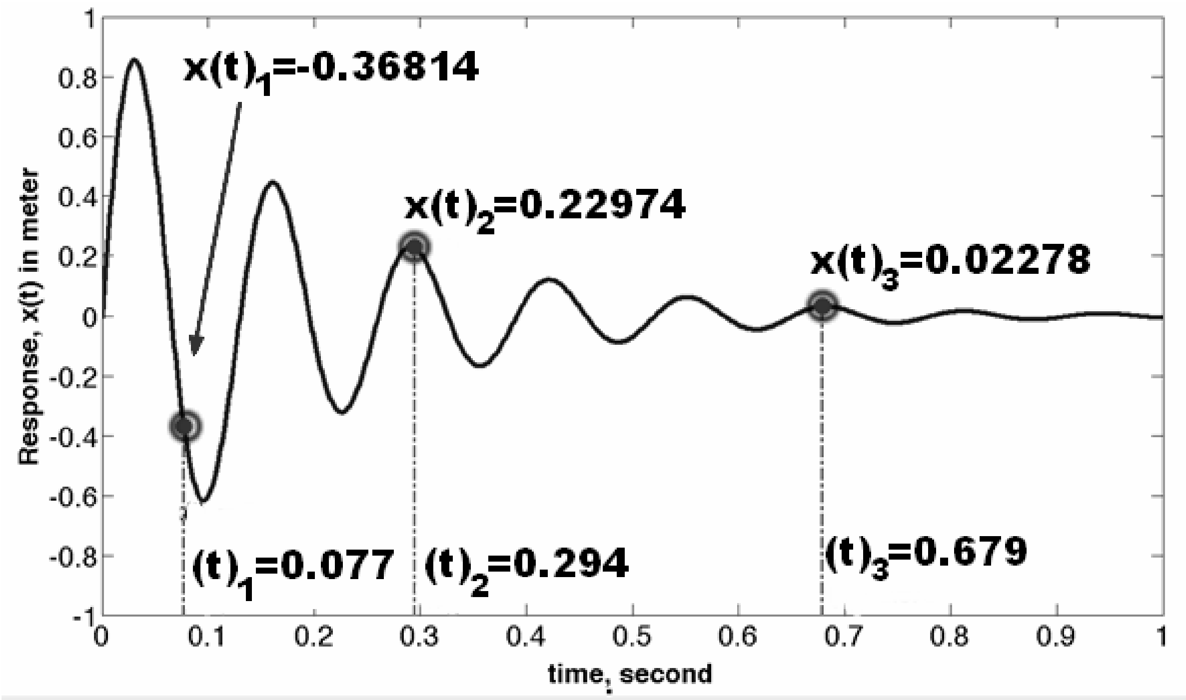


Figure 1

(40 marks)

2. [a] Prove the expression for steady state amplitude for a single degree of freedom system undergoing undamped force oscillation is equivalent to

$$A = \frac{\frac{F_o}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

(50 marks)

[b] A trainer aircraft shown in figure 2 has two external fuel tanks located at the tip of its wings. The length of the wing tip to the body is 5 meter. The wings are only susceptible to vertical oscillation with respect to the aircraft body. The mass of the fuel when the tanks are full is 300kg. The fuel tank itself weighs 50kg each. By considering only one side of the wing and assuming the wing structure has low damping and no mass, do the following;

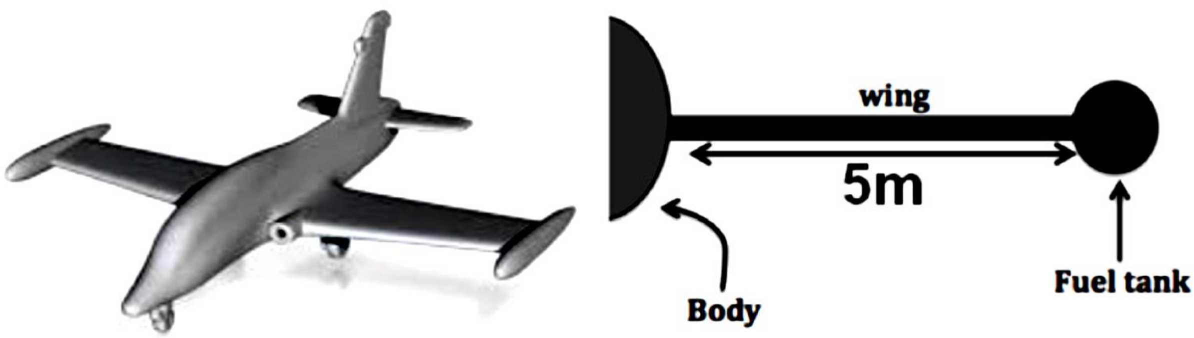


Figure 2

(i) Sketch a dynamic free body diagram of the system.

