THE COGNITIVE DIAGNOSTIC ASSESSMENT OF THE LEARNING OF ALGEBRAIC EXPRESSIONS FOR FORM TWO STUDENTS

By

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LIST OF ABBREVIATIONS

AHM	Attribute Hierarchy Method
CCR	Correct classification rate
CDA	Cognitive Diagnostic Assessment
CDM	Cognitive Diagnostic Model
CTT	Classical Test Theory
ECD	Evidence-Centered Design
IRT	Item Response Theory
MAD	Mean absolute difference
MCMC	Markov Chain Monte Carlo
NC-RUM	Non-compensatory Reparameterised Unified Model
PPM	Posterior probability of mastery
RSM	Rule space methodology
TRC	Test-retest consistency

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PENTAKSIRAN DIAGNOSTIK KOGNITIF UNTUK PEMBELAJARAN UNGKAPAN ALGEBRA DALAM KALANGAN PELAJAR TINGKATAN DUA

ABSTRAK

Pentaksiran diagnostik kognitif (PDK) berkemampuan untuk mentafsir kekuatan dan kelemahan dalam pembelajaran di samping memberi panduan untuk mengenalpasti ruang yang berpotensi untuk tujuan intervensi bagi pelajar yang menghadapi kesulitan dalam pembelajaran. Kajian ini bertujuan untuk membina satu bentuk PDK yang dijana dengan pembinaan item berdasarkan satu model pemprosesan kognitif yang eksplisit untuk mengukur atribut kognitif yang telah ditetapkan secara terperinci, serta untuk menentukan penguasaan atribut pelajar dalam pembelajaran ungkapan algebra. Satu pendekatan dengan tiga langkah digunakan untuk mereka pentaksiran dipatuhi: 1) pembinaan dan penilaian kualitatif model pemprosesan kognitif, 2) penilaian statistik kesesuaian dan ketepatan hubungan item-kepada-atribut yang dinyatakan dalam matrik-Q, dan 3) penggunaan Model Fusion untuk menganggar klasifikasi penguasaan atribut bagi setiap pelajar. Panel guru matematik yang berpengalaman telah meneliti objektif dan hasil pembelajaran Ungkapan Algebra dalam Spesifikasi Kurikulum Tingkatan Satu dan Tingkatan Dua, Penilaian Menengah Rendah (PMR) dan item-item buku teks untuk mengenalpasti dan menghuraikan atribut kognitif yang relevan bersama hirakinya, dan dua model pemprosesan kognitif telah dibina. Model-model tersebut disahkan melalui laporan lisan dan jawapan bertulis daripada satu sampel 30 orang pelajar Tingkatan Dua. Dua matrik Q telah dibina berdasarkan model-model pemprosesan kognitif tersebut untuk memspesifikasikan hubungan-hubungan item-kepada-atribut. Kesesuaian dan ketetapan matrik-matrik Q itu dinilai dengan menggunakan Model

Fusion. Data respon untuk setiap item yang mengukur *pengetahuan konsep* dan *kemahiran pemprosesan* dalam pembelajaran Ungkapan Algebra daripada satu sampel 2,395 orang pelajar Tingkatan Dua telah digunakan untuk menentukan parameter item dan parameter kebolehan pelajar, dan menganggar kebarangkalian penguasaan atribut dan klasifikasi penguasaan atribut. Keputusan mengesahkan bahawa pelajar menggunakan proses kognitif yang konsisten dengan jangkaan panel guru di mana semua atribut yang ditentukan dalam setiap profil atribut diperlukan untuk menyelesaikan item dengan betul. Kebarangkalian penguasaan atribut menunjukkan bahawa atribut K3 untuk *pengetahuan konsep* dan atribut S4 untuk *kemahiran pemprosesan* merupakan atribut yang paling susah dikuasai. Keputusan klasifikasi penguasaan atribut untuk setiap pelajar menunjukkan sama ada mereka adalah mahir ($PPM > 0.6$) atau tidak mahir ($PPM \leq 0.4$) untuk sesuatu atribut bagi menentukan kekuatan dan kelemahan mereka. Kajian ini menyumbang kepada perkembangan profesionalisme guru dalam pembinaan model pemprosesan cognitive dan tugasan penilaian yang dapat membantu guru untuk menilai dan memantau proses pengajaran dan pembelajaran.

