
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MAT102 - Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

[Arahan: Jawab **semua ten** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Determine whether the sequence $\left\{ \frac{1+\sqrt{2n}}{\sqrt{n}} \right\}$ is monotonic and bounded. [8 marks]

1. Tentukan sama ada jujukan $\left\{ \frac{1+\sqrt{2n}}{\sqrt{n}} \right\}$ berekannya dan terbatasi. [8 markah]

2. Test the convergence of the series.

(a) $\sum_{n=1}^{\infty} 5^{3n} 6^{1-n}$

(b) $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{n^2 + 1}$

[14 marks]

2. Uji penumpuan siri.

(a) $\sum_{n=1}^{\infty} 5^{3n} 6^{1-n}$

(b) $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{n^2 + 1}$

[14 marks]

3. Find the radius of convergence of power series $\sum_{n=0}^{\infty} \frac{(n+p)!}{n!(n+q)!} x^n$ where p and q are positive integers. [5 marks]

3. Dapatkan jejari penumpuan siri kuasa $\sum_{n=0}^{\infty} \frac{(n+p)!}{n!(n+q)!} x^n$ yang mana p and q adalah integer positif. [5 markah]

4. Use Maclaurin series to approximate the integral $\int_0^{0.2} \frac{\sin x^2}{x^2} dx$ to three decimal places.

[7 marks]

4. Menggunakan siri Maclaurin, anggarkan kamiran $\int_0^{0.2} \frac{\sin x^2}{x^2} dx$ kepada tiga tempat perpuluhan.

[7 markah]

5. Let $G(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Show that $G(x+1) = xG(x)$ for all $x > 0$.

[9 marks]

5. Biar $G(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Tunjukkan bahawa $G(x+1) = xG(x)$ bagi semua $x > 0$.

[9 markah]

6. (a) If $f(x, y) = \frac{xy^2 - 1}{y - 1}$, does $\lim_{(x,y) \rightarrow (1,1)} f(x, y)$ exist? Justify

(b) Show that $\lim_{x \rightarrow 0^+} x^\alpha \ln x = 0$ for any $\alpha > 0$.

[9 marks]

6. (a) Jika $f(x, y) = \frac{xy^2 - 1}{y - 1}$, adakah $\lim_{(x,y) \rightarrow (1,1)} f(x, y)$ wujud? Tentukan

(b) Tunjukkan bahawa $\lim_{x \rightarrow 0^+} x^\alpha \ln x = 0$ bagi sebarang $\alpha > 0$.

[9 markah]

7. Let \underline{u} be a unit vector and f is a differentiable function of x and y .

If $\nabla f(x_0, y_0) = \underline{i} - 2\underline{j}$ and $D_{\underline{u}}f(x_0, y_0) = -2$, find \underline{u} .

[8 marks]

7. Biar \underline{u} merupakan vektor unit dan f fungsi terbezakan x and y .

Jika $\nabla f(x_0, y_0) = \underline{i} - 2\underline{j}$ dan $D_{\underline{u}}f(x_0, y_0) = -2$, dapatkan \underline{u} .

[8 markah]

8. Find three positive numbers whose sum is 36 and the sum of their squares as small as possible.

[10 marks]

8. *Dapatkan tiga nombor positif yang mana hasil tambahnya ialah 36 dan jumlah kuasa duanya paling kecil.*

[10 markah]

9. (a) Let $w = (x_1^2 + x_2^2 + \dots + x_n^2)^k$ where $n \geq 2$. Use the chain rule to find the values of k such that

$$\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} + \dots + \frac{\partial^2 w}{\partial x_n^2} = 0.$$

- (b) The volume, V of a right circular cone of radius r and height h is given by

$$V = \frac{1}{3} \pi r^2 h.$$

Suppose that the height decreases from 20 in to 19.95 in and the radius increases from 4 in to 4.05 in. Compare the change in volume of the cone with an approximation of this change using a total differential.

[15 marks]

9. (a) *Biar $w = (x_1^2 + x_2^2 + \dots + x_n^2)^k$ yang mana $n \geq 2$. Gunakan petua rantai untuk mendapatkan nilai-nilai k sedemikian*

$$\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} + \dots + \frac{\partial^2 w}{\partial x_n^2} = 0.$$

- (b) *Isipadu, V kon bulat tegak berjajari r dan ketinggian h diberikan oleh*

$$V = \frac{1}{3} \pi r^2 h.$$

Katakan bahawa ketinggian berkurangan daripada 20 in ke 19.95 in dan jejari bertambah daripada 4 in ke 4.05 in. Bandingkan perubahan dalam isipadu kon dengan anggaran perubahan ini menggunakan pembeza keseluruhan.

[15 markah]

10. (a) Evaluate $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}} dx dy$

(b) Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the sphere $x^2 + y^2 + z^2 = 9$, and bounded below by the xy -plane.

[15 marks]

10. (a) Nilaiikan $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}} dx dy$

(b) Dapatkan isipadu pepejal yang di batasi oleh silinder $x^2 + y^2 = 4$, bahagian atasnya dibatasi oleh sfera $x^2 + y^2 + z^2 = 9$, dan bahagian bawahnya dibatasi oleh satah xy .

[15 markah]