UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2005/2006

November 2005

MST 564 – Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **TEN** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEPULUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions: Answer **all four** [4] questions.

[Arahan: *Jawab **semua empat** soalan].*

...2/-

1. (a) (i) Give the definition of a survivor function and its properties. What is the empirical definition for the survivor function?

[10 marks]

(ii) Does the function

$$S(t) = \begin{cases} 1-t & for\ 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$

satisfy the properties of a survivor function for some variable T? What is the distribution of T?

[20 marks]

(b) (i) Show that a lifetime random variable T with hazard function h satisfies

$$P(T > t+y \mid T > t) = \exp\left\{-\int_t^{t+y} h(u)du\right\}$$

(ii) Suppose that, at each age, the death rate of a person that smokes is twice that of a non-smoker.

A. If $h_s(t)$ denotes the hazard rate of a smoker at age t and $h_n(t)$ that of a non-smoker at age t, write an equation relating $h_s(t)$ and $h_n(t)$.

B. If the probability that an A-year-old individual reaches age B is given by $\exp\left\{-\int_A^B h(u)du\right\}$, show that: Of two individuals of the same age, one of whom is a smoker and the other a non-smoker, the probability that the smoker survives to any given age is the square of the corresponding probability of a non-smoker.

[40 marks]

(c) Given below is a table summarizing a clinical trial for the treatment of primary biliary cirrhosis, a potentially fatal liver condition.

Interval (years)	Total, n	Deaths, d	Losses, c
[0, 1)	191	18	14
[1, 2)	159	19	9
[2, 3)	131	13	9
[3, 4)	109	9	12
[4, 5)	88	14	7
[5, 6)	67	7	14
[6, 7)	46	10	12
[7, 8)	24	3	4
[8, 9)	17	2	3
[9, 10)	12	1	6
[10, 11)	5	0	3
[11, 12)	2	0	2

...3/-

(i) Construct further columns of this table giving the estimates for the hazard and the survival rates.

(ii) Use the table to estimate the probability of survival of the patients beyond 5 years.

(iii) For patients who have survived 5 years, estimate the probability of surviving a further year.

[30 marks]

1. (a) (i) *Beri definasi bagi satu fungsi survivor serta ciri-ciri fungsi tersebut. Apakah pula definasi empirik bagi fungsi survivor?*

[10 markah]

(ii) *Adakah fungsi*

$$S(t) = \begin{cases} 1-t & bagi\ 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$

memenuhi sifat-sifat bagi suatu fungsi survivor untuk pembolehubah T ? Apakah taburan bagi T ?

[20 markah]

(b) (i) *Tunjukkan bahawa pembolehubah rawak masahayat T dengan fungsi bahaya h memenuhi persamaan berikut;*

$$P(T > t+y \mid T > t) = \exp\left\{-\int_t^{t+y} h(u)du\right\}$$

(ii) *Andaikan bahawa, pada setiap umur, kadar kematian seseorang yang merokok adalah dua kali ganda berbanding dengan seseorang yang tidak merokok.*

A. *Jika $h_s(t)$ menandai kadar bahaya bagi seorang perokok pada umur t dan $h_n(t)$ menandai kadar bahaya bagi seorang yang bukan perokok pada umur t, tulis satu persamaan yang menghubungkan $h_s(t)$ dan $h_n(t)$.*

B. *Jika kebarangkalian bahawa seorang individu berumur A mencapai umur B adalah $\exp\left\{-\int_A^B h(u)du\right\}$, tunjukkan bahawa:*

bagi dua orang individu yang sama umur, seorang darinya adalah perokok dan seorang lagi bukan perokok, kebarangkalian bahawa perokok tersebut akan hidup sehingga satu umur yang diberi adalah kuasa dua kebarangkalian yang sepadan bagi yang bukan perokok.

