
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MAA 102 – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOUR pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

[Arahan: Jawab **semua sepuluh** [10] soalan].

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. Determine the convergence or divergence of the series

(a) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$. (Explain your answer)

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.1)^n}{n}$. (Explain your answer)

[15 marks]

1. Tentukan pencapahan atau penumpuan bagi siri berikut;

(a) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$. (Jelaskan jawapan anda)

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.1)^n}{n}$. (Jelaskan jawapan anda)

[15 markah]

2. Given a_n is a nonnegative number, if $\sum_{n=1}^{\infty} a_n$ is a convergent series, what can be said about $\sum_{n=1}^{\infty} \left(\frac{a_n}{n}\right)$? Justify your answer.

[5 marks]

2. Diberi a_n sebagai nombor positif, jika $\sum_{n=1}^{\infty} a_n$ adalah siri menumpu, apakah yang dapat di rumuskan tentang $\sum_{n=1}^{\infty} \left(\frac{a_n}{n}\right)$? Jelaskan jawapan anda.

[5 markah]

3. Show that if $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\left| \sum_{n=1}^{\infty} a_n \right| \leq \sum_{n=1}^{\infty} |a_n|$

[5 marks]

3. Tunjukkan sekiranya $\sum_{n=1}^{\infty} a_n$ menumpu secara mutlak, $\left| \sum_{n=1}^{\infty} a_n \right| \leq \sum_{n=1}^{\infty} |a_n|$.

[5 markah]

4. Find the first three nonzero terms in the Maclaurin series for given function $(1-x+x^2)e^x$ and the values of x for which the series converges absolutely.

[10 marks]

4. Dapatkan tiga sebutan positif pertama siri Maclaurin bagi fungsi $(1-x+x^2)e^x$ dan dapatkan nilai x di mana siri itu menumpu secara mutlak.

[10 markah]

5. Assume that $w = f\left(ts^2, \frac{s}{t}\right)$; $\frac{\partial f(x, y)}{\partial x} = xy$ and $\frac{\partial f(x, y)}{\partial y} = \frac{x^2}{2}$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.
[10 marks]

5. Andaikan $w = f\left(ts^2, \frac{s}{t}\right)$; $\frac{\partial f(x, y)}{\partial x} = xy$ dan $\frac{\partial f(x, y)}{\partial y} = \frac{x^2}{2}$. Dapatkan $\frac{\partial w}{\partial t}$ dan $\frac{\partial w}{\partial s}$.
[10 markah]

6. At the point $(1, 2)$, the function $f(x, y)$ has derivative of 2 in the direction toward $(2, 2)$ and a derivative of -2 in the direction toward $(1, 1)$.
(a) Find $f_x(1, 2)$ and $f_y(1, 2)$.
(b) Find the derivative of f in the direction toward the point $(4, 6)$.
[10 marks]

6. Pada titik $(1, 2)$, fungsi $f(x, y)$ mempunyai nilai terbitan 2 pada arah titik $(2, 2)$ dan nilai terbitan -2 pada arah titik $(1, 1)$.
(a) Dapatkan $f_x(1, 2)$ dan $f_y(1, 2)$.
(b) Dapatkan terbitan bagi f pada arah titik $(4, 6)$.
[10 markah]

7. A closed rectangular box is to have volume $V \text{ cm}^3$. The cost of the material used in the box is $a \text{ cents/cm}^2$ for top and bottom, $b \text{ cents/cm}^2$ for front and back, and $c \text{ cents/cm}^2$ for the remaining sides. What dimensions minimize the total cost of materials?
[15 marks]

7. Suatu kotak tertutup segiempat mempunyai isipada $V \text{ cm}^3$. Kos bahan untuk membuat kotak tersebut adalah $a \text{ sen/cm}^2$ untuk bahagian atas dan bawah kotak, $b \text{ sen/cm}^2$ bagi bahagian depan dan belakang kotak, dan $c \text{ sen/cm}^2$ bagi bahagian lain kotak. Apakah dimensi yang akan meminimalkan kos bahan membuat kotak ini?
[15 markah]

8. If $f(x, y) = \frac{10,000e^y}{\left(1 + \frac{|x|}{2}\right)}$ represents the population density of a certain bacterium on the xy -plane, where x and y are measured in centimeters, find the total population of bacteria within the rectangle $-5 \leq x \leq 5$ and $-2 \leq y \leq 0$.
[10 marks]

8. Jika $f(x, y) = \frac{10,000e^y}{\left(1 + \frac{|x|}{2}\right)}$ mewakili kepadatan sesuatu populasi bakteria pada satah - xy , dimana x dan y diukur dalam sentimeter, dapatkan jumlah populasi bakteria didalam suatu segiempat yang dibatasi oleh $-5 \leq x \leq 5$ dan $-2 \leq y \leq 0$.
[10 markah]
9. The region enclosed by lemniscate $r^2 = 2 \cos 2\theta$ is the base of a solid whose top is bounded by the sphere $z = \sqrt{2 - r^2}$. Find the volume of the solid.
[10 marks]
9. Rantau yang dilitupi oleh lemniskat $r^2 = 2 \cos 2\theta$ adalah dasar bagi suatu bahan padu dimana bahagian atasnya dibatasi oleh sfera $z = \sqrt{2 - r^2}$. Dapatkan isipadu bahan tersebut.
[10 markah]
10. Solve the exponential growth/decay initial value problem for y as a function of t with $P(x) = -k; Q(x) = 0; \frac{dy}{dt} = ky; y(0) = y_0$, where k is constant.
[10 marks]
10. Selesaikan masalah pertumbuhan/pereputan eksponen melalui masalah nilai awalan dimana y adalah fungsi t dan $P(x) = -k; Q(x) = 0; \frac{dy}{dt} = ky; y(0) = y_0$, k adalah pemalar.
[10 markah]