THE COGNITIVE DIAGNOSTIC ASSESSMENT OF THE LEARNING OF ALGEBRAIC EXPRESSIONS FOR FORM TWO STUDENTS

ABSTRACT

Cognitive diagnostic assessment (CDA) has the capacity to assess students' cognitive strengths and weaknesses in learning and serves guidance to identify potential areas of intervention for students struggling in learning. This study aims to develop a CDA designed with item development based on an explicit cognitive processing model for measuring the specified fine-grained cognitive attributes, and to determine the students' attributes mastery in algebraic expressions learning. A three-step approach to assessment design was followed: 1) the development and qualitative evaluation of cognitive processing models, 2) the statistical evaluation of the adequacy and accuracy of the item-to-attribute relation expressed in the Q-matrices, and 3) the use of Fusion Model to estimate individual students' attribute mastery classification. A panel of experience mathematics teachers reviewed the Form One and Form Two learning objectives and learning outcomes of the learning of algebraic expressions specified in the Curriculum Specifications, the high-stake examination (*PMR*) and text book items to identify and describe the relevant cognitive attributes and their hierarchy, and two cognitive processing models were developed. These models were validated using verbal reports and written responses from a sample of 30 Form Two students. Two Q-matrices were developed based on the cognitive processing models to specify the item-to-attributes relations. The adequacy and accuracy of Q-matrices were evaluated by using the Fusion Model. Response data for each item from a sample of 2,395 Form Two students measuring the *conceptual knowledge* and *processing skills* of algebraic expressions learning were used to determine the item parameters and student ability parameter, and estimate the attribute mastery probabilities and attribute mastery classification. Finding verified that students engaged in cognitive processes which are consistent with the panel's prediction where all attribute(s) specified in each attribute profile are required to solve the items correctly. Attribute mastery probability revealed that attribute K3 and attribute S4 are the most difficult attributes to master in *conceptual knowledge* and *processing skills* respectively. The results of the attribute mastery classification for each student revealed either they are master (PPM > 0.6) or non-master (PPM ≤ 0.4) of a particular attributes to identify their strengths and weaknesses in the learning of algebraic expressions. This study contributes to teachers' professional development in developing the cognitive processing model and assessment tasks to evaluate and monitor the teaching and learning processes.

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Over five and a half decades after independence, Malaysian education system has been revised repeatedly to meet the demand of the rising international education standard. In 2011, the Malaysian Government has come up with a Blueprint for the National Education System transforming the education system to a new perspective so that students develop skills needed for the 21st century competencies, that is, the knowledge, skills and attitudes necessary to be competitive in the 21st century workforce. The Malaysia Ministry of Education is committed to strengthen the quality of science, technology, engineering and mathematics education to meet the global challenges (Ministry of Education Malaysia, 2013).

Malaysia education emphasises the development of cognitive thinking skills in mathematics and in developing students to think logically and systematically in problem solving. Mathematics is a subject matter in which research in cognitive psychology and cognitive science has brought much change. A major reform in mathematics education curriculum has shifted from focusing on accuracy and procedural fluency to emphasising conceptual understanding and problem solving (Saxe, Gearhart, Franke, Howard & Crockett, 1997). The knowledge and concepts in mathematics are increasing in abstraction and cognitive complexity as one progresses to higher level of mathematics. When students make the transition from concrete arithmetic to the symbolic language of algebra, they develop abstract reasoning skills necessary to excel in mathematics and science.

Algebra is an intellectual gateway to abstract reasoning to success in the learning of mathematics. It is a way of moving beyond calculating and the language of information age (Steen, 1999). Although algebra has served as a gateway to higher mathematics, the gateway has been closed to many students (Kaput, 2008). Malaysian students' algebraic achievement is poor compare to the international benchmark. International Association for the Evaluation of Educational Achievement (Mullis, Martin, Foy & Arora, 2012) reported that less than half of the Malaysian students answered the four algebraic items correctly (47% for Item 2, 43% for Item 3, 36% for Item 6 and 3% for Item 10) in the 2011 Trends in International Mathematics and Science Study.

Malaysia Ministry of Education has reassessed the curriculum and formulated guidelines for holistic assessment to ensure that students are acquiring knowledge and skills necessary for their success in the 21st century and beyond. Students' achievement will be judged through school-based assessment besides the national examination which must be aligned with the curriculum to ensure that the students acquire the knowledge and skills (Ministry of Education Malaysia, 2013). In a classroom, assessment is carried out to gather accurate and detailed information about student achievement and use the assessment process and its feedback effectively to improve teaching and learning. Assessment in classroom should focus on helping students to improve their learning which is primarily diagnostic in purpose. Feedback must be timely and rich. It must inform students their state of learning and what they need to do to improve their learning along the learning continuum. It must also inform the teachers what they need to do to address learning gaps and how to improve their teaching.

Assessment is meant to elicit and *use* the information rather than just to *have* the information. An assessment must have high accountability to provide evidence of student mastery of content standards and objectives in the domain of learning. As mentioned by William, Lee, Harrison and Black (2004), curriculum, assessment, instruction and learning are inseparable to promote student learning. Assessment is linked to learning through teaching where assessment provides feedback about students' cognitive strengths and weaknesses in the given task to help them improve their learning.

