UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MAA 111 – Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:** Jawab **semua sepuluh**[10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (i) Show that $A=\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ is row-equivalent to I and then write A as a product of elementary row matrices.

(ii) Given

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + z &= 3 \\ x + y + \left(k^2-8\right)z &= k \end{aligned}$$

Determine the value(s) of k so that the above system has infinitely many solutions.

(iii) Prove that if A and B are $n\times n$ diagonal matrices then $AB = BA$.

[12 marks]

1. (i) Tunjukkan bahawa $A=\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ *adalah setara secara baris kepada I dan kemudian tulis A sebagai hasildarab matriks baris permulaan.*

(ii) Diberi

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + z &= 3 \\ x + y + \left(k^2-8\right)z &= k \end{aligned}$$

Tentukan nilai k supaya sistem di atas mempunyai bilangan penyelesaian yang tak terhingga banyaknya.

(iii) Buktikan bahawa jika A dan B adalah matriks pepenjuru $n\times n$ *maka* $AB = BA$.

[12 markah]

2. Show that

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix} = (b-a)(c-a)(d-a)(c-b)(d-b)(d-c).$$

(Explain all determinant properties that you use).

[8 marks]

2. *Tunjukkan bahawa*

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix} = (b-a)(c-a)(d-a)(c-b)(d-b)(d-c).$$

(Jelaskan semua sifat penentu yang anda gunakan).

[8 markah]

3. For any vector $v=(x,y,z)$ in $\mathbb{R}^3$, let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x,y,z)=(x+z,x+y,y-z)$.

(i) Show that T is a linear transformation

(ii) Find the basis and dimension of the image of T.

(iii) What is the dimension of the Kernel of T?

(iv) Does $v=(-1,-1,1)$ belong to the kernel of T?

[10 marks]

3. *Untuk sebarang vektor* $v=(x,y,z)$ *dalam* $\mathbb{R}^3$, *biar* $T:\mathbb{R}^3 \to \mathbb{R}^3$ *tertakrif dengan* $T(x,y,z)=(x+z,x+y,y-z)$.

(i) Tunjukkan bahawa T adalah suatu transformasi linear.

(ii) Cari asas dan dimensi imej T.

(iii) Apakah dimensi inti T?

(iv) Adakah $v=(-1,-1,1)$ *ahli kepada inti T?*

[10 markah]

4. For each of the following, determine whether or not S is a subspace of the indicated vector space. Prove your claim.

(i) $S=\left\{ax^3+bx^2+cx+d \mid ad>0\right\}$ in $P_3(\mathbb{R})$.

(ii) $S=\left\{(x_1,x_2,x_3,x_4) \mid x_1=x_2+2 \text{ and } x_3=3x_4\right\}$ in $\mathbb{R}^4$.

(iii) $S=\left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \middle| \, a,b \text{ are real numbers}\right\}$ in $M_{2\times 2}(\mathbb{R})$.

[7 marks]

4. *Untuk setiap yang berikut, tentukan sama ada S ialah suatu subruang dari ruang vektor yang ditunjukkan. Buktikan tuntutan anda.*

(i) $S = \{ax^3 + bx^2 + cx + d \mid ad > 0\}$ *dalam* $P_3(\mathbb{R})$.

(ii) $S = \{(x_1, x_2, x_3, x_4) \mid x_1 = x_2 + 2 \text{ dan } x_3 = 3x_4\}$ *dalam* $\mathbb{R}^4$.

(iii) $S = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \middle| \, a, b \text{ adalah nombor nyata} \right\}$ *dalam* $M_{2\times 2}(\mathbb{R})$.

[7 markah]

5. Given a set of vectors $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (1,2,3)\}$.

(i) Use determinant to show that $\{v_1, v_2, v_3\}$ is linearly independent. Deduce that $\{v_1, v_2, v_3\}$ is a basis for R^3.

(ii) Use the Gram-Schmidt process to transform $\{v_1, v_2, v_3\}$ into an orthonormal basis for R^3.

[15 marks]

5. *Diberi set vektor* $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (1,2,3)\}$.

