NUMERICAL INVESTIGATION OF HEAT AND MASS TRANSFER IN POROUS MEDIUM FIXED WITH VARIOUS GEOMETRIES

by

IRFAN ANJUM BADRUDDIN MAGAMI

Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

November 2006

ACKNOWLEDGEMENTS

I dedicate this work to my parents who have always encouraged, motivated and given me the moral values to lead a peaceful life. I thank my sisters and other family members for their unconditional support and motivation.

I thank my current and former supervisors Prof Z.A Zainal, Prof P A Aswatha Narayana and Prof K.N Seetharamu, whose invaluable guidance and advises have encouraged me through out the course. It was a phenomenal experience to work under them. It gives me pleasure to mention that the calm and friendly nature of Prof Z A Zainal has helped a lot as it created a conducive atmosphere to work and excel. My sincere gratitude to Prof P A Aswatha Narayana and Prof K.N Seetharamu for providing a homely environment and taking care of all our needs especially when we arrived in Malaysia.

My sincere gratitude to the dean of school of mechanical engineering for providing the facilities and all other staff of school for being cooperative during the course. I thank Universiti Sains Malaysia for funding this work. Last but not the least, thanks to all of my friends who have directly or indirectly helped me during the course.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

Ar	Aspect ratio
c_F	Form-drag factor
C_p	Specific heat
d	sphere diameter
D_p	particle diameter
$C, \overline{C}$	Species Concentration (dimensional and non dimensional respectively)
D	Mass diffusivity
g	Gravitational acceleration
G	Grashof number
h	Convective heat transfer coefficient
H	Inter-phase heat transfer coefficient
k	Thermal conductivity
K	Permeability of porous media
K'	Permeability parameter
L	Length
L_{ref}	Reference length
Le	Lewis number
n	Refractive index
N	Buoyancy ratio
p	Pressure
Pr	Prandtl number
$\overline{Nu}$	Average Nusselt number
q_r	Radiation flux
q_t	Total heat flux
r, z	Cylindrical co-ordinates

$\bar{r}, \bar{z}$ Non-dimensional co-ordinates

r_i, r_o Inner and outer radius

Ra Rayleigh number

R_d Radiation parameter

R_r Radius ratio

T Temperature

$\bar{T}$ Non-dimensional Temperature

u Velocity in x or *r* direction

u_0 Suction velocity

w Velocity in *z* direction

x, y Cartesian co-ordinates

$\bar{x}, \bar{y}$ Non-dimensional co-ordinates

Greek Symbols

α Thermal diffusivity

β_C Coefficient of concentration expansion

β_T Coefficient of thermal expansion

β_R Rosseland extinction coefficient

γ Modified conductivity ratio

ε Viscous dissipation parameter

σ Stephan Boltzmann constant

λ Power law exponent

ρ Density

ν Coefficient of kinematic viscosity

μ Coefficient of dynamic viscosity

φ Porosity

σ Stephan Boltzmann constant

ψ Stream function

$\bar{\psi}$ Non-dimensional Stream function

Subscripts

$_h$ Hot

c Cold

$_w$ wall

$_\infty$ Conditions at outer radius

$_f$ Fluid

s Solid

$_t$ Total

KAJIAN NUMERIKAL TERHADAP PERMINDAHAN HABA DAN JISIM DALAM BAHAN-ANTARA BERLIANG TETAP DENGAN PELBAGAI GEOMETRI

ABSTRAK

Usaha untuk memahami aliran haba dan jisim dalam bahanantara berliang telah meningkat berlipat ganda penydidikam dalam bahanantara berliang telah mengenalpasti beberapa aspek behantara berliang yang menghasilkan banyak penulisan jurnal. Walau bagaimanapun, terdapat banyak ruang yang tidak dapat dijawab berkaitan aliran haba dan jisim dalam bahanantara berliang. Kajian ini telah dejalankan untuk menyiasat pemindahan haba dan jisim dalam bahantara berliang dan meninjau ruang yang masih tidak dapat dijawab. Pelbagai fenomena seperiti pemindahan haba, pemindahan haba dan jisim, kesan disipasi likat, ketidakseimbangan keadaan terma antara fasa cecair dan pepejal bahantara berliang dikaji dengan mangambil kira tiga geometri iaitu bahantara berliang di sepanjang plat menegak, bahantara berliang di dalam ruang dan bahantara berliang dalam silinder menegak. Selian itu, kesan perolakan semulajadi dan sinaran juga diambil kira.

Analisa yang mendalam dijalankan dan kesan pelbagai parameter yang tiada-dimensi seperti parameter radiasi, nombor Prandtl, nombor Grashof yang diubahsuai, parameter ketelusan, nombor Rayleigh, nombor Lewis, nisbah apung, parameter disipasi likat , nilai pemindoran haba antara fasa, nisbah konduksi yang diubahsuai, nisbah bidang dan nisbah jejari dibincangkan berdasakan pemindahan haba dan jisim dalam bahantara berliang.