1.2 Current Assessment in School

Students' learning can be improved by assessment as assessment is able to help in identifying student's learning needs and monitoring student's growth and progress. Through assessment feedback, teachers can fine-tune their teaching and increase students' achievement. Shepard (2008) stated that assessment is considered as one of the key processes in the teaching and learning cycle that allows teachers and students to evaluate learning in addition to improve learning and teaching. Bloom (1968) had also stated clearly that assessment will have a positive effect on student learning and motivation when it is aligned with the process of teaching and learning. Assessment promotes learning by providing feedback on performance and helps students to identify their strengths and weaknesses.

Assessment practices in schools are customarily used for the purpose of measuring students' achievement on tests and examinations at the end of a learning context. The traditional methods of evaluating student learning usually occur at the end of the term or an academic year when it is too late to make any changes because most of the teaching and learning activities are completed. These assessments are designed for ranking, predicting and sorting that usually lack the detail needed to target specific improvement (Barton, 2002). Students only receive feedback such as a total score or a grade which summarises the average performance. This feedback only gives a general indication of the student's achievement. It does not provide specific inferences about student's strengths and weaknesses in the tested content areas.

Large-scale assessment or the national standardised examination uses a single test that may have a variety of formats (multiple choice questions or open-ended questions) to assess students' competence in a curriculum area. This assessment is designed to rank order schools and students for the purpose of accountability which generally is not a good instrument to help teachers to improve instruction or modify teaching approach to cater to the need of individual student. Due to the high-stake nature of national examination, the tendency of assessment *of* learning has caused teachers to react by focusing their teaching on the knowledge and skills assessed in the examination, which leads to the consequences of teaching to the test. Teachers tend to practice routine drilling in the mathematics teaching which resulted in rote learning on the part of the students. This approach to assessment is seen as passive and not having any impact on learning (Anderson, 1998; Sadler, 1989; Struyven, Dochy & Janssens, 2008) as it provides limited feedback on how to improve student performance and achievement.

An assessment is a tool to gather accurate information about students' current knowledge, skills and abilities. According to Pellegrino (2009), assessment is an instrument designed to observe students' behaviour which is outwardly invisible, and produce data that can be used to draw reasonable inferences about what the students know. As the current assessment practices provide limited feedback (Huff & Goodman, 2007), this approach to testing needs to be revised as the focus of assessment is now to provide more detailed information about student competence, that is, to diagnose the student's cognitive strengths and weaknesses in learning.

1.3 Assessment for Teaching and Learning

Student's cognitive strengths and weaknesses can be identified through a carefully designed assessment that measures specific conceptual understanding and procedural skills in a learning domain. New approaches are constantly developed by researchers to improve educational assessment by changing from *assessment of learning* to *assessment for learning* (Black & William, 1998a; Stiggins, 2001). More emphasis is now placed on the individual conceptual understanding and procedural skills or attributes that make up the ability in contrast to the types of general ability measured by the large-scale assessments. A national survey conducted by Huff and Goodman (2007) with Mathematics and English language teachers in the United States reported that majority (51% for state-mandated large-scale assessment and 53% for commercial large-scale assessment) of classroom teachers believed that large-scale assessment results do not provide sufficient information regarding students' cognitive strengths and weaknesses. Teachers do not have comprehensive information regarding student performance to enhance student achievement and students have no precise information as to how to develop and progress in their learning. As Gibbs and Simpson (2004) emphasised, educational assessment instruments should be designed and used to inform teachers about the individual student’s strengths and weaknesses, and also to identify the specific cognitive attributes that need to be strengthened.

Ideally, assessment should inform instruction, and provides teachers and students with a clear understanding about students’ cognitive strengths and weaknesses in thinking and learning, and what learning gaps that still exist in their knowledge attainment. Teachers need assessment that is able to promote commitment to learning goals and a shared understanding of the criteria of the assessment. This assessment must also be part of effective planning of instruction and learning, and provides constructive guidance about how to recognise and improve the strengths and weaknesses of all students. Hence, researchers have requested for more cognitively informed test design to better inform teaching and learning (Bennett, 1999; Mislevy, 1996; National Research Council [NRC], 2001; Pellegrino, Baxter & Glaser, 1999). Besides this, Huff and Goodman (2007) also discussed the demand for a more cognitively informed test design which is also known as cognitive diagnostic assessment (CDA) to inform teaching and learning by changing the way assessments are designed in K-12 education in the United States.

