UNIVERSITI SAINS MALAYSIA

First Semester Examination
2014/2015 Academic Session

December 2014/January 2015

**MAT 100 – Foundation Mathematics**
***[Asas Matematik]***

Duration : 3 hours
*[Masa : 3 jam]*

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Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions**: Answer **all nine** [9] questions.

*[**Arahan:** Jawab **semua sembilan** [9] soalan.*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. Let $A=\{1,2,3,4\}$ and $B=\{1,2,3,4,5\}$. For $x\in A$, let $P(x): x+4$ be prime. For $y\in B$, let $Q(y)$:10 divides $(5y+5)$.

(a) Find all $(x,\ y)\in A\times B$ such that $\sim\left(P(x)\vee Q(y)\right)$ is true.

[2 marks]

(b) Find all $(x,\ y)\in A\times B$ such that $\left(\sim P(x)\right)\wedge Q(y)$ is true.

[3 marks]

(c) Let $S=\{(x,\ y): x\in A, y\in B, P(x)\Rightarrow Q(y)$ be false$\}$. Find $|S|$.

[3 marks]

(d) For $x\in A$, let $R(x)$: 5 divides $x+4$. Find all $x\in A$ such that $P(x)\Leftrightarrow R(x)$ is true.

[3 marks]

*1. Biar* $A=\{1,2,3,4\}$ *dan* $B=\{1,2,3,4,5\}$. *Bagi* $x\in A$, *biar* $P(x): x+4$ *ialah nombor perdana. Bagi* $y\in B$, *biar* $Q(y)$:10 membahagi $(5y+5)$.

*(a) Cari semua* $(x,\ y)\in A\times B$ *sedemikian hingga* $\sim\left(P(x)\vee Q(y)\right)$ *adalah benar.*

*[2 markah]*

*(b) Cari semua* $(x,\ y)\in A\times B$ *sedemikian hingga* $\left(\sim P(x)\right)\wedge Q(y)$ *adalah benar.*

*[3 markah]*

*(c) Biar* $S=\{(x,\ y): x\in A, y\in B, P(x)\Rightarrow Q(y)$ be false$\}$. *Find* $|S|$.

*[3 markah]*

*(d) Bagi* $x\in A$, *biar* $R(x)$: *5 membahagi* $x+4$. *Cari semua* $x\in A$ *sedemikian hingga* $P(x)\Leftrightarrow R(x)$ *adalah benar.*

*[3 markah]*

2. Let $P(x, y, z)$ be an open sentence, where the domains of $x$, $y$ and $z$ are $A$, $B$ and $C$, respectively.

(a) State the quantified statement $\forall x \in A$, $\forall y \in B$, $\exists z \in C$, $P(x, y, z)$ in words for $P(x, y, z): x \geq yz$.

[2 marks]

(b) Determine whether the quantified statement in (a) is true when $A = \{4, 8\}$, $B = \{2, 4\}$ and $C = \{1, 2, 4\}$.

[3 marks]

*2. Biar $P(x, y, z)$ adalah suatu ayat terbuka dengan A, B dan C ialah domain kepada x, y dan z masing-masing.*

*(a) Nyatakan pernyataan terkuantifikasi $\forall x \in A$, $\forall y \in B$, $\exists z \in C$, $P(x, y, z)$ dalam perkataan bagi $P(x, y, z): x \geq yz$.*

*[2 markah]*

*(b) Tentukan sama ada pernyataan terkuantifikasi yang diberi di (a) adalah benar apabila $A = \{4, 8\}$, $B = \{2, 4\}$ dan $C = \{1, 2, 4\}$.*

*[3 markah]*

3. In the class, the truth of the following result has been discussed.

**Result** Let $A$ be a subset of the set of natural numbers and $P(A)$ is the power set of $A$. If $\{1, 2\} \in P(A)$, then $|P(A)| \geq 4$.

Immediately after the discussion, a student, Beng, has made the following claim,

"Whenever $|P(A)| \geq 4$, $\{1, 2\} \in P(A)$."

From the point of implication, prove or disprove the answer given by Beng.