(10 mark)

(ii) Derive the equation of motion of the system in terms of m, c and EI expression where there is no external force exist. The stiffness of the structure is given as

$$k = \frac{300EI}{L^3}$$

(10 marks)

(iii) Predict the undamped natural frequency of the wing structure when the fuel tank is empty, half full and full of fuel where EI = 1500Nm^2.

(10 marks)

(iv) Describe the trend of the steady state amplitude of the wing tip during flight if the aircraft takes off carrying full fuel load and lands with 20% fuel left. Assume the wing is experiencing 0.87Hz sinusoidal excitation with force of 7000N throughout the whole flight duration. Viscous damping is 1200N-s/m. Provide reasons for your answer.

(20 marks)

3. Consider a two-degree of freedom system shown in figure 3, where the pulley has no mass. Given mass, m = 1kg, the spring stiffness, k = 16N/m. By neglecting the gravitational acceleration, you must do the following:

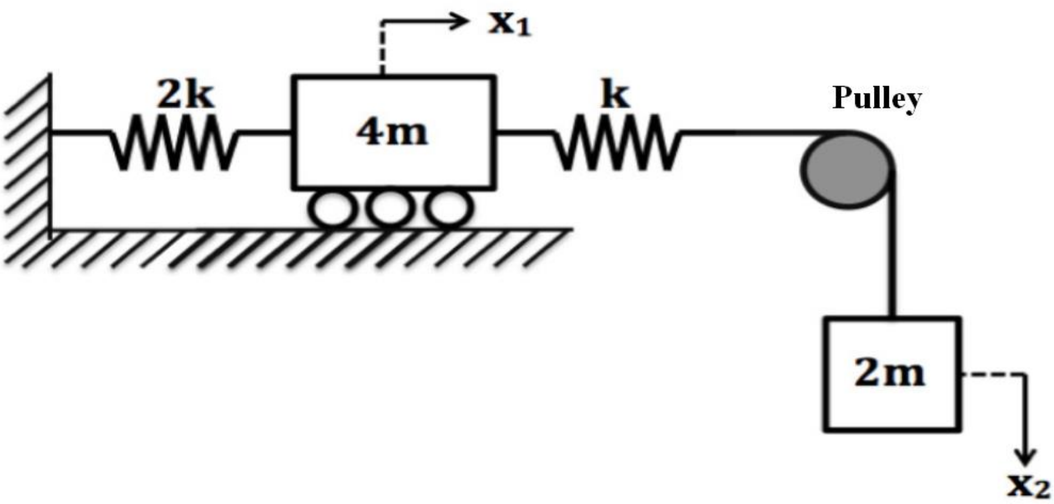


Figure 3

(i) Draw the free body diagram of the system.
(ii) Derive the equation of motion of the system in matrix form.
(iii) Determine the characteristic equation.
(iv) Calculate the natural frequencies.
(v) Calculate and draw the mode shapes.

(100 marks)

4. [a] With the help of Collar's triangle, write short notes on the following aeroelastic phenomena.

(i) Vortex Induced Oscillation
(ii) Limit Cycle Oscillation
(iii) Bending torsion flutter
(iv) Shock buffet

(40 marks)

[b] Within the scope of 2 dimensional incompressible flow regime, discuss in detail the quasi-steady aerodynamics and unsteady aerodynamics models that are used in aeroelastic analysis. List all the parameters that contribute to the aerodynamic forces for both models.

