UNIVERSITI SAINS MALAYSIA

First Semester Examination
2014/2015 Academic Session

December 2014/January 2015

MAA 111– Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **FIVE** pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ***LIMA*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all nine** [9] questions.

*[**Arahan:*** *Jawab* ***semua sembilan*** *[9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. If A and B are 3×3 matrices such that

$$A = E_2^1(2)E_3^1(-3)E_3^2(4) \quad \text{and} \quad B = E_2(-1)E_1^2(2)E_2^3(1)E_1^3(1),$$

find the matrix,

(a) A and B (b) AB

[8 marks]

1. Jika A dan B adalah matriks 3×3 *sedemikian hingga*

$$A = E_2^1(2)E_3^1(-3)E_3^2(4) \quad \textit{dan} \quad B = E_2(-1)E_1^2(2)E_2^3(1)E_1^3(1),$$

dapatkan matriks,

(a) A dan B (b) AB

[*8 markah*]

2. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2$, compute $\begin{vmatrix} 2b & 0 & 4d \\ 1 & 2 & -2 \\ a+1 & 2 & 2(c-1) \end{vmatrix}$. [6 marks]

2. Jika $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2$, *hitungkan* $\begin{vmatrix} 2b & 0 & 4d \\ 1 & 2 & -2 \\ a+1 & 2 & 2(c-1) \end{vmatrix}$. *[6 markah]*

3. Consider the linear system

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ x_3 &= 2 \\ (a^2-4)x_3 &= a-2. \end{aligned}$$

Find the value(s) of a so that the linear system

(a) is inconsistent

(b) has a unique solution

(c) has infinitely many solutions

[9 marks]

3. Pertimbangkan sistem linear

$$x_1 + x_2 + x_3 = 4$$
$$x_3 = 2$$
$$\left(a^2 - 4\right)x_3 = a - 2.$$

Dapatkan nilai a supaya sistem linear diatas

(a) tak konsisten

(b) mempunyai penyelesaian unik

(c) mempunyai penyelesaian yang tak terhingga

[9 markah]

4. Use the inverse of the coefficient matrix to solve the linear system

$$\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0. \end{aligned}$$

[9 marks]

4. Gunakan songsangan matriks pekali untuk menyelesaikan sistem linear

$$\begin{aligned} x + y + z &= 5 \\ x + y - 4z &= 10 \\ -4x + y + z &= 0. \end{aligned}$$

[9 markah]

5. Explain why the following are linearly dependent sets of vectors.

(a) $\mathbf{u}_1 = (-1, 2, -4)$ and $\mathbf{u}_2 = (3, -6, 12)$ in R^3.

(b) $\mathbf{u}_1 = (3, 2)$, $\mathbf{u}_2 = (-1, 5)$ and $\mathbf{u}_3 = (4, -7)$ in R^2.

(c) $\mathbf{p}_1 = 3 - 2x + x^2$ and $\mathbf{p}_2 = 6 - 4x + 2x^2$ in P_2.

[6 marks]

5. Terangkan kenapa yang berikut merupakan set vektor bersandar secara linear.

(a) $\mathbf{u}_1 = (-1, 2, -4)$ *dan* $\mathbf{u}_2 = (3, -6, 12)$ *dalam* R^3.

(b) $\mathbf{u}_1 = (3, 2)$, $\mathbf{u}_2 = (-1, 5)$ *dan* $\mathbf{u}_3 = (4, -7)$ *dalam* R^2.

(c) $\mathbf{p}_1 = 3 - 2x + x^2$ *dan* $\mathbf{p}_2 = 6 - 4x + 2x^2$ *dalam* P_2.

[6 markah]

6. Let

$$\mathbf{v}_1 = (1,-2,3,-4), \quad \mathbf{v}_2 = (2,1,-4,-3), \quad \mathbf{v}_3 = (-3,4,1,-2), \quad \mathbf{v}_4 = (4,3,2,1).$$

(a) Use Euclidean inner product to show that vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{v}_4$ form an orthogonal basis for R^4.

(b) Express $\mathbf{u} = (-1,2,3,7)$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{v}_4$.

[10 marks]

6. Biar

$$\mathbf{v}_1 = (1,-2,3,-4), \quad \mathbf{v}_2 = (2,1,-4,-3), \quad \mathbf{v}_3 = (-3,4,1,-2), \quad \mathbf{v}_4 = (4,3,2,1).$$

(a) Gunakan hasildarab terkedalam Euclidean untuk menunjukkan bahawa vektor $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ *dan* $\mathbf{v}_4$ *membentuk asas berortogon untuk* R^4.

(b) Ungkapkan $\mathbf{u} = (-1,2,3,7)$ *sebagai gabungan linear* $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ *dan* $\mathbf{v}_4$.

[10 markah]

7. Let

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}.$$

(a) Find the least squares solution of the linear system $\mathbf{Ax} = \mathbf{b}$.

(b) Hence, or otherwise, find the orthogonal projection of $\mathbf{b}$ on the column space of $\mathbf{A}$.

[15 marks]

7. Biar

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \text{dan} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}.$$

(a) Dapatkan penyelesaian kuasa dua terkecil bagi sistem linear $\mathbf{Ax} = \mathbf{b}$.

(b) Seterusnya, atau sebaliknya, dapatkan unjuran ortogon $\mathbf{b}$ *pada ruang lajur* $\mathbf{A}$.

[15 markah]

8. Let

$$\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{B}$.

(b) Determine the eigenspace associated with each eigenvalue. Is $\mathbf{B}$ diagonalizable? Why?

[15 marks]

8. Biar

$$\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix}.$$

(a) Nyatakan nilai-nilai eigen $\mathbf{B}$.

(b) Tentukan ruang eigen yang berkaitan dengan setiap nilai eigen tersebut. Adakah $\mathbf{B}$ *terpepenjurukan? Kenapa?*

[15 markah]

9. Let

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & 9 \\ -2 & 4 & -6 \end{pmatrix}.$$

(a) Find the solution space of the system $\mathbf{Ax} = \mathbf{0}$ (denote this as $N(\mathbf{A})$).

(b) Find the column space of $\mathbf{A}^T$ (denote this as $R(\mathbf{A}^T)$), and show that $R(\mathbf{A}^T) = N(\mathbf{A})^\perp$ where $N(\mathbf{A})^\perp$ denotes the orthogonal complement of $N(\mathbf{A})$.

[22 marks]

9. Biar

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & 9 \\ -2 & 4 & -6 \end{pmatrix}.$$

(a) Dapatkan ruang penyelesaian bagi sistem $\mathbf{Ax} = \mathbf{0}$ *(tulis ini sebagai* $N(\mathbf{A})$*).*

(b) Dapatkan ruang lajur $\mathbf{A}^T$ *(tulis ini sebagai* $R(\mathbf{A}^T)$*), dan tunjukkan bahawa* $R(\mathbf{A}^T) = N(\mathbf{A})^\perp$ *di mana* $N(\mathbf{A})^\perp$ *merujuk kepada pelengkap ortogon* $N(\mathbf{A})$.

[22 markah]

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