[4 marks]

*3. Dalam kelas, kebenaran bagi keputusan berikut telah dibincangkan.*

***Keputusan*** *Biar $A$ adalah suatu subset kepada set nombor asli dan $P(A)$ adalah set kuasa bagi A. Jika $\{1, 2\} \in P(A)$, maka $|P(A)| \geq 4$.*

*Sejurus selepas perbincangan, seorang pelajar, Beng, telah membuat tuntutan yang berikut,*

*"Setiap kali $|P(A)| \geq 4$, $\{1, 2\} \in P(A)$."*

*Dari segi implikasi, buktikan atau sangkal jawapan yang diberikan oleh Beng.*

*[4 markah]*

4. Each of the following proposed proofs contained a mistake. Evaluate the proofs by pointing out the mistakes.

[9 marks]

(a) **Result** Let $A$ and $B$ be two sets. Prove that $A \cap \overline{B} = A - B,$ where $\overline{A}$ is the complement of $A$.

*Proof* Let $x \in A \cap \overline{B}$.
Then $x \in A$ and $x \in \overline{B}$.
Then $x \in A$ and $x \notin B$.
Then $x \in A - B$.
Then $A \cap \overline{B} = A - B$.

(b) **Result** Let $A$ and $B$ be two $2 \times 2$ matrices. Show that $A + B = B + A$.

*Proof* Let $A = \left(a_{ij}\right)$ and $B = \left(b_{ij}\right)$ be two $2 \times 2$ matrices. For each $i, j \in \{1, 2\}$, we have

$$\begin{aligned} A + B &= a_{ij} + b_{ij} \\ &= b_{ij} + a_{ij} \\ &= B + A \end{aligned}$$

(c) **Result** Let $a \in □$. If $f$ is a function from $□$ to $□$, defined as $f(x) = ax + 1$, then $f$ is onto.

*Proof* Let $a = 1$. Then $f$ is a function from $□$ to $□$ defined as $f(x) = x + 1$. Note that $f$ is onto as for every $y \in □$, there exist a $x = y - 1 \in □$ such that $f(x) = y$. Thus, we have proven the given results.

4. *Setiap satu daripada bukti-bukti yang dicadangkan berikut terkandung satu kesilapan. Nilaikan setiap bukti ini dengan menjelaskan kesilapan tersebut.*

*[9 markah]*

*(a)* ***Keputusan*** *Biar* $A$ dan $B$ *merupakan dua set. Tunjukkan bahawa* $A \cap \overline{B} = A - B,$ *di mana* $\overline{A}$ *adalah pelengkap A.*

Bukti *Biar* $x \in A \cap \overline{B}$.
*Maka* $x \in A$ *dan* $x \in \overline{B}$.
*Maka* $x \in A$ *dan* $x \notin B$.
*Maka* $x \in A - B$.
*Maka* $A \cap \overline{B} = A - B$.

*(b)* ***Keputusan*** *Biar* $A$ *dan* $B$ *merupakan dua matriks* $2 \times 2$. *Tunjukkan bahawa* $A + B = B + A$.

Bukti *Biar* $A = \left(a_{ij}\right)$ *dan* $B = \left(b_{ij}\right)$ *merupakan dua matriks* $2 \times 2$. *Bagi setiap* $i, j \in \{1, 2\}$, *kita dapati bahawa*

$$\begin{aligned} A + B &= a_{ij} + b_{ij} \\ &= b_{ij} + a_{ij} \\ &= B + A \end{aligned}$$

*(c)* **Keputusan** *Biar* $a \in \square$. *Jika* $f$ *adalah suatu fungsi dari* $\square$ *ke* $\square$ *yang ditakrifkan sebagai* $f(x) = ax+1$, *maka* $f$ *adalah keseluruhan.*

Bukti *Biar* $a = 1$. *Maka* $f$ *adalah suatu fungsi dari* $\square$ *ke* $\square$ *yang ditakrifkan sebagai* $f(x) = x+1$. *Perhatikan bahawa* $f$ *adalah keseluruhan memandangkan bagi setiap* $y \in \square$, *wujudnya suatu* $x = y-1 \in \square$ *sedemikian hingga* $f(x) = y$. *Maka, kita sudah membuktikan keputusan yang diberi.*

5. Give the statement of the result that is proved in each of the following proofs:

[9 marks]

(a) *Proof* Assume that *x* is even and *y* is odd. Then we have $x = 2a$ and $y = 2b+1$, for some $a, b \in \square$. Thus, we have $x+y = 2a+2b+1 = 2(a+b)+1$ for some $a+b \in \square$. Hence, we have $x+y$ is odd.