(60 marks)

5. [a] An airfoil section with mass M_m and Inertia I_I shown in **Figure 5** undergoes a combined bending and torsion modes of oscillation. The stiffness for the two-degree of freedom system is provided by linear and torsional springs. Obtain the equation of motion of the system using Lagrange's approach assuming there are aerodynamic forces (Lift L and pitching moment M) acting on the airfoil.

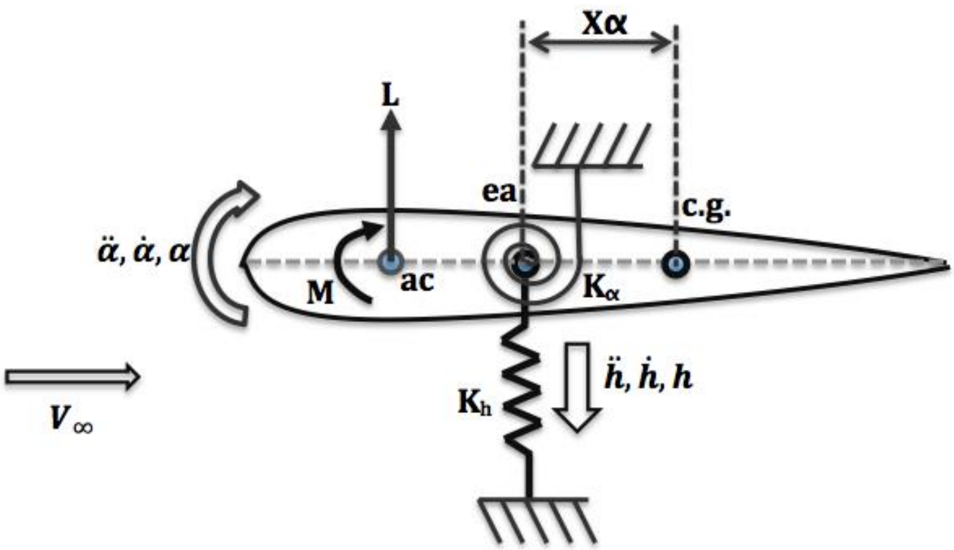


Figure 5

(40 marks)

[b] Give details on the five methods of structural excitation that could be employed for flight flutter testing. Which method you would recommend to be used for combat aircraft flight flutter test?

(60 marks)

6. Consider one-dimensional rod structure with uniform cross section shown in the **Figure 6**. By discretizing the structure into 2 bar elements, compute the following by using the information provided in Table 1.0.

   (i) Assembled stiffness matrix of the structure.
   (ii) Assembled mass matrix of the structure.
   (iii) Natural frequencies of the structure.
   (iv) Mode shapes of the structure.

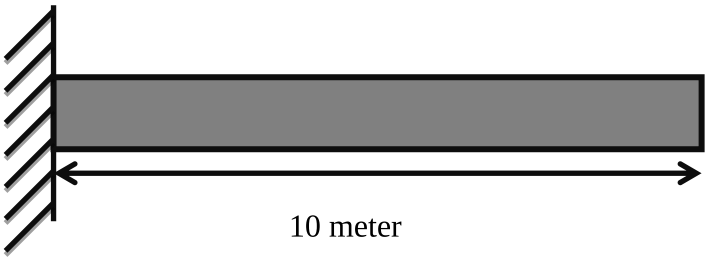


Figure 6

Table 1.0

	Value	**Unit**
Young Modulus	50	MPa
Cross sectional area	0.05	m
Density	7800	Kg/m^3

(100 marks)

1. [a] *Berikan jawapan kepada soalan berikut dalam bentuk perkataan. Lengkapkan jawapan dengan rajah jika perlu.*

(i) *Senaraikan tiga asas utama sistem getaran.*

(ii) *Apakah yang dimaksudkan dengan redaman likat. Bagaimana ia berfungsi?*

(i) *Tuliskan nota ringkas mengenai transformasi Fourier.*

(30 markah)

[b] *Plot sambutan masa getaran yang mengambarkan sistem satu darjah kebebasan yang menjalani getaran untuk setiap kes berikut;*

