UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MSG 253 – Queueing Systems and Simulation
[Sistem Giliran dan Simulasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>THIRTEEN</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>TIGA BELAS</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u> : Answer **<u>all three</u>** [3] questions.

*[**<u>Arahan</u>** : Jawab **<u>semua tiga</u>** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) A queueing process that is based on the birth and death process has the following birth and death rates:

$$\lambda_n = \frac{\lambda}{(n+1)}, \quad \text{for } n = 0, 1, 2, 3, \cdots$$

$$\mu_n = \mu, \quad \text{for } n = 1, 2, 3, \cdots$$

(i) Draw a rate-diagram to represent the queueing system.

(ii) Show that $P_n = \dfrac{\rho^n}{n!} e^{-\rho}$, where $\rho = \dfrac{\lambda}{\mu}$.

(iii) Next, show that $L_q = \rho - 1 + e^{-\rho}$.

[50 marks]

(b) Consider a single server queueing system where some customers balk (refuse to enter the system) and some customers who enter the system later get impatient and renege (leave without being served). Customers arrive according to a Poisson process with a mean rate of 4 per hour. An arriving customer who finds n customers already there will balk with the following probabilities:

$$\text{P}\{\text{balk} | n \text{ already there}\} = \begin{cases} 0 & \text{if } n = 0, \\ 0.5 & \text{if } n = 1, \\ 0.75 & \text{if } n = 2, \\ 1 & \text{if } n = 3. \end{cases}$$

Service times have an exponential distribution with a mean of 1 hour. A customer already in service never reneges, but the customers in the queue may renege. In particular, the remaining time that the customer at the front of the queue is willing to wait in the queue before reneging has an exponential distribution with a mean of 1 hour. For a customer in the second position in the queue, the time that she or he is willing to wait in this position before reneging has an exponential distribution with a mean of 2 hour.

(i) Construct the rate diagram for this queueing system.
(ii) Obtain the steady state distribution of the number of customers in the systems.
(iii) Find the expected fraction of arriving customers who are lost due to balking.
(iv) Find the expected number of customers in the system.

[50 marks]

1. (a) *Suatu proses giliran yang beasaskan kepada proses lahir dan mati mempunyai kadar kelahiran dan kematian seperti berikut:*

$$\lambda_n = \frac{\lambda}{(n+1)}, \text{ bagi } n = 0, 1, 2, 3, \cdots$$

$$\mu_n = \mu, \text{ bagi } n = 1, 2, 3, \cdots$$

(i) *Lukiskan gambar rajah kadar yang mewakili sistem giliran itu.*

(ii) *Tunjukkan bahawa* $P_n = \frac{\rho^n}{n!} e^{-\rho}$ *, di mana* $\rho = \frac{\lambda}{\mu}$.

(iii) *Kemudian, tunjukkan bahawa* $L_q = \rho - 1 + e^{-\rho}$.

[50 markah]

(b) *Pertimbangkan suatu sistem giliran satu pelayan dengan sebahagian daripada pelanggannya* enggan *(tidak memasuki sistem) dan sebahagian daripada pelanggan yang telah memasuki sistem pula menjadi tidak sabar dan belah (meninggalkan sistem sebelum dilayan). Ketibaan pelanggan berlaku mengikut proses Poisson dengan kadar min 4 sejam. Seorang pelanggan yang tiba dan mendapati seramai n pelanggan sudah berada di sana akan enggan dengan kebarangkalian berikut:*

$$P\{enggan \mid n \text{ sudah berada di sana}\} = \begin{cases} 0 & jika\ n = 0, \\ 0.5 & jika\ n = 1, \\ 0.75 & jika\ n = 2, \\ 1 & jika\ n = 3. \end{cases}$$

Masa layan adalah mengikut agihan eksponen dengan min 1 jam. Seorang pelanggan yang sedang dilayan tidak akan belah, tetapi pelanggan yang sedang menunggu mungkin akan belah. Secara khususnya, masa yang tinggal untuk terus menunggu sebelum belah bagi pelanggan yang berada paling hadapan barisan menunggu adalah mengikut agihan eksponen dengan min 1 jam. Bagi pelanggan yang berada di tempat kedua barisan menunggu pula, masa yang tinggal untuk terus menunggu sebelum belah adalah mengikut agihan eksponen dengan min 2 jam.