CDA is a form of assessment that could facilitate teachers in discovering individual student’s cognitive strengths and weaknesses. CDAs are capable of providing valuable feedback to teachers which help teachers to identify what knowledge and skills students have or have not mastered as well as to decide how teaching and learning needs to be adapted to the students (Huff & Goodman, 2007). Moreover, the information elicited from CDAs provides guidance to improve subsequent student performance and to motivate further learning.

1.4 Cognitive Diagnostic Information for Teaching and Learning

Teachers are searching for as much cognitive diagnostic information as possible to gauge students' cognitive strengths and weaknesses along the learning continuum. Results from Huff and Goodman (2007) survey showed that about 85% of the teachers are interested in receiving descriptions of specific conceptual understanding and procedural fluency each student demonstrated on a large-scale assessment. However, large-scale assessment that is administered at different grade levels only report students' overall performance with a single test score. Result from this assessment provides limited information to make inference about students' cognitive strengths and weaknesses to improve teaching and learning.

Although cognitive psychology is exerting its influence on assessment practices, the investigation of the underlying learning processes has been neglected in most large-scale assessment (Zhou, 2010, as cited in Alves, 2012). Assessment *for* learning in the day-to-day classroom instruction is important in bringing about students' mastery of the cognitive attributes in a learning domain. Feedback gathered from the assessment will be able to provide teachers with information about how students have learned and what they need to teach them. Teachers can use the assessment results to organise a good instructional plan for facilitating students' educational development or evaluating instruction to improve their classroom pedagogies.

CDA is an assessment instrument designed to measure specific knowledge structures and procedural skills in students, combines the theories of cognition of interest with statistical models to make inferences about students' mastery of attributes in a particular learning domain. Through fine-grained diagnostic reporting of students' attribute mastery profiles, CDA provides more detailed information concerning whether or not, or to what extent students have mastered each of a group of specific defined attributes, rather than assigning each student only one single test score (Sun, Suzuki & Toyota, 2013). These attributes affect students' learning performance and their understanding, and are needed to help students to master.

CDA can provide usable information that helps students improve their learning and also provide valuable information to teachers too. Individual student's attribute mastery probabilities are useful information for teachers to know about each student's detailed knowledge state so as to give them appropriate individual guidance for remedial work. Moreover, categorisation of students based on their knowledge states makes it clearer and easier to understand the situation about the whole class. These concise summaries of the results might be more useful than the information of each individual student's profile for teachers, especially for those teachers who are teaching many students and have difficulty to know and deal with student's problem individually (Sun et al., 2013). Therefore, CDAs which are developed from an explicit cognitive model of how students respond to test items have the means to help teachers remediate and to adjust instructional plans to meet each student's unique needs to master specific learning domain.

1.5 Diagnostic Model with Cognitive Features

CDA is described as an educational test for measuring students' conceptual understanding and procedural skills development for diagnostic purposes (Ketterlin-Geller & Yovanoff, 2009). CDA can be used to diagnose whether a student has mastered or yet to master the specific conceptual knowledge and procedural skills required to solve problems in a particular domain. The diagnostic information elicited will help the teachers to plan their instruction to build on the strengths and to remediate the weaknesses of students. Nichols (1994) and others (NRC, 2001; Pellegrino et al., 1999; Snow & Lohman, 1989) have argued that educational assessment designed from psychometric models are not optimal for informing instruction. The psychometrically designed assessment tasks were not developed from an explicit model of how students solve problems, and the scoring and reporting that are mainly used to rank order students are limited in their ability to reflect the complexity of the students’ cognitive strengths and weaknesses.

In order to gather cognitive diagnostic information, the use of a cognitive processing model which underlies the cognitive processes on problem solving with the specified cognitive attributes is required. According to Tatsuoka and Tatsuoka (1997), cognitive attributes are the knowledge and skills required for solving problems in a targeted domain. Cognitive processing model is formed by hierarchically ordering the identified attributes to describe the problem-solving strategies on assessment tasks. CDA makes use of a cognitive processing model to develop or identify items that measure specific structural knowledge, procedural skills or attributes. This model is then used to direct the analyses of the students’ item response patterns to promote specific test score inferences as stated by Gierl, Wang and Zhou (2007).

Cognitive processing model provides a frame of references as to how students' item responses are cognitively interpreted in terms of the hierarchical configuration of cognitive attributes in a learning domain. This facilitates explanation and prediction of students’ cognitive processes in item performances, including their cognitive strengths and weaknesses (Leighton & Gierl, 2007). Understanding students' knowledge acquisition and cognitive processes is essential for diagnosis since it can enhance test validity and reliability.