(i) *Resonans*

(ii) *Alun*

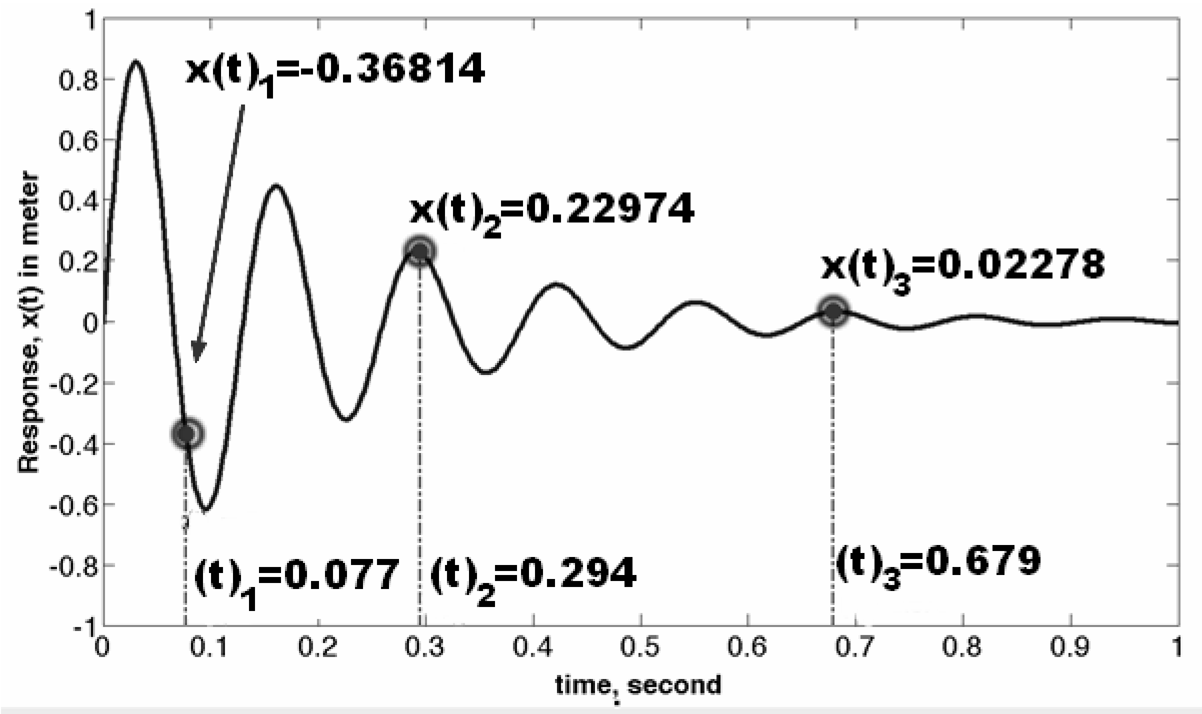
(iii) *Kurang redaman*

Sambutan getaran mesti di lakarkan di atas paksi berasingan dan dilabel dengan lengkap.

(30 markah)

[c] ***Rajah 1*** *menunjukkan sambutan getaran bebas untuk sistem satu darjah kebebasan yang mengalami getaran redaman disebabkan wujudnya redaman likat. Berdasarkan* ***Rajah 1*** *dan dengan menggunakan persamaan-persamaan yang berkenaan, kirakan:*

(i) *Frekuensi tabii teredam sistem tersebut dalam unit rad/s.*
(ii) *Nisbah redaman sistem tersebut.*
(iii) *Frekuensi tidak teredam sistem tersebut dalam unit rad/s.*
(iv) *Pekali redaman likat sistem tersebut dalam unit N-s/m jika jisim m = 1kg.*



Rajah 1

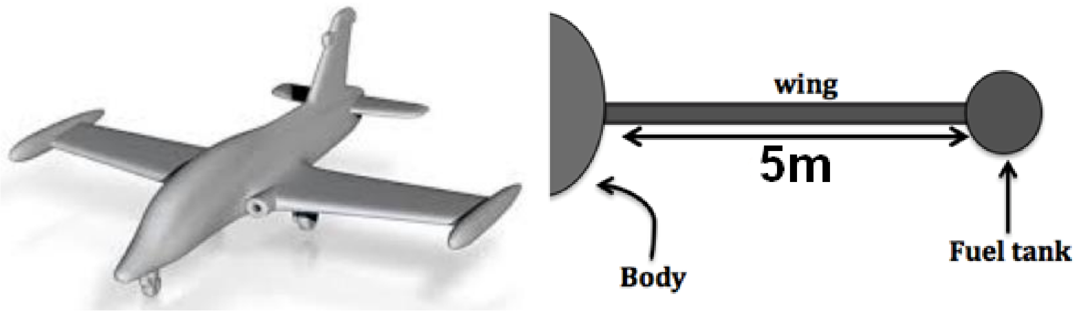
(40 markah)