(i) *Bentukkan gambar rajah kadar bagi sistem giliran ini.*

(ii) *Dapatkan agihan bilangan pelanggan di dalam sistem dalam keadaan mantap.*

(iii) *Tentukan nisbah jangkaan pelanggan yang tiba tetapi hilang disebabkan oleh keengganan.*

(iv) *Tentukan bilangan jangkaan pelanggan di dalam sistem.*

[50 markah]

2. (a) Cars arrive at a small petrol station to refuel according to a Poisson process with rate 30 per hour, and have an exponential service time distribution with mean 4 minutes. Since there are four petrol pumps available, four cars can refuel simultaneously, but unfortunately there is no room for cars to wait. Hence, if a car arrives when all pumps are busy, the driver leaves immediately. For each customer that is served, an average profit is made of RM7.

(i) Determine the probability that an arriving car is not refueled.
(ii) What is the expected number of pump that is being used?
(iii) What is the expected profit per day (consisting of eight hours)?

Suppose that the station manager has the opportunity to buy an adjacent parking lot. So there is room to wait for all cars that arrive when the four pumps are busy. Suppose all drivers decide to wait instead of leaving when this happens.

(iv) Determine the probability that an arriving car has to wait.
(v) What is the expected number of cars that will occupy the adjacent parking lot?
(vi) How long on average will a car occupy the adjacent parking lot?
(vii) What is the expected profit per day in this case?

[40 marks]

(b) Past records indicate that each of the 5 laser computer printers at the School of Mathematical Sciences, USM, needs repair after about 20 hours of use. Breakdowns have been determined to be Poisson distributed. The one technician on duty can service a printer in an average of 2 hours, following an exponential distribution. Printer downtime costs RM120 per hour. Technicians are paid RM25 per hour. Should the School hire a second technician?

[30 marks]

(c) A state agency that handles compensation for those who are unemployed is considering two options for processing applications. Option 1: Four clerks process applications in parallel from a single queue. Each clerk fills out the required form in the presence of the applicant based on information that is verbally related to the clerk. Processing time is exponentially distributed with a mean of 45 minutes. Option 2: Each applicant first fills out the form without the help of the clerk. The time required to accomplish this is exponentially distributed, with a mean of 65 minutes. When the applicant finishes the form, he or she joins a single queue to await a review by one of the four clerks. The time required to review a form is exponentially distributed with a mean of 5 minutes.

Given that the arrival of applicants is a Poisson process with a mean rate of 4.8 per hour, compare the two options with respect to the expected number of applicants in the system and the expected time in the system.

[30 marks]

2. (a) *Kereta tiba di sebuah stesyen petrol kecil untuk mengisi petrol mengikut proses Poisson dengan kadar 30 sejam, dan masa layan adalah megikut agihan eksponen dengan min 4 minit. Oleh kerana terdapat empat pam petrol sahaja, empat kereta dapat mengisi petrol dengan serentak. Malangnya, tidak ada ruang bagi kereta lain untuk menunggu. Dengan itu, jika sesebuah kereta tiba semasa kesemua pam sibuk digunakan, pemandu kereta berkenaan terpaksa meninggalkan stesyen itu dengan segera. Bagi setiap pelanggan yang dilayan, purata keuntungan yang diperolehi adalah RM7.*

   (i) *Tentukan kebarangkalian bahawa sesebuah kereta yang tiba tidak dapat diisi petrol.*
   (ii) *Berapakah bilangan purata pam yang digunakan?*
   (iii) *Berapah keuntungan jangkaan harian (melibatkan lapan jam)?*

   Katakan pengurus stesyen berpeluang membeli lot parkir bersebelahan. Dengan itu, ada ruang menunggu bagi kesemua kereta yang tiba semasa keempat-empat pam sedang digunakan. Katakan kesemua pemandu ada menunggu apabila keadaan ini berlaku.