[40 markah]

...4/-

(c) *Jadual di bawah meringkaskan satu ujian klinikal bagi rawatan primary biliary cirrhosis, satu keadaan hati yang merbahaya.*

Selang (tahun)	*Jumlah, n*	*Kematian, d*	*Kehilangan, c*
[0, 1)	*191*	*18*	*14*
[1, 2)	*159*	*19*	*9*
[2, 3)	*131*	*13*	*9*
[3, 4)	*109*	*9*	*12*
[4, 5)	*88*	*14*	*7*
[5, 6)	*67*	*7*	*14*
[6, 7)	*46*	*10*	*12*
[7, 8)	*24*	*3*	*4*
[8, 9)	*17*	*2*	*3*
[9, 10)	*12*	*1*	*6*
[10, 11)	*5*	*0*	*3*
[11, 12)	*2*	*0*	*2*

(i) Bina lajur seterusnya bagi jadual ini dan berikan anggaran bagi kadar bahaya dan kadar survival.

(ii) Guna jadual ini untuk menganggar kebarangkalian pesakit akan hidup melepasi 5 tahun.

(iii) Bagi pesakit yang telah hidup selama 5 tahun, anggar kebarangkalian pesakit tersebut akan hidup selama setahun lagi.

[30 markah]

2.(a) Suppose a lifetime distribution has the following probability density function

$$f(t) = \begin{cases} 2te^{-t^2} & , t > 0 \\ 0 & , otherwise \end{cases}$$

(i) Obtain the distribution function $F(t)$ and thus survivor function $S(t)$.

(ii) Also obtain the hazard function $h(t)$ and the function $H(t)/t$.

(iii) Is $F \in IFR$? $F \in IFRA$?

[30 marks]

(b) The data below are failure times of 12 components (in days):

1.4 1.6 2.5 3.5 4.0 4.5
6.0 6.5 7.2 8.0 8.8 10.0

16 components are put on test and the failure times are assumed to be distributed exponentially.

(i) Show that the maximum likelihood estimate for parameter λ is

$$\hat{\lambda} = \frac{12}{\sum_{i=1}^{12} t_{(i)} + 4t_{(12)}}$$

...5/-

(ii) Calculate the maximum likelihood estimate for the mean and find a 95% confidence interval for the mean.

(iii) Calculate the maximum likelihood estimate of the component reliability at 8 days.

[40 marks]

(c) If n items from an exponential population with parameter λ are placed on test and the test is terminated after r failures have occurred, show that

$$V\left(\sum_{i=1}^{n} x_i\right) = \frac{r}{\lambda^2}$$

where $x_i = \min\{t_i, c_i\}$, t_i is the time of the ith failure, and c_i is the censoring time for the ith item, $i = 1, 2, ..., n$.

[30 marks]

2.(a) *Andaikan taburan masahayat mempunyai fungsi ketumpatan kebarangkalian berikut;*

$$f(t) = \begin{cases} 2te^{-t^2} & , t > 0 \\ 0 & , \textit{di tempat lain} \end{cases}$$

(i) Dapatkan fungsi taburan $F(t)$ dan seterusnya fungsi survivor $S(t)$.

(ii) Dapatkan juga fungsi bahaya $h(t)$ dan fungsi $H(t)/t$.

(iii) Adakah $F \in IFR$? $F \in IFRA$?

[30 markah]

(b) Data berikut merupakan masa kegagalan 12 komponen (dalam hari):

1.4 1.6 2.5 3.5 4.0 4.5
6.0 6.5 7.2 8.0 8.8 10.0

16 komponen telah diuji dan andaikan masa kegagalan tertabur secara taburan eksponen.

(i) Tunjukkan bahawa anggaran kebolehjadian maksimum bagi parameter λ adalah

$$\hat{\lambda} = \frac{12}{\sum_{i=1}^{12} t_{(i)} + 4t_{(12)}}$$

(ii) Hitung anggaran kebolehjadian maksimum bagi min dan dapatkan satu selang keyakinan 95% bagi min tersebut.

(iii) Hitung anggaran kebolehjadian maksimum bagi kebolehpercayaan komponen pada 8 hari.

[40 markah]

...6/-

(c) *Jika n item dari satu populasi yang tertabur secara eksponen dengan parameter λ diletakkan dalam satu ujian dan ujian tersebut dihentikan selepas r kegagalan terjadi, tunjukkan bahawa*

$$V\left(\sum_{i=1}^{n} x_i\right) = \frac{r}{\lambda^2}$$

dengan $x_i = \text{minimum}\{t_i, c_i\}$, t_i *merupakan masa kegagalan ke-*i, *dan* c_i *adalah masa tertapis bagi item ke-* i, $i = 1, 2, ..., n$.

