
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MAA 101 - Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOUR pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all seven** [7] questions.

[Arahan: Jawab **semua tujuh** [7] soalan].

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. Given a function $f(x) = 1 + \sin x$.
- (a) Find the domain and the range of f .
 - (b) Determine whether f is even, odd or neither even nor odd.
 - (c) Find the inverse of f on the interval $\left[0, \frac{\pi}{2}\right]$.
 - (d) Sketch the graph of f on the interval $[-\pi, \pi]$.

[11 marks]

1. Diberi suatu fungsi $f(x) = 1 + \sin x$.
- (a) Cari domain dan julat bagi f .
 - (b) Tentukan samada f genap, ganjil atau bukan kedua-duanya.
 - (c) Cari songsang bagi f pada selang $\left[0, \frac{\pi}{2}\right]$.
 - (d) Lakarkan graf f pada selang $[-\pi, \pi]$.

[11 markah]

2. Find the following limit if it exists.
- (a) $\lim_{x \rightarrow 4} \frac{3 - \sqrt{x+5}}{x-4}$.
 - (b) $\lim_{x \rightarrow 0^+} (1 + \sin x)^{\frac{1}{\tan x}}$.
 - (c) $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right)$. Use Squeeze Theorem

[12 marks]

2. Cari had berikut jika wujud.
- (a) $\text{had}_{x \rightarrow 4} \frac{3 - \sqrt{x+5}}{x-4}$.
 - (b) $\text{had}_{x \rightarrow 0^+} (1 + \sin x)^{\frac{1}{\tan x}}$.
 - (c) $\text{had}_{x \rightarrow 0} x^4 \text{kos}\left(\frac{2}{x}\right)$. Guna Teorem Menyepit.

[12 markah]

3. Find $\frac{dy}{dx}$ for the following.
- (a) $y = \cos 2x - \ln(x^2 + 1)$.
 - (b) $e^{2x} = \sin(x + 3y)$.
 - (c) $y = \int_3^{x^2} \frac{t^2}{t-2} dt$.

[11 marks]

3. Dapatkan $\frac{dy}{dx}$ yang berikut .

(a) $y = \cos 2x - \ln(x^2 + 1)$.

(b) $e^{2x} = \sin(x + 3y)$.

(c) $y = \int_3^{x^2} \frac{t^2}{t-2} dt$.

[11 markah]

4. (a) Suppose f is a continuous function $[0,4]$, $f(0)=1$ and $2 \leq f'(x) \leq 5$ for all x in $(0,4)$. Show by the Mean Value Theorem that $9 < f(4) < 21$

(b) Suppose that $C(x) = x^2 - 20x + 20000$ is the average cost of manufacturing x items. Find the number of production that will minimize the average cost.

[10 marks]

4. (a) Katakan f suatu fungsi selanjur pada $[0,4]$, $f(0)=1$ dan $2 \leq f'(x) \leq 5$ untuk semua x dalam $(0,4)$. Tunjukkan dengan Teorem Nilai Min bahawa $9 < f(4) < 21$

(b) Katakan $C(x) = x^2 - 20x + 20000$ ialah purata kos menghasilkan x barangan. Cari bilangan barang yang dihasilkan supaya kos purata adalah minimum.

[10 markah]

5. Let $f(x) = \frac{1}{x^2 - 9}$.

Find

- (a) the domain, critical number(s) and all asymptotes of f .
- (b) the interval on which f is increasing or decreasing.
- (c) the local maximum and minimum points of f (if any).
- (d) the intervals of concavity and the inflection point(s) (if exists).

Hence, sketch the graph of f .

[20 marks]

5. Biarkan $f(x) = \frac{1}{x^2 - 9}$.

Cari

- (a) domain, nombor genting dan semua asimptot bagi f .
- (b) selang yang mana f menokok atau menyusut.
- (c) titik maksimum dan minimum tempatan bagi f (jika ada).
- (d) selang kecekungan f dan titik lengkok balas (jika wujud).

Seterusnya, lakarkan graf f .

[20 markah]

6. Solve the following integral.

(a) $\int \frac{x^3 - x}{(x^4 - 2x^2 + 3)^2} dx.$

(b) $\int \frac{\sqrt{x^2 - 4}}{x} dx.$ Use substitution $x = 2 \sec \theta.$

(c) $\int \frac{2}{x^2(x-1)} dx.$

[17 marks]

6. Selesaikan kamiran berikut.

(a) $\int \frac{x^3 - x}{(x^4 - 2x^2 + 3)^2} dx.$

(b) $\int \frac{\sqrt{x^2 - 4}}{x} dx.$ Guna penggantian $x = 2 \sec \theta.$

(c) $\int \frac{2}{x^2(x-1)} dx.$

[17 markah]

7. (a) Given that the curves $y = 4 - x^2$ intersect with the line $y = -x + 2$ at $x = -1$ and 2.

(i) Sketch the graph of the curve and the line. Show the intersection points.

(ii) Compute the bounded area in part (i).

(b) Compute the volume of the solid generated by rotating the region between the curves $x = 3y^2 - 2$, x -axis and $x = y$ about

(i) x -axis.

(ii) line $x = 1$.

(c) Set up an integral for the surface area generated by rotating the curve $y = \sqrt{x+1}$, $1 \leq x \leq 5$ about x -axis.

[19 marks]

7. (a) Diberi suatu lengkung $y = 4 - x^2$ bersilang dengan garis $y = -x + 2$ pada $x = -1$ dan 2.

(i) Lakarkan graf lengkung dan garis tersebut. Tunjukkan titik persilangan.

(ii) Hitung luas kawasan yang dibatasi di bahagian (i).

(b) Hitung isipadu bongkah kisanan yang terhasil apabila rantau di antara lengkung $x = 3y^2 - 2$, paksi- x dan $x = y$ dikisarkan terhadap

(i) paksi- x .

(ii) garis $x = 1$.

(c) Nyatakan kamiran untuk luas permukaan yang terhasil apabila lengkung $y = \sqrt{x+1}$, $1 \leq x \leq 5$ dikisarkan terhadap paksi- x .

[19 markah]