CDAs have drawn increasing attention from researchers and teachers to address a need for providing score users with pedagogically useful assessment information (Rupp, Templin & Henson, 2010). CDA approach makes inferences about students' attribute mastery based on their test items responses to diagnose their cognitive strengths and weaknesses. Therefore, teachers can adjust or change their instruction and intervention to improve student learning with the elicited cognitive diagnostic information.

1.6 Problem in Learning Algebra

The expectation that students will be more competent through the cognitive performance emphasised the development of strong content knowledge in core subjects like mathematics (Ministry of Education Malaysia, 2013). For students, algebraic learning holds a great weight in learning as it is fundamental in all areas of mathematics because it provides the tools (the language and structure) for representing and analysing quantitative relationships, modelling situations, solving problems, and stating and proving generalisations. However, learning algebra is an unpleasant experience for many students because it involves manipulating symbols that are not meaningful to them. Students struggle with introductory algebra and teachers have little guide in assisting their students to learn this important component of mathematics (Linsell, 2007).

Filloy and Rojano (1989) proposed that arithmetical thinking evolves very slowly from concrete processes into more abstract, algebraic thinking. When students experience difficulties in learning algebra, teachers will naturally wonder this is due to developmental constraints (insufficient developed mental structures) or whether the students have simply not achieved the necessary preparation. Little is known about the effect of students' numeracy on the learning of the early algebra, or about the strategies that students use to solve equations. Without adequate knowledge about students' mastery state of basic concepts or operations, teachers could underestimate the complexity of the individual students' learning process of algebra.

Researchers in Malaysia found that secondary school students have poor mastery of algebraic conceptual understanding and procedural skills, and are not able to solve algebraic problems. The finding by Nor Hasnida Che Ghazali and Effandi Zakaria (2011) using a survey method on 132 Form Two students revealed that 54.5% of the students achieved a score of 2.0 – 4.2 out of the total score of eight and were categorised as having a low level of conceptual understanding. Chow (2011) conducted a survey on 72 Form Two students revealed that the students faced difficulties and had misconceptions in basic understanding of letters and the manipulation of these letters or variables, used of rules of manipulation to solve equations, used of knowledge of algebraic structures and syntax to form equation, and generalisation of rules for repetitive patterns or sequences of shapes. Lim's (2008) study which involved 265 Form Two students found that students made exponent errors, misinterpretation of symbolic notation errors, conjoin errors and subtraction of negative integer errors in simplifying algebraic expression items. Lim and Noraini Idris (2006) assessed the ability in solving linear equations among nine Form Four students of varying levels of achievement revealed that the low achievers were unable to generalise the linear pattern in the form of algebraic expression or linear equation. In another study involving 123 Form Four students, Teng (2002) found that most students were unable to manipulate and interpret algebraic notation when solving linear equation. Another study by Ong (2000) with 139 Malaysian urban Form Four students' understanding of algebraic notation revealed that students made various errors, including conjoining the numerical and algebraic elements and wrong concatenation in their interpretation of letter as specific unknown, a generalised number and a variable.

These studies show the seriousness of the existing problems in the teaching and learning of algebra. A diagnostic tool or cognitive diagnostic assessment (CDA) is needed to elicit information about students' cognitive strengths and weaknesses which will enable the teachers to guide the students in algebraic learning. Without the information regarding students' cognitive processes in solving problems, it is an arduous task to help students in mastering complex topic such as algebra.

1.7 Research Rationales

Assessment has been recognised as a powerful tool to improve teaching and learning (Black & Wiliam, 1998a; Goodrum, Hackling & Rennie, 2001). New methods of assessment have been developed to evaluate the ways students interpret mathematical problems and construct strategies in problem solving, including the domain of algebra (Curriculum Development Centre [CDC], 2003). Mathematics teachers have used open-ended items in their assessment to gain insight into students' cognitive skills in problem solving and their understanding of mathematical concepts. In the assessment of students' learning progress, information about student attainment in the learning outcomes intended in the curriculum is necessary. Unfortunately, the formative and summative assessments that teachers regularly administer in their classrooms provide limited information about students' cognitive abilities (Archbald & Grant, 2000; Harrera, Murray & Cabral, 2007; Saxe et al., 1997). These assessments provide minimal direct and immediate feedback to the teachers and students.

Teachers need more information about the cognitive strengths and weaknesses of specific knowledge and skills individual student demonstrated on assessment to improve their instructional planning. The information gathered is different from what the current standardised assessment provides. Teachers need more guidance and information in assessing students' cognitive strengths and weaknesses to gain insight into their cognitive abilities. The results of Huff and Goodman's (2007) survey in 2006 also revealed that teachers strongly viewed assessment as being the best way to gauge students' strengths and weaknesses in learning. However, researches have shown that classroom assessment practices do not always provide accurate and valid information that measure a full complement of cognitive attributes (Notar, Zuelke, Wilson & Yunker, 2004; Stiggins, 2001). Efforts to help teachers incorporate CDA principles into the design of classroom assessment practices would therefore be seen to provide some obvious benefits.