[30 markah]

3.(a) An electronic unit consists of two components, $C1$ and $C2$, arranged in series. Suppose that T_1 denotes the lifetime of $C1$ and T_2 denotes the lifetime of $C2$. Assume that T_1 and T_2 are independent.

(i) If T_1, T_2 are identically distributed as the $\exp(\lambda)$ random variables, determine the probability density function of the lifetime of the unit.

(ii) If $T_i \sim \exp(\lambda_i)$, $i = 1, 2$, determine the probability density function of the lifetime of the unit.

(iii) If T_1, T_2 are identically distributed as the $\exp(\lambda)$ random variables and two such units are connected in parallel, determine the probability density function of the lifetime of the resulting system.

(iv) Discuss briefly the above results applied to n components.

[60 marks]

(b) Consider the survivor function of the Weibull baseline distribution

$$S_0(t) = e^{-(\lambda t)^{\beta}} \quad , t \geq 0$$

(i) Show that if $T_1 = \dfrac{T_0}{g(x)}$, then for the accelerated life model, T_1 has the form of a Weibull survivor function with shape parameter β.

(ii) Show that this model can also be regarded as proportional hazard model and state the form of $g(x)$.

[40 marks]

3.(a) Satu unit elektronik mengandungi dua komponen, C1 dan C2, disusun bersiri. Andaikan bahawa T_1 menandai masa hayat C1 dan T_2 menandai masa hayat C2. Andaikan bahawa T_1 dan T_2 adalah tak bersandar.

(i) Jika T_1, T_2 merupakan pembolehubah rawak yang tertabur secara secaman dengan taburan eksponen(λ), tentukan fungsi kebarangkalian ketumpatan bagi masa hayat unit tersebut.

...7/-

(ii) *Jika* $T_i \sim \text{eksponen}(\lambda_i)$, $i = 1, 2$, *tentukan fungsi kebarangkalian ketumpatan bagi masa hayat unit tersebut.*

(iii) *Jika* T_1, T_2 *merupakan pembolehubah rawak yang tertabur secara secaman dengan taburan eksponen*(λ), *dan dua unit tersebut disusun secara selari, tentukan fungsi kebarangkalian ketumpatan bagi masa hayat sistem yang diperoleh.*

(iv) *Bincangkan secara ringkas keputusan yang diperoleh di atas sekiranya terdapat n komponen.*

[60 markah]

(b) *Pertimbangkan fungsi survivor bagi taburan baseline Weibull*

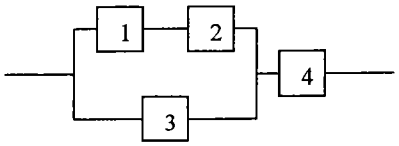
$$S_0(t) = e^{-(\lambda t)^{\beta}} \quad , t \geq 0$$

(i) *Tunjukkan bahawa jika* $T_1 = \dfrac{T_0}{g(x)}$ *, jadi bagi suatu model hayat pecut,* T_1 *mempunyai suatu bentuk taburan survivor Weibull dengan parameter bentuk* β.

(ii) *Tunjukkan bahawa model ini juga boleh dianggap sebagai model bahaya berkadaran dan nyatakan bentuk fungsi* $g(x)$.

[40 markah]

4.(a) Consider the four-component system with block diagram shown below.



(i) Find the structure function, $\phi(\mathbf{x})$.

(ii) Redraw the block diagram as the parallel arrangement of series subsystems and as the series arrangement of parallel subsystems.

(iii) Assume that component 4 is not included in the above system. The reliabilities of components 1 and 2 are $p_1 = 0.8$, $p_2 = 0.4$ respectively. Find the reliability of component 3, p_3, in the system to achieve a system reliability of 0.76.