Secara am, didapati kesan radiasi memainkan peranan penting dalam kadar peralihan keseluruhan dari permukaan ketiga-tiga geometri yang dikaji. Didapati bahawa, pengaruh oleh keadaan ketidakseimbangan adalah kuat pada nilai fasa peralihan haba yang rendah dan nisbah keberaliran. Bagi kes, plat mendatar pula,

keadaan keseimbangan dicapai pada $H = 1000$ apabila $\gamma = 1$. Namun, keadaan keseimbangan dicapai pada $H = 100$ apabila γ dinaikkan ke 25. Diperhatikan bahawa nilai kritkal bagi parameter Pembebasan tenaga likat berubah ke nilai yang lebih tinggi apabila parameter radiasi dinaikkan. Bagi kes empatsegi lubang nombor Sherwood naik sebanyak 6.28 kali apabila nombor Lewis diubah dari 1 hingga 25. Dalam silinder, nilai kritikal bagi ε adalah 0.031 apabila R_d dinaikkan ke 1.

NUMERICAL INVESTIGATION OF HEAT AND MASS TRANSFER IN POROUS MEDIUM FIXED WITH VARIOUS GEOMETRIES

ABSTRACT

The effort to understand the heat and fluid flow behavior inside the porous medium has increased to many folds in the recent few decades. The intensified research in porous medium has generally addressed many aspects of porous medium resulting into enormous number of publications. In spite of the endeavoring effort, there are many gaps left unanswered related to the heat and fluid flow behavior inside the porous medium. The present study is undertaken to investigate the heat and mass transfer in a porous medium and thus explore some of those unanswered areas. The different phenomenon such as, heat transfer, heat and mass transfer, viscous dissipation effect, the thermal non-equilibrium condition between fluid and solid phases of the porous medium are investigated with respect to three basic geometries i.e. porous medium adjacent to a vertical plate, porous medium enclosed in a cavity and porous medium filled in vertical annular cylinder. The combined effect of natural convection and the radiation is investigated in all above mentioned phenomenon.

The rigorous in-depth analysis is carried out using FEM and the effect of various non-dimensional parameters such as radiation parameter, Prandtl number, modified Grashof number, permeability parameter, Rayleigh number, Lewis number, Buoyancy ratio, viscous dissipation parameter, inter-phase heat transfer coefficient, modified conductivity ratio, aspect ratio, radius ratio etc are discussed with respect to the heat and mass transfer inside the porous medium.

In general, it was found that the effect of radiation is significant in the overall heat transfer rate from the surface in all the three geometries being studied. It was found that the non-equilibrium effect is considerably strong at small values of the inter-

phase heat transfer coefficient and the modified conductivity ratio. In case of vertical plate the equilibrium state is reached at $H = 1000$ when $\gamma = 1$. However the equilibrium state is reached at $H = 100$ as γ is increased to 25. It was observed that the critical value of the viscous dissipation parameter shifts to higher value when radiation parameter is increased. In case of square cavity the Sherwood number increased by 6.28 times when Lewis number is varied from 1 to 25. In annular porous medium, the critical value of ε is found to be 0.031 at $R_d = 0$, However it shifted to 0.036 when R_d is increased to 1.

CHAPTER 1
INTRODUCTION

1.1 Background

The interest in the study of different aspects of porous medium has increased to many folds in recent decades. This immense amount of interest has resulted into myriad research papers addressing the wide range of problems involving porous medium such as understanding of structure of porous medium, deformation of porous medium, natural convective heat transfer etc. The study of porous medium is one of the classical subjects in science. There are various definitions of porous medium but, it can be defined in simple terms as, a medium which allows the fluid to flow through itself.

Typically a porous medium is composed of solid matrix, which is interconnected to form a web like network. This network of the medium is arranged in such a way that the pores are created in between the solid. These pores are responsible for the flow of fluid through the porous medium. The pore size in the medium is an important feature, which describes the physique of the porous medium. The relative size of the solid area and the area occupied by pores is expressed in terms of non-dimensional variable called porosity. In simple words, porosity is the ratio of the volume occupied by the pores to the total volume of the medium. The porosity of naturally occurring porous medium is generally lesser than 0.6.

There are plenty of porous mediums which are available naturally and also many are artificial as well. The well-known examples of naturally existing porous media are, soil, sand, limestone, coal, wood, human body organs etc. The examples of artificial porous medium are concrete, cigarette fiberglass, cloth etc. The study of porous media involves many aspects like the structure of solid matrix, the deformation of solid matrix etc. One such important aspect is the study of heat and fluid flow behavior inside the medium. The fluid flow could be either a single-phase flow or multiphase flow. This study can be carried out at two different levels namely macroscopic level and microscopic level. The microscopic study deals with the properties in terms of pore size or solid matrix size where as the macroscopic study focuses on the properties over a representative average volume. The present study deals with the analysis of heat and fluid flow inside the porous medium by utilizing macroscopic approach.

Henry Darcy was the pioneer in this field who gave a relation to determine the flow characteristics of a porous medium in 1856, which is commonly termed as Darcy law. He developed this law while investigating the hydrologic network of a place named Dijon in France. His law became the foundation-stone for flow study in porous media. By and large, his law is still much popularly used and plenty of researchers are using the same to predict fluid behavior in the porous medium. Darcy investigated the phenomenon of water filtering but gave limited physical reasoning. Later on it was Dupuit who used the theoretical principles to explain it physically.

Apart from Darcy, there are other researches whose contributions have been significant. Forchheimer modified the Darcy equation and introduced a new term accounting for the flow resistance. Brinkman studied the effect of viscous shear stresses due to surface of porous medium adjacent to the fluid. The present day research revolves around these three models or combinations of these models to account the flow behavior under different situations.