CDA and its psychometric models are able to provide pedagogically useful assessment information about the cognitive strengths and weaknesses of students in the learning of algebra. This diagnostic information will further aid the intervention program to improve and maintain students' interest to explore the abstract world of algebra that presents many obstacles and misconceptions for students. As algebraic expressions is the introductory topic in the learning of algebra, it is vital for teachers to gain insight into students' cognitive strengths and weaknesses in this sub-domain so that weaknesses can be overcome before progressing to more advance and abstract topics.

CDA design ensures that the cognitive attributes of interest are explicitly targeted during items and test development. The information that reflects students' cognitive strengths and weaknesses elicited from CDA is able to guide teachers to help the students in their knowledge and performance in the algebraic expressions learning. CDA may help to increase the accuracy and reliability of the determination of the students' cognitive attribute profiles, and can be utilised to improve both the teaching and the learning processes.

1.8 Problem Statement

Researchers have indicated that students faced problem in developing algebraic understanding and solving algebraic problem (Boulton-Leiws, Cooper, Atweh, Pillay & Wilss, 1998; Chow, 2011; Effendi Zakaria, 2011; Kieran, 1992; Linchevski & Herscovics, 1996; Nickson, 2000; Nor Hasnida & Lim, 2008; Ong, 2000; Teng, 2001; Welder, 2012). It was realised that students at both levels of lower and upper secondary school displayed inability to handle problems that involved formulation and/or manipulation of algebraic expressions and equations. Since mathematics is hierarchical in nature, understanding of higher order mathematical concepts is dependent on proper understanding of related lower order concepts. It is imperative that a study is carried out to examine students' cognitive strengths and weaknesses in the learning of the conceptual knowledge and procedural fluency of algebraic expressions at the early stages of algebra learning. Reports from research projects conducted by mathematics educators such as Kuchemann (1981), Kieran

(1989), Linchevski and Herscovicks (1996) have indicated that students experience serious problems in understanding pre-algebraic concepts. Nickson (2000) also pointed out students encounter difficulties when solving problems that involve manipulation of algebraic expressions and equations.

Assessment has a positive impact on learning as it provides information to teachers that can be used to plan their teaching. Teachers believe that the best way to gauge students' achievement in a learning domain is through assessment. Teachers need an assessment tool that can gather specific information of students' unobserved cognitive strengths and weaknesses in thinking and learning. According to Huff and Goodman (2007), when a cognitive processing model provides a framework for both the design of the CDA and the design of instructions surrounding the assessment, learning and teaching are optimised.

Teachers' classroom assessment practices are not always well integrated with instruction as they could be, and do not provide valid and detailed diagnostic information about students' strengths and weaknesses of the cognitive processes. Normally, the test specifications for assessments in classroom only specify content requirements and no explicit consideration is given to the type of cognitive attributes that underlie a curriculum. This lack of explicit consideration to cognitive attributes in the development of items for an assessment has also been reported in the study of O'Neil, Sireci and Huff (2004). Although teachers can predict the overall performance of the students through their own classroom assessment, the results do not tell them much about their students' cognitive processes in item performance (Lorsbach, Tobin, Briscoe & Lamaster, 1992) as students tend to focus on recall to get through the task (Duscl & Gitomer, 1997). Moreover, most teachers associate diagnostic information with reporting at the individual achievement level with limited information of students' structural knowledge, procedural skills and abilities elicited from assessment (Huff & Goodman, 2007).

Traditional diagnostic assessment provides information that is used by teachers and students to determine what students already know and can do with respect to the content specification and expectation. A great deal of research in Malaysia has been done to investigate a variety of misconceptions and difficulties in learning algebra (Chow, 2011; Lim, 2008; Lim & Noriani Idris, 2006; Nor Hasnida Che Ghazali & Effendi Zakaria, 2011; Ong, 2000; Teng, 2002), but very little consideration is given to the assessment of students' cognitive strengths and weaknesses in learning algebra. The question of how to assess the students' cognitive processes in algebraic learning may still be new and not easy to deal with for many teachers. CDA which is capable of assessing students' cognitive strengths and weaknesses is not well understood by the teachers and CDA is reported by Russell, Qualter and McGuigan (1995) as 'being seriously in need of development'.