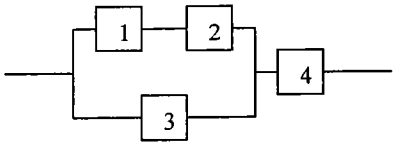
[60 marks]

...8/-

(b) A study was done to compare treatments for prostatic cancer. The two treatments used in the study were a placebo and 1.0 mg of diethylstilbestrol (TREAT). The survival time, in months, for 38 patients with stage III cancer was recorded. Information on other prognostic factors were also recorded, which include the age (in years) of the patients at trial entry (AGE), their serum haemoglobin level in gm/100ml (SHB), the size of their primary tumour in cm^2 (SIZE) and the value of the Gleason index (INDEX); the more advanced the tumour, the greater the value of the index. Based on the output given in the Appendix, which factors affect the survival of the patients? Write the conclusion that can be made from this study at $\alpha = 0.10$.

[40 marks]

4.(a) *Pertimbangkan suatu sistem empat-komponen dengan gambarajah blok seperti berikut;*



(i) *Dapatkan fungsi struktur,* $\phi(\mathbf{x})$.

(ii) *Lukis semula gambarajah blok bagi sistem di atas sebagai suatu susunan selari subsistem bersiri dan kemudian sebagai susunan bersiri subsistem selari.*

(iii) *Andaikan komponen 4 tidak termasuk ke dalam sistem di atas. Kebolehpercayaan bagi komponen 1 dan 2 adalah masing-masing* $p_1 = 0.8, p_2 = 0.4$ *. Dapatkan kebolehpercayaan bagi komponen 3,* p_3*, dalam sistem tersebut untuk mencapai kebolehpercayaan sistem sebesar 0.76.*

[60 markah]

(b) *Satu kajian telah dijalankan untuk membanding rawatan bagi kanser prostatik. Dua rawatan yang diguna dalam kajian ini ialah placebo dan 1.0 mg. diethylstilbestrol (TREAT). Masa hidup , dalam bulan, bagi 38 orang pesakit kanser peringkat III telah dicatit. Maklumat faktor prognostik yang lain juga dicatit, termasuk umur (dalam tahun) pesakit semasa memasuki kajian (AGE), paras serum haemoglobin dalam gm/100ml (SHB), saiz ketumbuhan dalam* cm^2 *(SIZE) dan nilai indeks Gleason (INDEX); semakin ketumbuhan membesar, semakin besar nilai indeks. Berdasarkan output yang diberi dalam Lampiran, faktor manakah yang memberi kesan terhadap masa hidup pesakit? Nyatakan kesimpulan yang boleh dibuat dari kajian ini pada* $\alpha = 0.10$..

[40 markah]

...9/-

Case Processing Summary

		N	Percent
Cases available in analysis	Event[a]	6	15.8%
	Censored	30	78.9%
	Total	36	94.7%
Cases dropped	Cases with missing values	0	.0%
	Cases with non-positive time	0	.0%
	Censored cases before the earliest event in a stratum	2	5.3%
	Total	2	5.3%
Total		38	100.0%

a. Dependent Variable: TIME

Block 0: Beginning Block

Omnibus Tests of Model Coefficients

-2 Log Likelihood
36.349

Block1: Method = Enter

Omnibus Tests of Model Coefficients[a,b]

-2 Log Likelihood	Overall (score)			Change From Previous Step		
	Chi-square	df	Sig.	Chi-square	df	Sig.
22.173	14.992	5	.010	14.176	5	.015

Omnibus Tests of Model Coefficients[a,b]

Change From Previous Block		
Chi-square	df	Sig.
14.176	5	.015

a. Beginning Block Number 0, initial Log Likelihood function: -2 Log likelihood: -36.349