1.2 Applications of porous medium

One may ask the question as what is the need of studying heat and fluid flow behavior in the porous medium. Well, the answer is that the porous medium is involved in numerous applications covering a large number of engineering disciplines. Some of these applications are:

1. Combustion processes
2. Heat exchanger applications
3. Solar energy collectors
4. Regenerators and recuparators
5. Insulation of buildings and other mechanical devices etc
6. Packed and circulating bed combustors and reactors
7. Energy storage and conversion methods
8. Containment transport in groundwater and exploitation of geothermal resources.
9. Drying problems such as vegetable, grains, ceramic, wood, brick etc
10. Flow through the different organs of animal body for example lungs
11. Problem involving nuclear energy such as heat removal from the pebble-bed nuclear reactors, multishield structures used in the insulation of nuclear reactors, nuclear waste disposal etc
12. Seepage of water through river bed
13. Migration of pollutants into the soil and aquifier.
14. Study to know the solidification of alloys
15. Heat transfer and fluid flow in bio-digesters
16. Chemical reactors and petroleum recovery processes catalytic reactors etc.
17. Flow of moisture through porous industrial materials

Apart from these applications, the porous media approach is becoming popular in approximating the flow behavior in many other phenomena such as flow through turbo machinery and liquid metal flow in alloy casting etc. Understanding of heat and fluid flow behavior in the porous medium will help to design the systems in a better way so as to increase the efficiency of systems involving porous medium. For instance, the study can help to find out the occurrence of maximum and minimum heat transfer rate with respect to geometrical parameters of the cylindrical porous medium, using which, the system can be designed so as to have maximum or minimum heat transfer rate.

1.3 Mathematical modeling and simulation study

As in other fields, the study of various problems related to porous media can either be carried out by experimental setup or by mathematical modeling and simulations. The mathematical modeling and thus the simulation study have many advantages over experimental study. For instance, the experimental setup requires a huge amount of investment which many times becomes a deciding factor whether to carry out the study or abandon it. As far as the time is concerned, the experimental study requires an epoch. Contrary to experimental study, the time required for simulation study is substantially low. The great thing about simulation is that it compresses the time and thus allows investigating the different operational conditions and answers the many "what if" scenarios which otherwise becomes impossible in experimental study for various reasons. Simulation study also avoids the need of human being exposed to various hazardous environments, which are part and parcel of experimental study.

1.4 Heat transfer in porous medium

The natural convection in the porous media occurs due to differential temperature maintained at different surfaces of porous medium. The differential temperature makes the fluid absorb the thermal energy and get heated. The warm heated fluid moves upward due to difference in density. This sets up the natural convective current, which assists in the heat transfer phenomenon. There is an immense amount of effort being put by myriad researchers to study the natural convection in saturated porous medium, bounded or enclosed by different geometries. Thus the literature on this particular aspect of the porous medium is abundant.

Radiation heat transfer in the porous media plays an important role when the natural convection is relatively small. The keen observation through the available literature reveals that the radiation heat transfer has received little attention as compared to natural convection heat transfer. One reason for this can be understood in a way that the inclusion of radiation term in the governing energy equation of porous medium increases the complexity of the partial differential equations. But when we look at the realistic aspects of heat transfer, then one can find that there are various applications wherein radiation heat transfer takes place and thus cannot be ignored. For instance the combustion processes, high temperature heat exchangers, solar energy collectors, the radiative drying of papers and other porous materials, manufacturing systems that use laser to melt the powder etc. In these and many such applications, the porous medium either absorbs or emits the radiant energy. Therefore, it is of great importance to study the heat transfer behavior in the porous medium bounded by different geometries and thus predict the influence of natural convection and radiation heat transfer. Thus the effect of radiation heat transfer is investigated in the present study.

It is also seen that the thermal buoyancy driven natural convection has been studied in great detail but combined heat and mass transfer through the porous medium has been studied to lesser extent. The combined heat and mass transfer occur in many practical phenomenon such as seawater and mantle flow in the crust of the earth, migration of moisture through air in fibrous insulation, underground spreading of chemical waste and pollutants etc. Thus there is a need to explore the knowledge and understanding, of combined heat and mass transfer through the porous media. This is another area, which require an attention and thus focused in the current study.

1.5 Heat transfer modeling approaches

There are two different approaches in modeling the heat transfer in porous medium, namely thermal equilibrium and thermal non-equilibrium modeling. In case of thermal equilibrium modeling, the fluid and solid phases of porous medium are assumed to be in thermal equilibrium and thus the local temperature of solid and fluid in the medium is same. In such type of modeling, only one energy equation is required to predict heat transfer behavior. In case of thermal non-equilibrium modeling, the fluid and solid phases are not in thermal equilibrium condition. In this case, two energy equations i.e. one for fluid and another for solid matrix are necessary. The two energy equations of thermal non-equilibrium modeling are coupled by a convection term between fluid and solid phases. This convection term allows the thermal energy to be exchanged between fluid and the solid matrix of the porous medium depending upon their temperature level. Overview of the available literature pertaining to thermal aspects of porous media indicates that enormous amount of work has been carried out considering the thermal equilibrium between fluid phase and solid matrix of the porous media. It can be said that a handful of work has been done using the thermal non-equilibrium approach as compared to that of thermal equilibrium approach. Even though the concept of non-equilibrium modeling was introduced long back by Schmann in 1929, only very-recent years have seen an increased use of thermal non-equilibrium model. There are various applications, which involve thermal non-equilibrium condition between fluid phase and solid matrix of the porous media where the equilibrium model does not predict the heat transfer behavior accurately. For instance, the thermal boundary layer flows where cold fluid entrains into the boundary layer which cools the fluid phase relative to the solid phase, the situations where the rapid changes in heating occurs when speed of thermal front advancing is different in fluid and solid phase of porous medium, heat pipes etc. Thus it is essential to investigate the heat transfer in porous media by making use of thermal non-equilibrium model for various problems involving porous medium. In present study, the thermal non-equilibrium approach has been given a special attention.