Teachers need a mechanism that can provide them with detailed information on individual student's strengths and weaknesses on specific knowledge structure, procedural skills and abilities. This information ought to direct the emphasis on student needs and also give meaningful interpretation to the score obtained by students. Assessment instruments need to be designed in such a way to inform teachers about the individual student's cognitive profile, and to highlight the specific cognitive attributes that need to be strengthened. CDAs are capable to provide information about students' cognitive and learning processes to influence meaningful student learning.

1.9 Research Aim

The aim of this study is to develop a CDA design with item development guided by an explicit cognitive processing model for measuring the specific fine-grained cognitive attributes, and to determine the students' attribute mastery in algebraic expression learning. Initially, this study investigates whether the teachers' conceptualisation of the students' cognitive processes and problem solving strategies as presented in the cognitive processing model are consistent with the actual cognitive processes and problem solving strategies students used when responding to the test items.

Next, Q-matrix which specifies the cognitive blueprint or specification for the cognitive test was generated. The purpose of this blueprint is to specify the attribute-to-item relationships where items were developed to measure specific attribute(s) outline in the hierarchy of cognitive processing model. Rupp and Templin (2008) claimed that Q-matrix embodies the development of the assessment instrument in used and in addition determining the quality of the resulting diagnostic information. The adequacy and accuracy of the attribute-to-item specification entries ('1' if the attribute is required to solve the item and '0' if the attribute is not required to solve the item) in the Q-matrix is determined using the Fusion Model.

This study then statistically analyse the students' response data using the Fusion Model to provide differentiated cognitive profile of individual student which underlies the mastery state of the specified cognitive attributes required in the algebraic expressions learning. These statistically driven classifications of mastery and non-mastery state according to multiple latent attributes competences provide detailed information about students' cognitive strengths and weaknesses in the learning of algebraic expressions. From the test performance, inferences about students' cognitive processes are made to explain the conceptual understanding and procedural fluency of algebraic expressions learning as test performances are directly linked to the information about students' cognitive strengths and weaknesses.

1.10 Research Objectives

This study focuses on the development of a CDA to measure specific knowledge structure and procedural skills in students to provide diagnostic information about their cognitive strengths and weaknesses in the learning of algebraic expressions. The objectives of this study are to:

(i) construct expert-based cognitive processing models and validate using students' verbal reports and written responses on item performances to accentuate the cognitive processes and knowledge structures in the learning of algebraic expressions.

(ii) construct Q-matrices to develop test items that measure specific attributes outlined in the cognitive processing models.

(iii) to identify students' mastery state.

(iv) estimate individual student's cognitive profiles for the tested attributes of the algebraic expressions learning.

1.11 Research Questions

The research questions for this study are as follows:

(i) Are the student-based cognitive processing models consistent with the expert-based cognitive processing models for *conceptual knowledge* and *processing skills* in algebraic expressions learning?

(ii) To what extent the attribute-to-item relations specified in the Q-matrices is adequate and accurate?

(iii) What are the hierarchy of the attribute difficulty for the *conceptual knowledge* and *processing skills* in algebraic expressions learning?

(iv) What are the students' cognitive strengths and weaknesses for the tested attributes of *conceptual knowledge* and *processing skills* in algebraic expressions learning?

1.12 Significance of the Study

This study explores a new way of designing test items that integrates the curriculum, instruction and assessment to diagnose students' cognitive strengths and weaknesses of algebraic expressions learning. The findings of this study will contribute in the classification and interpretation of students' learning performance. With the identification of students' cognitive strengths and weaknesses using test items that were developed with reference to the validated cognitive processing models, teachers can achieve better understanding about the students' performance characteristics or attribute attainment. This interpretable diagnostic feedback will also help students to take actions to close the gap between their current achievement level and their desired learning goals (Black & William, 1998a).

Findings from this study in developing and validating the cognitive processing models may have its implication on professional development among teachers to develop assessment tasks to evaluate and monitor the learning progress of students. According to Alderson (2005), descriptive diagnostic test information is needed for improving instructional designs and guiding students' learning. As the importance of algebra is acknowledged by most teachers and educators, it is a fact that the fundamental nature of algebraic concept is complex where teachers have been seeking answers on what they can do to help students master the conceptual knowledge and procedural skills in learning algebra. Teachers may develop assessment tasks for different learning sub-domains or units from the findings, and guides for this study will further aid them in the teaching profession.

This study also provides valuable references for practicing researchers who are interested in the study of cognitive processing models diagnosis and the research of the development of CDA. CDA is a relatively new approach that provides formative diagnostic feedback through a fined-grained reporting of students' attribute mastery profiles (DiBello, Roussos & Stout, 2007; Tatsuoka, 1983). Given the increasing researches and operational interests in CDA and cognitive diagnostic models (CDMs), this study provides some guidelines and recommendations for potential CDM users to keep abreast with the current demand in cognitive diagnosis for the teaching and learning of mathematics, particularly algebra. This study is able to contribute to the Malaysian assessment reform policy which supports the adoption of formative assessment approach to improve students' mastery of 21st century competencies such as learning how to learn, thinking about their own thinking, and knowing how to plan, monitor and evaluate own thinking and understanding.