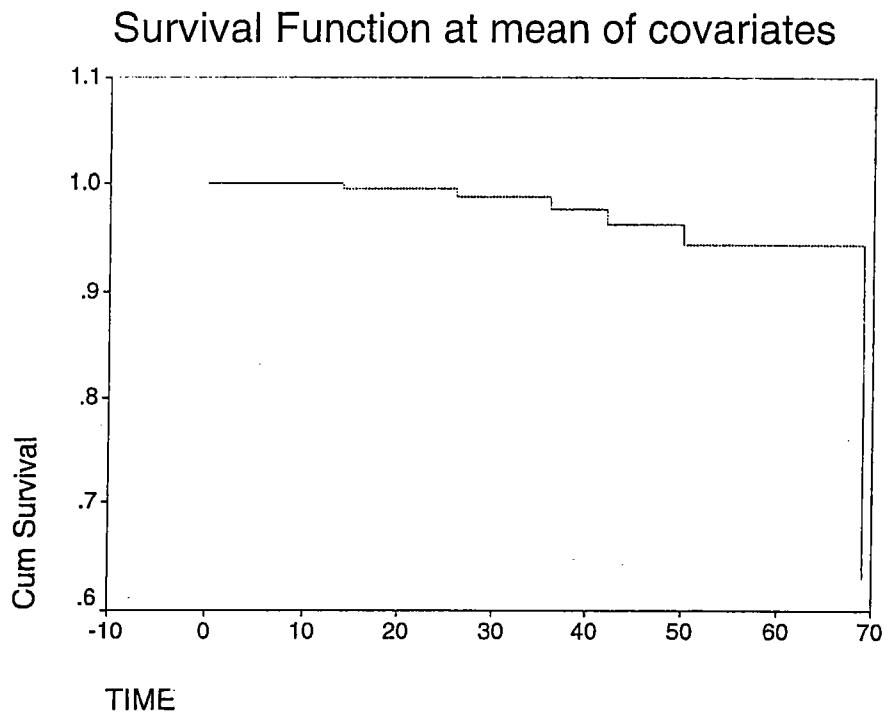
b. Beginning Block Number 1. Method: Enter

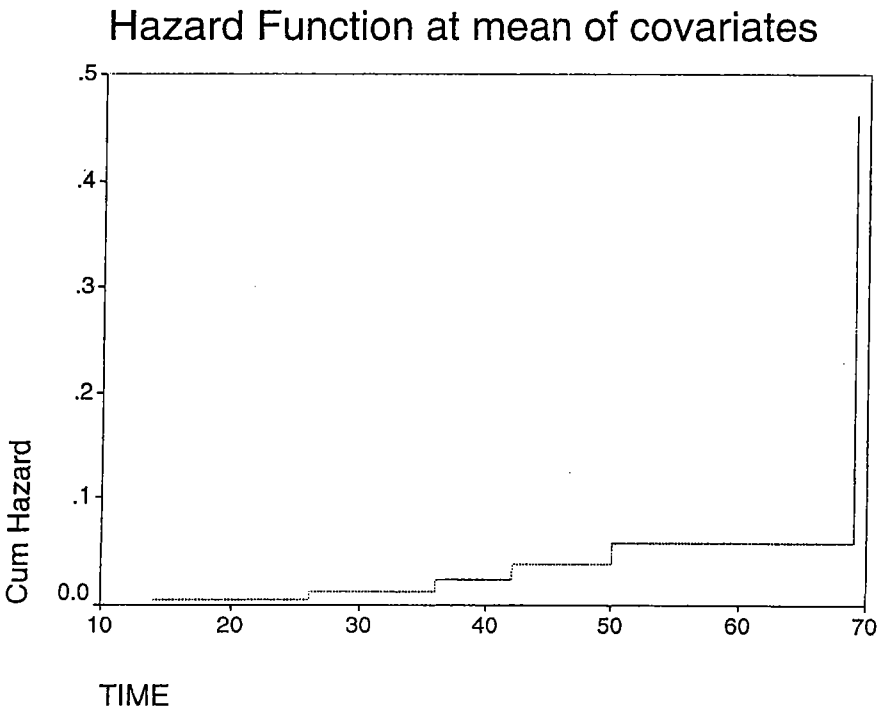
Variables in the Equation

	B	SE	Wald	df	Sig.	Exp(B)
AGE	.044	.072	.373	1	.541	1.045
SHB	-.022	.453	.002	1	.961	.978
SIZE	.094	.052	3.254	1	.071	1.099
INDEX	.723	.350	4.273	1	.039	2.061
TREAT	-1.182	1.210	.954	1	.329	.307

...10/-

	Mean
AGE	68.278
SHB	14.000
SIZE	10.750
INDEX	9.139
TREAT	1.528





- 000 O 000 -