(b) *Proof* Let $P(n): 1+2+\cdots+n = \dfrac{n(n+1)}{2}$. Since $1 = \dfrac{(1+1)}{2}$, we have $P(1)$ is true.

Suppose that $P(k)$ is true for some $k \in \square$. Note that

$$\begin{aligned} 1+2+\cdots+k+(k+1) &= (1+2+\cdots+k)+(k+1) \\ &= \frac{k(k+1)}{2}+(k+1) \\ &= \frac{k^2+k+2k+2}{2} \\ &= \frac{k^2+3k+2}{2} \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

Hence, by the Principle of Mathematical Induction, we have proven that $P(n)$ is true for all positive integers.

(c) *Proof* Assume, to the contrary, that there is a smallest positive real number, say *r*. Since $0 < \frac{r}{2} < r$, it follows that $\frac{r}{2}$ is a positive real number that is smaller than *r*. This, however, is a contradiction.

5. *Berikan pernyataan bagi setiap keputusan yang dibuktikan berikut:*

*[9 markah]*

(a) Bukti *Andaikan x adalah suatu nombor genap dan y adalah suatu nombor ganjil. Maka kita dapat* $x = 2a$ *dan* $y = 2b+1$, *bagi suatu* $a, b \in \square$ . *Seterusnya kita dapati* $x + y = 2a + 2b + 1 = 2(a+b)+1$ *bagi suatu* $a + b \in \square$ . *Maka, kita dapati* $x + y$ *adalah suatu nombor ganjil.*

(b) Bukti *Biar* $P(n): 1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$. *Memandangkan* $1 = \dfrac{(1+1)}{2}$, *kita dapati* $P(1)$ *adalah benar.*

*Andaikan* $P(k)$ *adalah benar bagi suatu* $k \in \square$ . *Perhatikan bahawa*

$$
\begin{aligned}
1 + 2 + \cdots + k + (k+1) &= (1 + 2 + \cdots + k) + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k^2 + k + 2k + 2}{2} \\
&= \frac{k^2 + 3k + 2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}
$$

*Maka, dengan Prinsip Aruhan Matematik, kita telah menunjukkan bahawa* $P(n)$ *adalah benar bagi setiap integer positif.*

(c) Bukti *Andaikan yang bertentangan, wujudnya suatu nombor nyata terkecil r. Memandangkan* $0 < \dfrac{r}{2} < r$, *maka* $\dfrac{r}{2}$ *adalah suatu nombor nyata yang lebih kecil daripada r. Ini adalah suatu percanggahan.*

6. Prove by contrapositive the following result:

Let $n \in \square$ . If $3n - 8$ is odd, then $n$ is odd.

[4 marks]

6. *Buktikan keputusan berikut secara kontrapositif:*

*Biar* $n \in \square$ . *Jika* $3n - 8$ *adalah nombor ganjil, maka n adalah nombor ganjil.*

*[4 markah]*

7. Use mathematical induction to prove that, for all natural numbers *n*,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

[4 marks]

*7. Gunakan aruhan matematik untuk membuktikan bahawa bagi setiap nombor asli n,*

$$P(n): 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

*[4 markah]*

8. Let $p(x) = x^3 + x + 10.$

(a) Show that $x = 1 + 2i$ is a root of $p(x)$, where $i = \sqrt{-1}$.

[2 marks]

(b) By using the fact that $p(x)$ is a polynomial with real coefficients, show by contradiction that $p(x)$ will have another complex root.

[4 marks]

*8. Biar $p(x) = x^3 + x + 10.$*

*(a) Tunjukkan $x = 1 + 2i$ adalah suatu punca bagi $p(x)$, di mana $i = \sqrt{-1}$.*

*[2 markah]*

*(b) Dengan menggunakan fakta bahawa $p(x)$ adalah suatu polinomial dengan koefisien nyata, tunjukkan melalui percanggahan bahawa $p(x)$ akan mempunyai satu lagi punca kompleks.*

*[4 markah]*

9. Let $x, y \in \square$ . Show that $|(x-1)(y-2)| = |x-1||y-2|$.

[8 marks]

*9. Biar $x, y \in \square$ . Tunjukkan $|(x-1)(y-2)| = |x-1||y-2|$.*

*[8 markah]*

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