(i) *Guna penentu untuk tunjukkan yang* $\{v_1, v_2, v_3\}$ *adalah tak bersandar linear. Simpulkan bahawa* $\{v_1, v_2, v_3\}$ *adalah asas bagi* R^3.

(ii) *Guna proses Gram-Schmidt untuk menukarkan* $\{v_1, v_2, v_3\}$ *kepada suatu asas ortonormal bagi* R^3.

[15 markah]

6. Given two bases

$$A = \{(2,0,1), (1,2,0), (1,1,1)\} \text{ and } B = \{(6,3,3), (4,-1,3), (5,5,2)\}$$

of R^3.

(i) Find the coordinate of $v = (4,-9,5)$ with respect to *A*, v_A and with respect to *B*, v_B.

(ii) Obtain the change of basis matrix from *B* to *A*, $I_{A\leftarrow B}$.

(iii) Verify that $v_A = I_{A\leftarrow B} v_B$.

[12 marks]

6. *Diberi dua asas*

$$A = \{(2,0,1), (1,2,0), (1,1,1)\} \text{ dan } B = \{(6,3,3), (4,-1,3), (5,5,2)\}$$

bagi R^3.

(i) *Cari koordinat bagi* $v = (4, -9, 5)$ *berkenaan dengan A,* v_A *dan berkenaan dengan B,* v_B.

(ii) *Dapatkan matriks pertukaran asas dari B ke A,* $I_{A \leftarrow B}$.

(iii) *Tentusahkan yang* $v_A = I_{A \leftarrow B} v_B$.

[12 markah]

7. Given that the system $AX = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 7 \\ 3 \\ 10 \\ -2 \end{bmatrix}$$

is inconsistent. Find the least squares solution to the system. (State the normal equation and show your working in solving the problem).

[10 marks]

7. *Diberi sistem* $AX = \boldsymbol{b}$*, yang mana*

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & -1 \end{bmatrix} \quad \textit{dan} \quad \boldsymbol{b} = \begin{bmatrix} 7 \\ 3 \\ 10 \\ -2 \end{bmatrix}$$

adalah tak konsisten. Cari penyelesaian kuasa dua terkecil terhadap sistem ini. (Nyatakan persamaan normal dan tunjukkan jalan kerja anda dalam menyelesaikan masalah ini).

[10 markah]

8. Given that $T : \mathbb{R}^4 \to \mathbb{R}^2$ defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3, 2x_1 + x_2 + x_4)$$

is a linear transformation.

(i) Find the standard matrix A for T.

(ii) Find a basis for the null space of A.

(iii) From your results in (ii), what can you deduce about the kernel of T?

[10 marks]

8. *Diberi* $T : \mathbb{R}^4 \to \mathbb{R}^2$ *tertakrif dengan*

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3, 2x_1 + x_2 + x_4)$$

adalah suatu transformasi linear.

(i) Cari matriks piawai A bagi T.

(ii) Cari asas bagi ruang nol A.

(iii) Dari hasil anda dalam (ii), apa yang boleh anda simpulkan tentang inti T?

[10 markah]

9. (i) Given

$$B = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}.$$

Determine whether B is diagonalizable or not.

(ii) Prove that if a matrix C is nonsingular and λ is an eigenvalue of C, then λ^{-1} is an eigenvalue of C^{-1}.

[6 marks]

9. (i) Diberi

$$B = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}.$$

Tentukan sama ada B terpepenjurukan atau tidak.

(ii) Buktikan bahawa jika suatu matriks C tak singular dan λ ialah nilai eigen bagi C, maka λ^{-1} ialah nilai eigen bagi C^{-1}.

[6 markah]

10. Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(i) Obtain the characteristic equation of A.

(ii) Find A^{10} using the method of diagonalization.

(iii) Verify the Cayley-Hamilton theorem for A.

[10 marks]

10. Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(i) Dapatkan persamaan cirian bagi A.

(ii) Cari A^{10} menggunakan kaedah pempepenjuruan.

(iii) Tentusahkan teorem Cayley-Hamilton bagi A.

[10 markah]

- ooo O ooo -