1.6 Problem statement

Generally the radiative heat transfer in porous medium is neglected to avoid the complexity of mathematical modeling but the presence of radiation plays a vital role in overall heat transfer rate and cannot be neglected. Also, the thermal equilibrium model does not give accurate prediction of heat transfer rate in many situations when the solid and fluid phases of porous medium differ in temperature. Thus the present study focuses on this complex phenomenon and an effort is made to understand the heat and fluid flow behavior inside the porous medium bounded or fixed in various geometries. The investigation is carried out with respect to heat transfer, heat and mass transfer, viscous dissipation and thermal non-equilibrium condition. Figure 1.1 illustrates the summary of research work being carried in the present study.

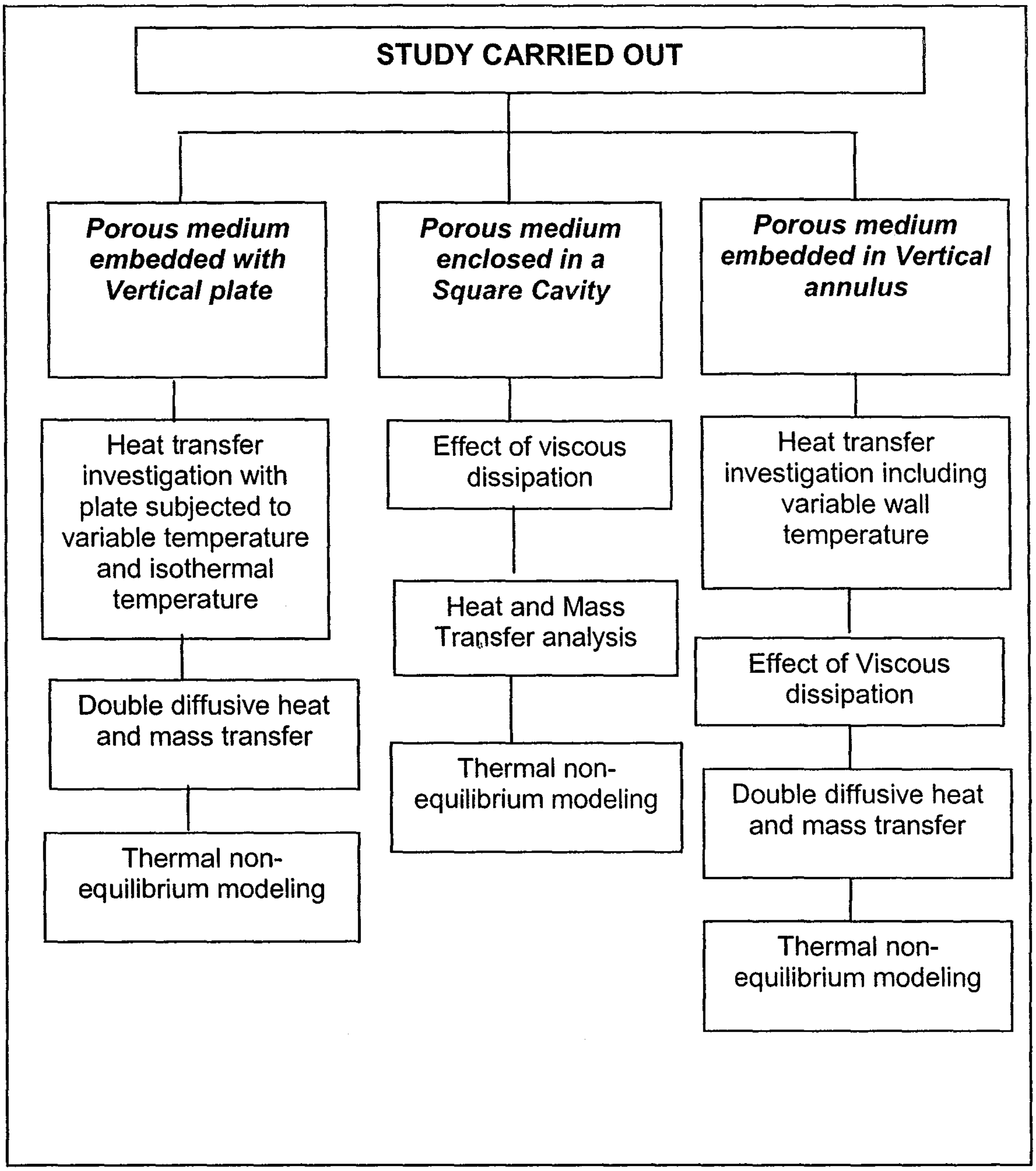

Figure 1.1. Summary of work carried out

1.7 Organization of the thesis

The thesis is organized in such a way that it provides a continuous and smooth flow of information to the reader, regarding the heat and mass transfer in porous medium. There are total of 7 major chapters which are subdivided into suitable sections.