   (iv) *Tentukan kebarangkalian bahawa sesebuah kereta yang tiba terpaksa menunggu.*
   (v) *Berapakah bilangan jangkaan kereta yang ada berada di lot parkir bersebelahan?*
   (vi) *Berapa lamakah pada puratanya sesebuah kereta akan berada di lot parkir bersebelahan?*
   (vii) *Berapakah jangkaan keuntunggan harian bagi kes ini?*

[40 markah]

(b) *Rekod yang lepas menunjukkan bahawa setiap satu daripada 5 pencetak computer laser di Pusat Pengajian Sains Matematik, USM, perlu dibaiki selepas 20 jam penggunaan. Kerosakan didapati berlaku mengikut agihan Poisson. Seorang juruteknik yang bertugas boleh membaiki pencetak dalam masa purata 2 jam, mengikut agihan eksponen. Kos ketidak-operasian pencetak adalah RM120 sejam. Juruteknik dibayar RM25 sejam. Patutkah Pusat Pengajian mengupah seorang lagi juruteknik?*

[30 markah]

(c) *Sebuah perbadanan kerajaan negeri yang mengendalikan urusan subsidi kepada mereka yang menganggur sedang menimbangkan dua opsyen untuk memproses permohonan. Opsyen 1: Empat kerani akan memproses permohonan secara selari daripada satu barisan menuggu. Setiap kerani akan mengisi borang untuk pemohon berasaskan kepada maklumat yang diberikan secara lisan kepadanya. Masa pemprosesan adalah mengikut agihan eksponen dengan min 45 minit. Opsyen 2: Setiap pemohon mengisi sendiri borang yang disediakan. Masa yang diperlukan ialah mengikut agihan eksponen dengan min 65 minit. Apabila borang siap diisi, pemohon akan memasuki satu barisan menunggu untuk menuggu giliran penyemakan borang oleh salah seorang daripada empat kerani yang ada. Masa penyemakan boring ialah mengikut agihan eksponen dengan min 5 minit.*

Jika ketibaan pemohon adalah mengikut proses Poisson dengan kadar min 4.8 sejam, bandingkan dua opsyen itu berasaskan kepada bilangan jangkaan pemohon di dalan sistem dan jangkaan masa di dalam sistem.

[30 markah]

3. (a) To facilitate customers with only check cashing needs, a certain bank has assigned one special teller to perform only this service. About 40% of arriving customers are of this nature. The remaining 60% have additional transactions to process and must use one of the other four tellers in the bank. Customers arrive at intervals described by a uniform distribution with a mean of 66 seconds and a range of ± 10 seconds, and either join the check-cashing teller line or the single line for the other four tellers. Check cashing requires 120 ± 30 seconds, and all other back business takes 360 ± 120 seconds per customer.

(i) Write a GPSS World program to simulate a 7-hour banking day from 9:00am to 4:00pm. Measure the statistics for the queues.
(ii) For the arrival of those customers who have only check cashing needs, if the special teller is busy and one of the four regular tellers is free, these customers are allowed to go to the regular tellers. How do you modify the GPSS program in (i) for this?
(iii) Suppose that upon the arrival of those customers who need the regular service, if the four regular tellers are busy and the special teller is free, these customers are allowed to go to the special teller, how do you modify your GPSS program in (ii)?