1.13 Limitation of the Study

There are several limitations associated with the current study that constraint the generalisability of the results. The main limitation is the selection of schools as the research sample. Given administrative restrictions, random sampling was not possible. The sample in this study was selected using convenience sampling (Gay, Mills & Airasian, 2006) as the students were chosen by the schools' administrators. The target sample was to include students from classes that comprised of low, intermediate and high abilities. Even though request has been made to the school principals to include students from varying abilities for heterogeneity of sample, the school administrators only approved of the use of specific classes and the selection of students was done without detailed academic background provided. The sample covered only students from Kedah and Northern Perak due to time constraint. This sample is relatively small for generalisation of findings.

Although various approaches were taken in developing and validating the cognitive processing models in this study, the completeness and hierarchy in specifying the necessary attributes is one's concern. There is still a possibility that other alternate hierarchical structures are available (Gierl, Roberts, Alves & Gotzmann, 2009; Gierl et al., 2009) because teachers may use different instructions and students may use varying learning strategies in answering an item. Dishonest answers were detected in students' script even though subject teachers were requested to supervise the test administration. This might affect the accuracy of the findings.

1.14 Definition of Terminologies

1.14.1 Cognitive Diagnostic Assessment (CDA)

CDA is a diagnostic assessment approach that is aimed at providing formative diagnostic feedback through a fine-grained reporting of students' attribute mastery profiles (DiBello, Roussos & Stout, 2007). The CDA approach combines theories of cognition with statistical models to identify and evaluate the students' cognitive attributes as specified in a cognitive processing model to make inferences about the students' mastery state for the tested attributes. In this study, CDA is developed to identify the mastery level of students in the learning of algebraic expressions.

1.14.2 Cognitive Attributes

Cognitive attribute is a description of the conceptual knowledge and procedural skills needed to perform a task in a specific domain (Gierl et al., 2009). In this study, cognitive attributes are referred to the *conceptual knowledge* and *processing skills* utilised by the students to correctly solve items related to algebraic expressions learning.

1.14.3 Cognitive Processing Model

Cognitive processing model represents the students' knowledge structures and cognitive processes in responding to an item. It reflects the hierarchical dependency among specified attributes as these attributes are interrelated and developed sequentially. This model provides an interpretive framework that can guide item development so test performance can be linked to specific cognitive inferences about students' knowledge structure, cognitive processes and strategies. This study adapted the attribute hierarchy method (Leighton & Gierl, 2007) to develop the cognitive processing model for algebraic expressions learning.

1.14.4 Q-Matrix

Q-matrix is an item-by-attribute binary matrix indicating the attribute(s) required by each item (Tatsuoka, 1985) so that responses to items can reveal the attribute mastery configuration of the students. Q-matrix is crucial for parameter estimations as it represents the loading factors of a CDM.

1.14.5 Cognitive Diagnostic Models (CDMs)

CDMs are probabilistic, confirmatory multidimensional latent-variable models with a simple or complex loading structure developed to diagnose the presence or absence of multiple fine-grained attributes required for correctly solving items in a test (de la Torre, 2009). CDMs are used to analyse item response data in such a way that multivariate classifications of students can be made on the basis of their latent attribute mastery patterns. This study utilised the Fusion Model, a non-compensatory CDM to classify the students' mastery profiles in algebraic expressions learning.

1.14.6 Fusion Model

Fusion model is an IRT-based attributes-diagnosis model that defines the probability of observing student *j* responses to an item *i* in term of student ability parameters and item parameters (Hartz & Roussos, 2008). This probability is represented as $P\left(X_{ij} = x \middle| \theta_j, \beta_i\right)$, where $X_{ij} = x$ is the response of student *j* to item *i*, θ_j is the vector of student *j* ability parameters, and β_i is a vector of item *i* parameters. The item *i* = 1, ..., *I* relate to a set of cognitive attributes *k* = 1, ..., *K* as specified in a Q-matrix.

1.14.7 Algebraic Expressions

An algebraic expression is a mathematical expression that consists of variables, constants or numbers and operators where the value of this expression can change, as defined in the Form One Curriculum Specification (CDC, 2003). Algebraic expressions do not contain relation symbols like the equal sign. Expressions are simply numbers, unknowns, and operations strung together. In this study, the operations on algebraic expressions are limited to the basic arithmetic operations of addition, subtraction, multiplication and division to transform the original expression to its simpler equivalent forms.