Chapter 1 gives a brief background and the relevant importance of the heat and mass transfer in porous medium. Chapter 2 deals with the open-literature pertaining to heat and mass transfer in porous medium and highlights briefly, the different aspects of porous medium being addressed by other researchers.

Chapter 3 presents the mathematical formulations of all the cases being studied and also discusses the solution methodology adopted to solve these equations. All the aspects of formulation and solution are discussed in this chapter.

Chapters 4-6 present the results obtained in the current study. These chapters give the meticulous details of investigation of different phenomenon in porous medium. Chapter 4 describes the results of porous medium adjacent to a vertical flat plate where as chapter 5 and 6 discusses about porous medium enclosed in a square cavity and vertical annular cylinder respectively. Chapter 7 gives the conclusive remarks of the present work.

CHAPTER 2
LITERATURE REVIEW

2.0 Introduction

As stated earlier, the study of heat and fluid flow in a saturated porous media is one of those subjects in which research has been going on since 150 years. This long history of research has led to emergence of myriad research papers. The research work in this field has been well documented. This chapter is aimed at providing some of the related information regarding the research being carried out pertaining to heat and mass transfer in porous media, by different researchers across the globe.

It is noteworthy to start the literature review with the work of Henry Darcy whose research is considered to be the milestone in this field. Darcy (1856) conducted experiments in France to understand more about water filtering. He used silica sand as the filtering media for experimental purpose. His experimental setup was comprised of a simple vertical column where he could measure the flow rate of water and the pressure difference in the vertical column. The experimental results were presented in terms of water flow rate and the pressure difference across the silica sand. Based on these experimental results Darcy proposed a relation that relates the flow rate with the pressure difference, length of the porous media, cross sectional area and a constant. Darcy's relation has been used extensively since its emergence, to predict the flow characteristics in the porous medium.

2.1 Heat transfer in a porous medium adjacent to vertical plate

2.1.1 Thermal equilibrium model

Cheng and Menkowycz (1977) have studied the free convection from a vertical surface in porous medium. Hossain and Pop (2000) have studied the radiation effect on free convection over a vertical flat plate embedded in a porous medium with high porosity. In this study, the vertical surface was maintained at isothermal wall temperature. They used two different methods to solve the equations i.e. Keller box method and the local non-similarity method. The results were given in terms of local skin friction coefficient and local Nusselt number, velocity profile and the temperature profile for various non-dimensional parameters. Their study suggests that the velocity profile decrease with the increase in non-dimensional Darcy parameter whereas the temperature profile increases with increase of Darcy parameter. Hossain and Pop (1997) have also studied the radiation effect of Darcy free convection flow along an inclined surface in porous medium.

Chamkha (1997) investigated the radiation assisted natural convection in uniform, porous medium supported by a vertical flat plate. He used local non-similar technique to solve the governing partial differential equations. This study revealed that the increase in Prandtl number decreases the tangential velocity and temperature by confining them to smaller region nearer to the vertical plate. It was also observed that the boundary friction decreases with increase in Prandtl number. However Nusselt number increases owing to increase in Prandtl number.

Chamkha et al (2002) also studied the Natural convection boundary-layer flow due to solar radiation of an absorbing and electrically-conducting fluid over a semi-infinite, inclined flat plate embedded in a porous medium with variable porosity. Their study suggests that the increase in Prandtl number cause reduction in the wall slopes of the tangential velocity profiles and the peak values in the temperature profile

Bakier (2001a, 2001b) has considered thermal radiation effect on mixed convection from a vertical surface in saturated porous media. The governing equations were solved using fourth order Runge-Kutta method. Bakier concludes that the radiation parameter has considerable influence on the augmentation of surface heat transfer rate and the effect of increasing buoyancy is to increase velocity at the surface and augment the heat transfer rate from the surface.

Kim and Vafai (1989) have studied the buoyancy driven flow about a vertical plate for constant wall temperature and heat flux. The governing equations were solved in two different ways i.e. numerically and also analytically by the method of matched asymptotic expansion along with the modified Oseen method. They found that the heat transfer rate depends on modified Rayleigh number when the thermal boundary layer thickness is greater than the viscous boundary layer, but heat transfer rate depends on the product of Rayleigh number and porosity if the thickness of viscous boundary layer is greater than the thermal boundary layer.

Kumari and Nath (2004) have investigated the effect of radiation on non-Darcy mixed convection flow over a non-isothermal horizontal surface immersed in a saturated porous medium. Here Keller box finite difference method was employed to solve the equations. They observed that the Nusselt number and velocity at the wall for the stagnation flow are more than those of parallel flow. It was also noted that the Nusselt number and velocity at the wall increases with aiding flow but decreases with increase in non-Darcy parameter.

Mahmud and Fraser (2003) have analysed the mixed convection radiation interaction in a vertical porous channel. It was found that the higher value of radiation parameter suppresses the velocity profile at centerline of vertical porous channel. It was also observed by Mahmud and Fraser that the radiation and mixed convection parameters have a more dominating influence on entropy generation than porous magnet parameter.

Macroscopic study of pipes filled with small diameter silicon carbide fibers is carried out by Martin et al (1998). Numerical study is presented to investigate radiation effect between fiber and tube wall, conduction within fibers and convection from the fibers to the surrounding fluid. They observed that the inclusion of radiation leads to significant local non-thermal equilibrium effect.