[50 marks]

(b) Ships arrive at a harbor at an inter-arrival time of either 1, 2 or 3 hours, each with equal chances of happening. There are three berths to accommodate them. They also need the service of a crane for unloading and there are two cranes. After unloading, 10% of the ships stay to refuel before leaving; the others leave immediately. Ships do not need the cranes for refueling. The unloading time is either 2, 3, 4 or 5 hours with probabilities of 0.1, 0.3, 0.4 and 0.2, respectively. The refueling time is either 1, 1½ , or 2½ hours, each with equal chances of happening. The answers to the following questions are to be provided:

i) What is the average time that a ship spent at the harbor?
ii) What proportion of time are the cranes idle?
iii) What is the average time that a ship spent waiting for a berth to be available?

(Use the enclosed 2-digit random number table with the first column for *inter-arrival time*, second column for *unloading time*, the third column for *determining whether to refuel or not* and the fourth column for *refueling time.*)

[50 marks]

3. (a) *Bagi memudahkan pelanggan yang hanya berurusan menunaikan cek, sebuah bank telah mengumpukkan satu teller khas hanya untuk urusan berkenaan. Lebih kurang 40% daripada pelanggan adalah di dalam kategori ini. Lebihan 60% pelanggan memerlukan urusan lain untuk diproses dan mereka mesti menggunakan salah satu daripada empat teller lain yang disediakan. Pelanggan tiba mengikut selang yang bersifat agihan seragam dengan min 66 saat dan berjulat ± 10 saat, dan mereka memasuki sama ada barisan menunggu untuk teller menunaikan cek ataupun satu barisan menunggu untuk empat teller yang lain. Menunaikan cek memerlukan 120 ± 30 saat, dan semua urusan lain memerlukan 360 ± 129 saat bagi setiap pelanggan.*

(i) *Tuliskan satu aturcara GPSS World untuk mensimulasi satu hari bekerja bank selama 7 jam bermula 9:00 pagi sehingga 4:00 petang. Tentukan statistik bagi semua barisan menunggu.*

(ii) *Bagi pelanggan yang tiba untuk urusan menunaikan cek sahaja, jika teller khas sedang sibuk dan salah satu daripada empat teller biasa adalah bersenang, mereka dibenarkan berurusan dengan teller biasa. Bagaimanakah aturcara GPSS di dalam (i) perlu diubahsuai untuk ini?*

(iii) *Katakan bahawa semasa ketibaan pelanggan yang memerlukan urusan biasa, jika keempat-empat teller biasa adalah sibuk dan teller khas sedang bersenang, mereka dibenarkan berurusan dengan teller khas itu. Bagaimanakah aturcara GPSS di dalam (ii) perlu diubahsuai untuk ini?*

[50 markah]

(b) *Kapal tiba di pelabuhan dengan masa antara ketibaan ialah sama ada 1, 2, atau 3 jam, dengan setiap satunya sama mungkin akan berlaku. Tiga pelantar disediakan untuk kegunaan kapal. Kapal juga memerlukan kren untuk pemunggahan dan terdapat dua kren di perlabuhan itu. Selepas siap pemunggahan, 10% daripada kapal akan terus berlabuh untuk mengisi minyak; yang lainnya akan terus beredar. Semasa pengisian minyak, kren tidak diperlukan. Masa memunggah ialah sama ada 2, 3, 4 atau 5 jam dengan kebarangkalian masing-masingnya ialah 0.1, 0.3, 0.4 dan 0.2. Masa untuk mengisi minyak pula ialah 1, 1½ atau 2½ jam, dengan setiap satunya sama mungkin akan berlaku. Jawapan kepada soalan-soalan berikut mestilah diberikan:*

i) *Berapakah masa purata sesebuah kapal berada di pelabuhan?*

ii) *Berapakah peratusan masa bersenang kren?*

iii) *Berapakah masa purata sesebuah kapal menuggu untuk menggunakan pelantar?*

(Gunakan sifir nombor rawak 2-digit yang disertakan dengan lajur pertama untuk masa antara ketibaan, *lajur kedua untuk* masa memunggah, *lajur ketiga untuk* menentukan sama ada perlu mengisi minyak ataupun tidak *dan lajur keempat untuk masa mengisi minyak)*