2. [a] *Buktikan ungkapan yang mewakili amplitud untuk sistem getaran paksa satu darjah kebebasan tidak teredam ialah seperti berikut;*

$$A = \frac{F_o / k}{1 - \left(w / w_n \right)^2}$$

(50 markah)

[b] *Satu pesawat latihan seperti yang ditunjukkan dalam rajah 2 mempunyai dua tangki minyak yang terletak pada setiap hujung sayap. Jarak diantara hujung sayap dan badan pesawat ialah 5m. Sayap pesawat tersebut mampu melakukan getaran menegak berdasarkan rujukan pada badan pesawat. Jisim bahan api apabila kedua-dua tangki penuh diisi ialah 300kg. Setiap tangki pesawat tersebut mempunyai jisim sebanyak 50kg. Dengan mengambil kira hanya satu sayap dan menganggap sayap tersebut mengalami redaman yang rendah serta tiada jisim,lakukan yang berikut;*



Rajah 2

(i) *Lakarkan rajah badan bebas sistem tersebut.*

(10 markah)

(ii) *Terbitkan persamaan gerakan sistem dengan menggunakan ungkapan m,c dan EI untuk sistem yang tidak mengalami tindakan daya luaran. Kekakuan struktur sayap pesawat diberi sebagai;*

$$k = \frac{300EI}{L^3}$$

(10 markah)

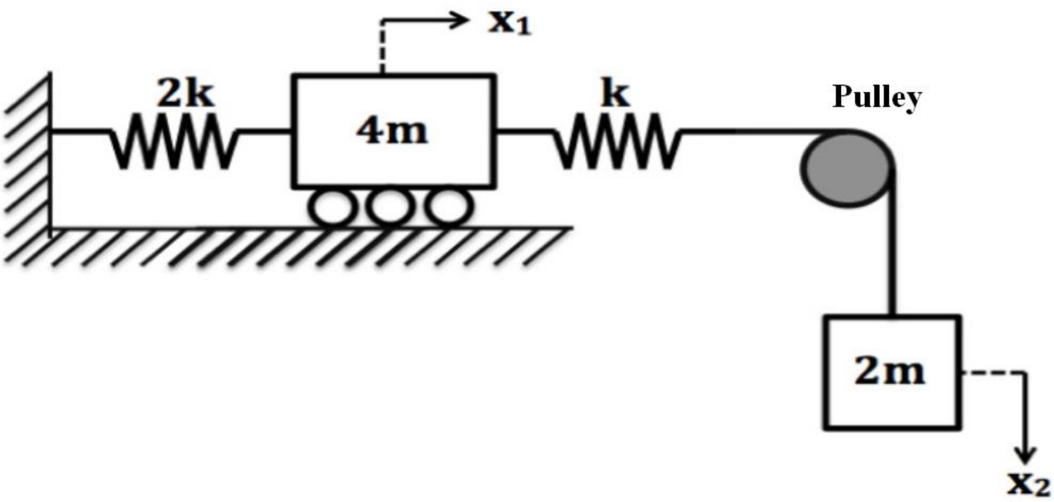
(iii) *Jangkakan frekuensi tabii tidak teredam struktur sayap pesawat tersebut apabila tangki bahan api kosong, separuh penuh dan penuh. Nilai EI = 1500Nm^2.*

(10 markah)

(iv) *Terangkan tren amplitud yang dialami oleh hujung sayap pesawat jika pesawat berlepas membawa tangki yang penuh diisi dengan bahan api dan membuat pendaratan bersama 20% baki bahan api. Anggapkan sayap pesawat dikenakan 0.87Hz daya harmonik sebanyak 7000N semasa pesawat terbang. Struktur pesawat mengalami redaman likat sebanyak 1200N-s/m. Berikan alasan kepada jawapan anda.*