The thermal radiation effects on power law fluid over a horizontal plate embedded in a porous medium was analysed by Mohammadein and El-Amin (2000). In this paper Ostwald-de Waele power-law model was used to characterize the non-Newtonian fluid behavior. The equations were solved using fourth order Runge Kutta method with shooting technique. It was found that the power law index reduces the temperature profile inside the porous medium.

Israel et al (2003) have studied the Influence of viscous dissipation and radiation on an unsteady free convection flow past an infinite vertical plate in a porous medium with time-dependent suction. They discussed the effect of material properties on the temperature and velocity profile. They conclude that the increased viscous dissipation leads to increased temperature profile, increase in the magnetic field decreases the temperature profile on cooling and increased cooling of the plate and viscous dissipation results in increased velocity profile.

El-Hakiem (2000) has studied the magneto hydrodynamic free convection and thermal radiation flow of an electrically conducting viscous incompressible fluid through porous medium embedded with vertical isothermal wall. They found that velocity decreases with increase in magnetic parameter.

Raptis (1998) considered radiation and free convection through a porous medium bounded by vertical plate with suction velocity. In this paper an analytical solutions for velocity and temperature distribution inside the porous medium was presented. Raptis (2000) also gave the local similarity transformation for the boundary layer flow through a porous medium. But in this case the radiation was not considered.

Al-Odat (2004) has investigated the unsteady free convection over a vertical plate embedded in a porous medium with suction velocity. Here Non-Darcy model is used to describe the fluid flow. It was found that the increase in inertial and suction parameters caused a significant decrease in the fluid velocity and slight increase in the temperature, however the skin friction coefficient was reduced due to increase in the inertial parameter.

Talukdar et al (2004) have investigated combined radiation and convection heat transfer in a porous channel bounded by isothermal parallel plates. In this paper, the authors have considered different cases of boundaries such as those maintained at temperature higher or lower than the medium. The heat and fluid behavior has been discussed with respect to porous medium shape parameter γ. Their findings can be concluded as: Nusselt number for the hot-plate condition decreases along the axial length, reaches a minimum value and then increases again whereas for cold plate condition, the total Nusselt number decreases along the axial length and reaches an asymptotic value.

For the hot-plate condition, the effect of γ on the radiation Nusselt number is smaller near the entrance region, but it increases along the axial length. The opposite is the case with the convection Nusselt number. For the cold-plate condition, the effect of γ decreases gradually in the streamwise direction.

Radiative effects on magneto hydrodynamic natural convection flows in saturated porous media were studied by Mansour and El-Shaer (2001). In this paper a boundary layer solution was presented to study the effects of joule heating on magneto hydrodynamic natural convection flow for four different cases namely an isothermal surface, a uniform heat flux surface, a plane plume and flow generated from a horizontal line energy source and a vertical adiabatic surface.

Elbashbeshy and Bazid (2004) have considered the problem of starching surface with internal heat generation and suction or injection in a porous medium. The following conclusions were drawn from this study: the heat source parameter leads to increase in skin friction, the heat transfer rate decreases with the heat source parameter. It was also observed that the thermal boundary layer thickness increases with injection and decreases with suction.

Seddeek (2000) has analysed the effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary against the stationary fluid with radiation. He observed that the velocity decreases with the increase of variable viscosity parameter. However the temperature profiles increased due to increase in variable viscosity parameter. Seddeek (2002) also studied the effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow. Here the plate was moved with constant velocity and the viscosity was assumed to vary as an inverse linear function of temperature. Their results showed that the variation of viscosity with temperature has substantial effect on the drag, heat transfer characteristics as well as the velocity field within the boundary layer over a continuously moving plate. Seddeek and Abdelmeguid (2006) have considered effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux and found that the temperature profile increases with increase in thermal diffusivity. Liao and Pop (2004) have given explicit analytical solution for similarity boundary layer equations applicable for convective viscous flows past a suddenly heated vertical plate in a porous medium as well as stretching wall.

Aly et al (2003) have modeled thin vertical fin as fixed semi-infinite vertical surface in a porous medium and thus analysed mixed convection boundary layer flow. Hung et al (1999) investigated the non-Darcy free convection along a non-isothermal vertical surface in a thermally stratified porous medium. They observed that the thermal stratification reduces the heat transfer rate from vertical surface but higher value of temperature exponent increases the heat transfer rate. It was also observed that the non-Darcy flow affects significantly the velocity and temperature fields inside the porous medium. Kumar and Shalini (2004) have considered non-Darcian free convection flow in vertical wavy surface in a thermally stratified porous medium.

There is plentiful literature available addressing the different aspects of porous media adjacent to the vertical plate but still there are certain gaps left untouched such as variable wall temperature effect when combined radiation and convection is taking place etc.

The heat transfer in saturated porous medium has been studied widely thus abundant amount of literature pertaining to heat transfer can be found in various journals. On the other hand, combined heat and mass transfer in porous media has not received as much attention. The double diffusive convection, which is also referred as thermosolutal, deals with the combined heat and mass transfer in the porous medium. This kind of convection is driven by buoyancy forces. Double diffusive convection occurs due to the influence of temperature gradient on the solute. It may be noted that the double diffusive heat and mass transfer appears in many practical applications such as seawater flow and mantle flow in earths crust, contamination in the ground water and study of geothermal reservoirs etc. The current state of research pertaining to double diffusive heat and mass transfer through the porous medium is summerised in the following pages.