[50 markah]

APPENDIX 1 / *LAMPIRAN 1*

Formulas for Queueing Theory:

1. *M/M/1*:

$$\rho = \lambda / \mu$$

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0,1,2,\ldots$$

$$L = \frac{\lambda}{\mu-\lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W = \frac{1}{\mu-\lambda} \quad , \quad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P[w > t] = e^{-t/w}$$

$$P[w_q > t] = \rho e^{-t/w}$$

2. *M/M/s*:

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left[\frac{(\lambda/\mu)^s}{s!}\frac{1}{(1-\rho)} \quad + \quad \sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!} \right]^{-1}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!}P_0 & , \quad \text{if } 0 \le n \le s \\ \dfrac{(\lambda/\mu)^n}{s!s^{n-s}}P_0 & , \quad \text{if } n \le s \end{cases}$$

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2}P_0$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = W_q + 1/\mu$$

$$L = L_q + \lambda/\mu$$

$$P[w_q > t] = e^{-\mu t}\left[1 + \frac{P_0\left(\dfrac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\left(\frac{1-e^{\mu t(s-1-\lambda/\mu)}}{s-1-\lambda/\mu} \right) \right]$$

$$P[w_q > t] = \left[1 - P\{w_q = 0\}\right]e^{-s\mu(1-\rho)t}$$

$$\text{where} \quad P\{w_q = 0\} = \sum_{n=0}^{s-1} P_n$$

APPENDIX 2 / *LAMPIRAN 2*

3. *M/M/s*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$P_n = \begin{cases} P_0 \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } 0 \le n \le s \\ P_0 \binom{M}{n} \left(\frac{n!}{s^{n-s} s!}\right) \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } s < n \le M \\ 0 & , \quad \text{if } n > M \end{cases}$$

$$L = P_0 \left[\sum_{n=0}^{s-1} n \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} n \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$L_q = L - s + P_0 \sum_{n=0}^{s-1} (s-n) \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n$$

$$W = \frac{L}{\lambda(M-L)} \quad , \quad W_q = \frac{L_q}{\lambda(M-L)}$$

4. *M/G/1*:

$$P_0 = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$L = \rho + L_q$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = w_q + \frac{1}{\mu}$$

5. $M / E_k / 1$:

$$L_q = \frac{1+k}{2k} \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W_q = \frac{1+k}{2k} \frac{\lambda}{\mu(\mu-\lambda)}$$

$$W = W_q + 1/\mu$$

$$L = \lambda W$$

APPENDIX 3 / *LAMPIRAN 3*

6. *M/M/1/k*:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & (\rho \neq 1) \\ \dfrac{1}{k+1} & (\rho = 1) \end{cases}$$

For $\rho \neq 1$

$$L = \frac{\rho\left[1-(k+1)\rho^k + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)}$$

$$L_q = L - (1-P_0) = L - \frac{\rho\left(1-\rho^k\right)}{1-\rho^{k+1}}$$

$$W = L/\lambda' \quad , \quad \lambda' = \mu\left(L - L_q\right)$$

$$W_q = W - 1/\mu = L_q/\lambda'$$

For $\rho = 1$

$$L = \frac{k}{2}$$

7. *M/M/s/k*:

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (0 \leq n < s) \\ \dfrac{1}{s^{n-s}s!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (s \leq n \leq k) \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}\,\frac{1-\left(\dfrac{\lambda}{s\mu}\right)^{k-s+1}}{1-\dfrac{\lambda}{s\mu}}\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} \neq 1\right) \\ \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}(k-s+1)\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} = 1\right) \end{cases}$$

$$L_q = \frac{P_0 (s\rho)^S \rho}{s!(1-\rho)^2}\left[1-\rho^{k-s+1} - (1-\rho)(k-s+1)\rho^{k-s}\right]$$