(20 markah)

3. *Sistem dua darjah kebebasan ditunjukkan dalam **Rajah 3** mengandungi roda takal yang tidak mempunyai jisim. Anggapkan m =1kg dan keanjalan spring bersamaan dengan k = 16N/m. Dengan mengabaikan pecutan graviti, anda mesti:*



Rajah 3

(i) *Lakarkan rajah badan bebas kedua dua jisim.*
(ii) *Terbitkan persamaan gerakan sistem dalam bentuk matriks.*
(iii) *Tentukan persamaan ciri sistem tersebut.*
(iv) *Kirakan frekuensi-frekuensi tabii.*
(v) *Kirakan dan lakarkan bentuk bentuk mod.*

(100 markah)

4. [a] *Dengan berpandukan segitiga Collar, tulis nota ringkas mengenai fenomena-fenomena aeroleastik berikut;*

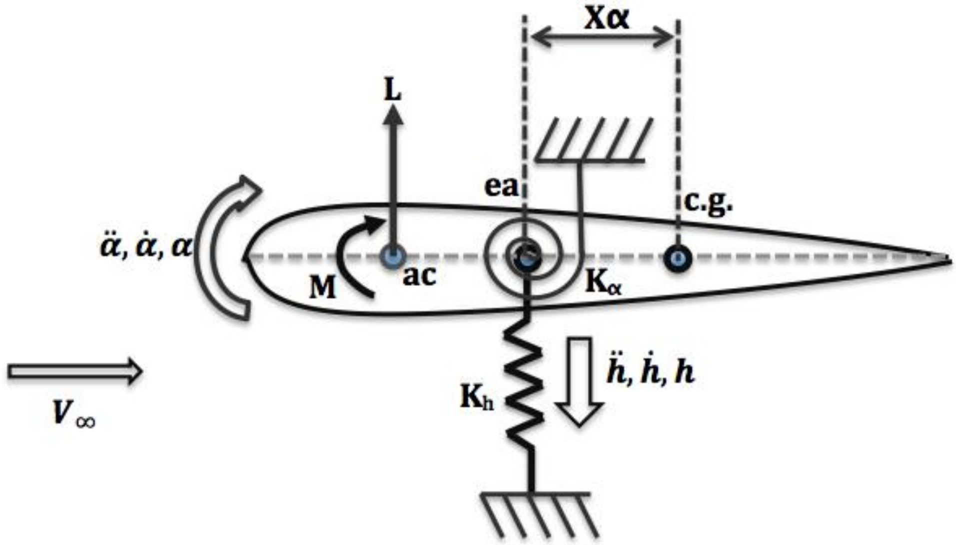
   (i) Getaran induksi vorteks
   (ii) Getaran litar terhad
   (iii) Kibaran lentur kilas
   (iv) Gelombang kejut paluan

(40 markah)

[b] *Dalam cakupan 2 dimensi aliran tak boleh mampat, bincangkan dengan terperinci model aerodinamik kuasi mantap dan model aerodinamik tak mantap yang digunakan dalam analisa aeroelastik. Senaraikan semua parameter yang menyumbang kepada penghasilan daya aerodinamik untuk kedua-dua model tersebut.*

(60 marks)

5. [a] *Satu kerajang sayap dengan jisim M_m dan Inertia I_I ditunjukkan dalam* ***Rajah 5*** *mengalami gerakan lenturan dan kilasan. Keanjalan untuk sistem dua darjah kebebasan tersebut dibekalkan oleh spring linear dan kilasan. Terbitkan persamaan gerakan sistem tersebut dengan menggunakan kaedah persamaan"Lagrange" dengan mengandaikan terdapat daya-daya aerodinamik (daya angkat L dan momen angguk M) bertindak ke atas kerajang sayap tersebut.*



Rajah 5

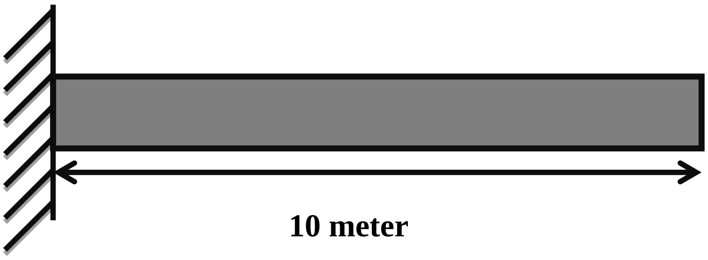
(40 markah)