Bejan and Khair (1985) have studied the double diffusive natural convection in an infinite porous medium. They presented Nusselt number and Sherwood number by using order of magnitude analysis of boundary layer equations. Later on Lai and Kulacki (1991) considered this problem for constant wall temperature and constant wall flux. The effect of wall inclination on a two-layer structure was analysed by Jang and Chang (1988a,b) by employing similarity approach of Bejan and Khair.

Angirasa et al (1997) have considered combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium adjacent to vertical plate. Yih (1997) analysed the transpiration effect on coupled heat and mass transfer due to mixed convection over a vertical plate embedded in a saturated porous medium and it was found that the local Nusselt and Sherwood numbers increase with suction and decrease with blowing. As stated earlier, there is scarcity of literature pertaining to double diffusion is porous medium bounded with vertical plate. Thus there are various areas in heat and mass transfer in porous media with vertical plate, which are to be addressed.

Few researchers also studied the waviness of the vertical surface adjacent to fluid saturated porous media. Cheng (2000) has analysed the heat and mass transfer near a vertical wavy surface having constant wall temperature and concentration. The effect of waviness was transformed from the boundary conditions into the governing equations itself. This work revealed that the local Nusselt number oscillates about a mean value due to waviness and the wavelength of oscillation is half of that of wavy surface. it was also noted that the increase in amplitude-wavelength ratio tends to increase the amplitude of local Nusselt number.

2.1.2 Thermal non-equilibrium modeling

The thermal non-equilibrium modeling in the porous medium was first demonstrated by Shumann (1929). In his model, the flow of incompressible fluid was considered but longitudinal conduction terms in both liquid and solid equations were neglected. The recent years have seen an increased use of the two-temperature thermal non-equilibrium modeling in the porous medium. This can be attributed to the availability of better computational facilities in terms of high-speed computers to solve the complex equations. Following are the some of noticeable works being carried out to study the thermal non-equilibrium status in porous medium adjacent to vertical plate

Rees and Pop (1999) have studied the effect of thermal non-equilibrium model in the porous medium bounded by vertical isothermal surface. They found that the use of thermal non-equilibrium model lead to capture the behavior which was substantially different from those of thermal equilibrium model. Recently Saeid (2004a) has applied two-temperature non-equilibrium model to investigate the mixed convection heat transfer from a vertical porous layer. He used finite volume method to solve the governing equations and concluded that the heat transfer behavior cannot be predicted accurately when the inter-phase heat transfer parameter and modified conductivity ratio are smaller. Thus a non-equilibrium modeling is the best choice at smaller values of above stated parameters. As stated earlier, the endeavoring effort to use two-temperature model is relatively new and thus there are various areas of interest such as radiation effect, viscous dissipation etc, which are still wide open to be explored.

Saeid and Mohamad (2005) have used non-equilibrium model to investigate the effect of sinusoidal plate temperature in a saturated porous medium embedded with vertical plate. The governing equations were solved with the help of implicit finite difference method. This study revealed that there are some points in the porous medium which have temperature higher than that of the vertical plate thus indicating a reverse heat transfer from porous medium to the heated plate. This phenomenon is observed when the amplitude and the frequency of the plate temperature are high.

2.2 Heat transfer in porous cavities

2.2.1 Thermal equilibrium model

In the following pages a concise review of the research carried out with respect to the heat transfer in porous cavities is presented. In this type of geometry, the porous medium is completely confined inside a close chamber having four solid boundaries. This situation arises in some of the building insulation, geothermal and oil extraction applications etc. The cavities are generally studied by heating the left vertical wall to a constant temperature and cooling the right vertical wall.

Ali (2005) has studied analytically and numerically the natural convection in a rectangular porous medium. In this case the vertical walls of the cavity were maintained at different temperatures and the horizontal walls were adiabatic. He considered two important parameters for the study, which are the Rayleigh number and the aspect ratio of cavity. He found that the flow is parallel to the cavity and the isotherms are stratified in the core of the cavity when the aspect ratio is large.

Baytas (2000) has attended the problem of inclined porous cavities with focus on the natural convection and entropy generation in the porous medium by having maintained two opposite walls at hot and cold temperatures and other two walls at adiabatic condition. The angle of inclination of the cavity was taken as one of the study parameter and results were presented mainly for entropy generation and the angle of inclination from 0^{0} to 360^{0}. It was shown that the average Nusselt number is minimum when the angle of inclination of the cavity is 270^{0}.

Baytas and Pop (1999) have analysed the free convection in an oblique enclosure filled with porous medium. They simplified the solution by transforming the oblique cavity into a rectangular cavity using a non-linear axis transformation and then solved the new coordinate system by finite difference method. Their results suggests that at sharp corners of the cavities the flow breaks down into series of vertices and the subvertices system grown in size with increased Rayleigh number and the inclination angle.