APPENDIX 4 / *LAMPIRAN 4*

$$L = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(\rho s)^n}{n!}$$

$$W = \frac{L}{\lambda'} \quad , \quad \lambda' = \lambda(1 - P_k)$$

$$W_q = W - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda'}$$

8. *M/M/s/s*:

$$P_n = \frac{(\lambda/\mu)^n / n!}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad \text{for} \quad (0 \le n \le s)$$

$$P_s = \frac{(s\rho)^s / s!}{\sum_{i=0}^{s} (s\rho)^i / i!} \quad \text{where} \quad \left(\rho = \frac{\lambda}{s\mu}\right).$$

$$L = \frac{\lambda}{\mu}(1 - P_s) \quad , \quad W = \frac{L}{\lambda'} \text{ where } \lambda' = \lambda(1 - P_s)$$

9. $M/M/\infty$:

$$P_n = \frac{(\lambda/\mu)^n e^{-\lambda/\mu}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \lambda/\mu$$

$$W = \frac{1}{\mu}$$

APPENDIX 5 / *LAMPIRAN 5*

10. *M/M/1*: state-dependent service

$$\mu_n = \begin{cases} \mu_1 & (1 \le n \le k) \\ \mu & (n \ge k) \end{cases}$$

$$P_0 = \left[\frac{1-\rho_1^k}{1-\rho_1} + \frac{\rho\rho_1^{k-1}}{1-\rho} \right]^{-1} \quad (\rho_1 = \lambda/\mu_1, \rho = \lambda/\mu < 1)$$

$$L = P_0 \left[\frac{\rho_1 \left[1 + (k-1)\rho_1^k - k\rho_1^{k-1} \right]}{(1-\rho_1)^2} + \frac{\rho\rho_1^{k-1}\left[k - (k-1)\rho \right]}{(1-\rho)^2} \right]$$

$$L_q = L - (1 - P_0)$$

$$W = \frac{L}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1-P_0}{\lambda}$$

$$P_n = \begin{cases} \left(\frac{\lambda}{\mu_1} \right)^n P_0 & (0 \le n < k) \\ \frac{\lambda^n}{\mu_1^{k-1}\mu^{n-k+1}} P_0 & (n \ge k) \end{cases}$$

11. *M/M/1*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

$$P_n = \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 1, 2, \ldots, M$$

$$L = M - \frac{\mu}{\lambda}\left[1 - P_0 \right]$$

$$L_q = M - \frac{\lambda + \mu}{\lambda}(1 - P_0)$$

$$W = \frac{L}{\lambda'} \quad , \quad W_q = \frac{L_q}{\lambda'} \quad \text{where } \lambda' = \lambda(M - L)$$

APPENDIX 6 / *LAMPIRAN 6*

TWO-DIGIT RANDOM NUMBER TABLE

03	26	48	92	38	96	41	04	35	84
71	44	81	46	44	47	07	20	58	04
33	75	06	41	87	72	63	88	59	54
53	71	27	13	37	45	89	61	30	26
41	15	43	91	46	81	57	39	34	86
16	18	75	11	26	80	93	97	29	33
88	50	00	56	70	19	90	00	93	95
13	10	08	15	29	33	75	70	43	05
15	72	73	69	27	75	72	95	99	56
64	10	99	02	18	26	78	69	19	12
98	66	53	86	34	71	09	88	56	08
43	05	06	19	91	78	03	65	08	16
69	82	02	61	98	50	74	84	60	41
06	40	10	24	68	42	39	97	25	55
34	86	83	41	33	83	85	92	32	29
46	05	92	36	82	04	67	05	18	69
28	73	59	56	43	88	61	17	07	48
35	53	49	39	98	14	16	76	69	10
90	90	18	27	75	08	75	17	55	68
62	32	97	16	33	66	02	34	62	26

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