[b] *Senaraikan lima kaedah pengujaan struktur sayap yang boleh dilakukan untuk ujian kibaran terbang. Berikan butiran lengkap. Cadangkan kaedah terbaik untuk digunapakai semasa ujian kibaran terbang untuk pesawat tempur.*

(60 markah)

6. ***Rajah 6*** *menunjukan struktur rod satu dimensi yang mempunyai keratan rentas yang sama. Dengan mendiskrestasi struktur tersebut kepada 2 unsur "bar" serta menggunakan* ***Jadual 1****, kirakan;*

(i) Matrik keanjalan struktur tersebut.
(ii) Matrik jisim struktur tersebut.
(iii) Frekuensi-frekuensi tabii struktur tersebut.
(iv) Bentuk-bentuk mod struktur tersebut.



Rajah 6

Jadual 1.0

	Value	**Unit**
Young Modulus	50	MPa
Cross sectional area	0.05	m^2
Density	7800	Kg/m^3

(100 markah)

-000000000-

APPENDIX 1 / *LAMPIRAN 1*

Fundamental Equations in Vibration

1. $\zeta = \dfrac{c}{2m\omega_n}$;

2. $x(t) = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$; $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

$$C = \sqrt{x_0^2 + \frac{(\dot{x}_0 + \zeta\omega_n x_0)^2}{(1-\zeta^2)\omega_n^2}} \quad ; \quad \psi = \tan^{-1}\frac{\sqrt{1-\zeta^2}\,\omega_n x_0}{\dot{x}_0 + \zeta\omega_n x_0}$$

3. For $F(t) = me\omega^2 \sin\omega t$

$$X = \frac{me\omega^2}{\sqrt{(k-M\omega^2)^2 + (c\omega^2)^2}}; \quad \phi = \tan^{-1}\left[\frac{c\omega}{k-M\omega^2}\right]$$

$$\frac{F_T}{F_0} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

4. For base excitation

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2} \qquad \phi = \tan^{-1}\left[\frac{2\zeta r^3}{1+(4\zeta^2-1)r^2}\right]$$

$$\frac{F_T}{kY} = r^2\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

5. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\dfrac{1}{\Delta(\omega)}$; $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

6. For cantilever beam with load P at the free end, $\delta_{max} = PL^3(3EI)^{-1}$

7. For cantilever beam with uniform load w, $\delta_{max} = wL^4(8EI)^{-1}$

APPENDIX 2/*LAMPIRAN 2*

Vibration-related Formulas

1. $\zeta = \dfrac{c}{2\omega_n m}$

2. $x(t) = e^{-\zeta\omega_n t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$

3. $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

4. $x_p = X\sin(\omega t - \varphi)$, $X = \dfrac{F_0/k}{\left((1-r^2)^2 + (2\zeta r)^2\right)^{1/2}}$, $\varphi = \tan^{-1}\dfrac{2\zeta r}{1-r^2}$

5. $\dfrac{X}{Y} = \left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$ $\quad \phi = \tan^{-1}\left[\dfrac{2\zeta r^3}{1+(4\zeta^2-1)r^2}\right]$

6. $\dfrac{F_T}{kY} = r^2\left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$

7. $TR = \left[\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$

8. $\dfrac{mX}{m_0 e} = \dfrac{r^2}{\left[(1-r^2)^2+(2\zeta r)^2\right]^{1/2}}$

9. $A^{-1} = \dfrac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

10. $\det(A) = ad - bc$

11. $f_d = \dfrac{1}{T}$

12. $\omega_d = \dfrac{2\pi n}{\Delta T}$

13. $\delta = \dfrac{1}{n}\ln\left(\dfrac{y_0}{y_n}\right)$

14. $\xi = \dfrac{1}{\sqrt{1+\left(\dfrac{2\pi}{\delta}\right)^2}}$

Finite Element method

Bar Element

$$[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[m] = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$