Saeid (2005) has considered the case of sinusoidal bottom wall temperature with top cooled and adiabatic vertical walls of a porous medium enclosed in a cavity. The heated wall was assumed to have temperature varying in a sinusoidal wave around a mean temperature greater than that of the top cooled wall. The main finding of this work was that the average Nusselt number increased when the length of the heat source or the amplitude of the temperature variations increased. In another work, Saeid (2005) investigated similar problem of square porous cavity but boundary conditions were swapped such that the temperature differential was maintained along vertical surfaces. A sinusoidal temperature variation was applied on left vertical surface along with cooled right surface. The bottom and top surfaces were adiabatic. Saeid (2004b) also studied the effect of viscous dissipation on free convection in a porous cavity and presented results in terms of local and average Nusselt number at vertical hot and cold walls of the cavity. Saeid and Pop (2004) have investigated the transient free convection in a square porous cavity by employing the finite volume method to solve the governing equations. They observed that the time required to reach the steady state is longer at low Rayleigh number and low at higher Rayleigh number.

Above-mentioned research dealt with plane boundaries of the cavity. Some researchers also addressed the problem of wavy nature of the cavities. Misirlioglu et al (2005) have investigated the free convection in a wavy cavity filled with a porous medium. The waviness was assumed to occur on the vertical walls but the horizontal walls were considered to be plane. The parameters considered were aspect ratio and the waviness of on the surface. They found that at high Rayleigh number and moderate values of aspect ratio and surface waviness, the local Nusselt number from vertical wall is negative at certain portion of the cavity.

Bourich et al. (2004a) have considered the problem of thermosolutal convection in a saturated porous enclosure. The boundary conditions considered were isothermal heating and cooling of bottom and upper horizontal surface whereas the vertical boundaries were subjected to concentration differentials. Their study resulted in a set of correlations for Nusselt and Sherwood numbers for parameters such as Rayleigh number, Lewis number and buoyancy ratio. Same authors Bourich et al. (2004b) have also studied the double diffusive natural convection in a porous enclosure partially heated from the bottom wall. The heated boundary was half of the total boundary length at the bottom wall. They conclude that the buoyancy ratio on the dynamic behavior of fluid is dependent on the position of the isothermal heater at bottom wall and asymmetric bicellular flow was observed when the dimensionless position of heater was at 0.5.

Gobin et al (2005) have considered the natural convection driven by thermal and solutal buoyancy forces in an enclosure partially filled with porous layer. In this paper the effect of porous layer thickness on the heat and mass transfer is analysed. They showed that the presence of the porous layer has strong influence on the heat transfer as well as flow structure. Some of the other notable work with respect to heat and mass transfer in porous medium can be found in the papers of Trevesion and Bejan (!985, 1990), Charrier –Mojtabi et al.(1997)

Mahidjiba et al. (2000) have analysed the double diffusive convection in a rectangular porous cavity subjected to mixed boundary conditions. In this case the boundary conditions were such that the horizontal walls were subjected to heat flux and also differential concentration, whereas both vertical walls were adiabatic and impermeable. The other notable works from last quarter of 20th century in the porous media enclosed in a square cavity are those of Walker and Homsy (1978), Bejan (1979)

Beckerman et al. (1986) Gross et al.(1986), Monolo and Lage (1992) Moya et al (1987) etc

2.2.2 Thermal non-equilibrium modeling of cavities filled with porous medium

The application of non-equilibrium model to porous enclosure has started in very recent years. There are very few articles available in this regard. Baytas and Pop (2002) have use non-equilibrium model to investigate the fluid and heat transfer characteristics in a square porous cavity. They employed cell centered finite volume method to solve the governing equations and found that the equilibrium state is reached at higher values of the inter-phase heat transfer coefficient and also when the conductivity ratio parameter takes higher value.

Khashan et al (2006) have carried out numerical simulation of natural convection heat transfer in a porous cavity heated from below and employed the non-equilibrium model. They conclude that the flow circulation and the local thermal non-equilibrium is enhanced due to increased Rayleigh number. It was also observed that the decrease in Darcy number reduces the flow circulation and decreases the average Nusselt number. However the increase in Biot number improved the local thermal equilibrium between the fluid and solid phases.

Baytas (2003) has addressed the problem of natural convection in a square enclosure filled with heat generating porous medium by considering non-equilibrium approach. It was assumed that the solid phase generates the heat. The governing partial differential equations were solved by using finite volume method. He observed that at small value of modified conductivity ratio, the temperature difference between solid and fluid phase are negative at upper part of the cavity and positive at the lower part of the cavity.

2.3 Porous medium embedded with cylindrical geometry

2.3.1 Thermal equilibrium model

The situation of porous medium embedded with cylindrical geometry arises in many industrial applications and thus it is of fundamental interest to engineers and scientist. Such applications are often found in power plants, oil and gas distribution lines, electrical cables, nuclear waste storage, solar collectors etc to name but few.

Minkowycz and Cheng (1976) have analysed the free convection boundary layer flow about a vertical cylinder embedded in saturated porous medium. In this study, the temperature of the wall was assumed to vary along the height of the cylinder in a power law fashion. Similar type of problem was considered in detail but for linearly varying wall temperature by Merkin (1986). Here the finite difference method was used to solve the governing equations. Bassom and Rees (1996) have extended the work of Merkin for a range of exponential values of power law wall temperature to study the free convection from a heated vertical cylinder in a fluid saturated porous medium by employing the Keller box method. Yucel (1984) investigated the influence of injection or withdrawal of a fluid on free convection about a vertical cylinder in a porous medium whereas Kumari et al. (1985) have discussed about finite difference and improved perturbation solutions for free convection on a vertical cylinder embedded in a saturated porous medium. Convective heat transfer in the vertical annulus filled with a porous medium has been investigated by