UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2013/2014 Academic Session

June 2014

MSG 389 - Engineering Computation II
[Pengiraan Kejuruteraan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Consider a large tank holding 1000 liter of pure water into which a salt solution begins to flow at a constant rate of 6 liter/min. The solution inside the tank is kept well stirred, and is flowing out of the tank at a rate of 6 liter/min. If the concentration of salt in the solution entering the tank is 0.1 kg/liter, determine when the concentration of salt will reach 0.05 kg/liter?

[40 marks]

(b) Consider the initial value problem

$$y'(x) + 3xy + y^2 = 0 \qquad\qquad y(1) = 0.5, h = 0.1 \;.$$

Solve the initial value problem to determine *y* (1.3) using:

(i) Euler method
(ii) Second order Taylor series method
(iii) Second order Runge Kutta method
(iv) Fourth order Runge Kutta method
(v) Euler method as predictor method and Heun method as corrector method
(vi) Midpoint method

[60 marks]

1. (a) Pertimbangkan sebuah tangki besar mengandungi 1000 liter air tulen di mana larutan garam mula mengalir pada kadar yang tetap 6 liter / min. Campuran di dalam tangki itu dikacau seragam, dan mengalir keluar dari tangki pada kadar 6 liter / min. Jika kepekatan garam dalam campuran yang memasuki tangki itu adalah 0.1 kg / liter, tentukan masa yang diperlukan untuk kepekatan garam mencapai 0.05 kg / liter?

[40 markah]

(b) Pertimbangkan masalah nilai awalan

$$y'(x) + 3xy + y^2 = 0 \qquad\qquad y(1) = 0.5, h = 0.1 \;.$$

Selesaikan masalah nilai awalan ini bagi menentukan y (1.3) menggunakan:

(i) Kaedah Euler
(ii) Kaedah siri Taylor tertib kedua
(iii) Kaedah Runge Kutta tertib kedua
(iv) Kaedah Runge Kutta tertib keempat
(v) Kaedah Euler method sebagai penganggar dan kaedah Heun sebagai pembaiki
(vi) Kaedah titik tengah

[60 markah]

2. (a) Find the approximate value of y(0.5) for the initial value problem

$$y' = x + y, \quad y(0) = 1,$$

using the multistep method

$$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$$

with h=0.1. Compute the starting value using Runge-Kutta fourth order method with the same step length *h=0.1*.

[50 marks]

(b) Solve the initial value problem

$$y' = x^2 + y^3, \quad y(1) = 0,$$

on the interval [1, 1.6] using the predictor-corrector method,

$$\text{Predictor: } y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

$$\text{Corrector : } y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

with the step length h=0.2. Perform three corrector iterations per step. Compute the starting value using Taylor series order method with the same step length *h*.

[50 marks]

2. (a) Dapatkan nilai anggaran y(0.5) *bagi masalah nilai awalan*

$$y' = x + y, \quad y(0) = 1,$$

dengan menggunakan kaedah multi langkah:

$$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$$

dengan h=0.1. *Dapatkan nilai awalan menggunakan kaedah Runge-Kutta tertib keempat dengan saiz langkah yang sama h=0.1.*

[50 markah]

(b) Selesaikan masalah nilai awalan

$$y' = x^2 + y^3, \quad y(1) = 0,$$

dalam selang [1, 1.6] menggunakan kaedah penganggar dan pembaiki,

$$\textit{Penganggar} : y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

$$\textit{Pembaiki} : y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

dengan saiz langkah h=0.2. *Lakukan tiga langkah pembaiki bagi setiap lelaran. Dapatkan nilai permulaan dengan mengunakan kaedah siri Taylor dengan saiz langkah, h yang sama.*

[50 markah]

3. (a) Set up the equations for a value of *u* that satisfies the Laplace equation of a regular plate with $0 \le x \le 4$ and $0 \le y \le 2$. Use the conditions:

$$\Delta x = \Delta y = 1$$
$$T(0, y) = 5$$
$$T(x,0) = 0 \quad \text{and} \quad T(x,2) = 10$$
$$\frac{\partial T}{\partial x} = 3 \quad \text{at} \quad x = 4$$

[40 marks]

(b) Solve the boundary value problem

$$y'' - 4y' + 3y = 0, \qquad y(0) = 1, y(1) = 0 .$$

using second order finite difference method with

(i) h=1/2

(ii) h=1/3.

[Hint: $y_i'' = \frac{1}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right), y_i' = \frac{1}{2h}(y_{i+1} - y_{i-1})$]

[60 marks]

3. *(a)* *Bangunkan persamaan untuk nilai u yang memenuhi persamaan Laplace bagi satah biasa dengan* $0 \le x \le 4$ *dan* $0 \le y \le 2$. *Gunakan syarat :*

$$\Delta x = \Delta y = 1$$
$$T(0, y) = 5$$
$$T(x,0) = 0 \quad \textit{dan} \quad T(x,2) = 10$$
$$\frac{\partial T}{\partial x} = 3 \quad \textit{pada} \quad x = 4$$

[40 markah]

(b) *Selesaikan masalah nilai sempadan*

$$y'' - 4y' + 3y = 0, \qquad y(0) = 1, y(1) = 0 .$$

mengunakan kaedah beza terhingga peringkat kedua dengan

(i) *h=1/2*

(ii) *h=1/3.*

[Petunjuk: $y_i'' = \frac{1}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right), y_i' = \frac{1}{2h}(y_{i+1} - y_{i-1})$ *]*

[60 markah]

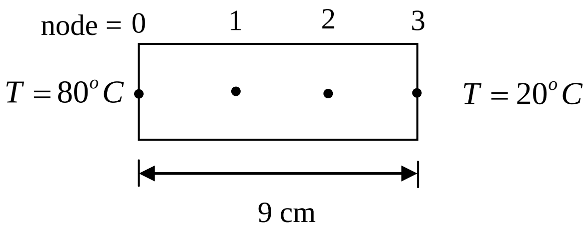
4. (a) The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad .$$

If $\alpha = 0.8 cm^2 / s$, the initial temperature of rod is $40^\circ C$, and the rod is divided into three equal segments, find the temperature at node 1 (using $\Delta t = 0.1s$) for t=0.2 sec by using:

(i) Crank-Nicolson method

(ii) Forward Time Centered Space (FTCS) scheme



[70 marks]

(b) A typical steady-state heat flow problem is the following: A thin steel plate is a unit square in m^2. If two of its edges are held at $0^\circ C$ and the other two are held at the temperatures shown below:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \; 0 < x < 1, 0 < y < 1 \; .$$

$$u(0,x) = 100x$$

$$u(y,0) = 100y$$

$$u(x,1) = 100x$$

$$u(1,y) = 100y$$

Write down the five point formula for the following coordinates: (1,1), (2,1), (3,1), (1,2), (2,2), (3,2), (1,3), (2,3) and (3,3).

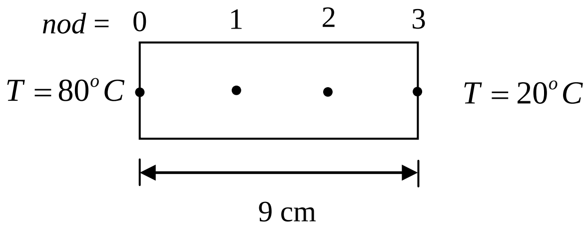
[30 marks]

4. (a) *Masalah pembezaan separa bagi suhu suatu rod nipis yang panjang diberikan oleh*

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad .$$

Sekiranya $\alpha = 0.8 cm^2 / s$, *suhu awalan rod adalah* $40^\circ C$, *dan rod dibahagikan kepada tiga bahagian yang seragam, dapatkan suhu di nod* 1 *(mengunakan* $\Delta t = 0.1s$*) untuk* t=0.2*s dengan menggunakan:*

(i) Kaedah Crank- Nicolson
(ii) Skema Beza Ke Depan Terhadap Masa Beza Pusat Terhadap Ruang



[70 markah]

(b) *Aliran haba kaedah mantap biasa diberikan sebagai : Satu plat besi yang nipis adalah satu unit persegi dalam* m^2. *Sekiranya dua penghujuna diletakkan pada suhu* $0^\circ C$ *dan dua penghujung lagi diletakkan dalam suhu berikut :*

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < 1, 0 < y < 1 \ .$$

$$u(0, x) = 100x$$
$$u(y, 0) = 100y$$
$$u(x, 1) = 100x$$
$$u(1, y) = 100y$$

Tuliskan formula lima titik untuk koordinat berikut: (1,1), (2,1), (3,1), (1,2), (2,2), (3,2), (1,3), (2,3) dan (3,3).

[30 markah]

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