UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSG 388 – Mathematical Algorithms for Computer Graphics
[Algoritma Matematik untuk Grafik Komputer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Describe the outcome of applying Chaikin's subdivision algorithm repeatedly onto a polyline of $n > 2$ vertices.

(b) Given a cubic Bézier curve $\boldsymbol{P}(t)$ with control points $(1,\ 0)$, $(1,\ 1)$, $(2,\ 1)$ and $(2,\ 0)$. The curve is split to three curve segments of cubic at $t = 1/3$ and $t = 2/3$, find the new control points of the curve segments.

(c) Let $\{(-8,\ 4),\ \boldsymbol{V},\ \boldsymbol{W}\}$ and $\{\boldsymbol{V},\ \boldsymbol{W},\ (8,\ 4)\}$ be two sets of control points for the quadratic Bézier curves $\boldsymbol{P}(t)$ and $\boldsymbol{Q}(t)$ respectively, where $\boldsymbol{V}$, $\boldsymbol{W} \in \mathbb{R}^2$ and $t \in [0,\ 1]$. Suppose $\boldsymbol{P}(0.5) = (-3,\ 7)$ and $\boldsymbol{Q}(0.5) = (3,\ 7)$, evaluate the cubic Bézier curve of control points $(-8,\ 4)$, $\boldsymbol{V}$, $\boldsymbol{W}$ and $(8,\ 4)$ at $t = 0.5$.

[100 marks]

1. (a) Huraikan hasil mengaplikasikan algoritma subdivisi Chaikin berulang kali ke atas satu poligaris berbucu $n > 2$.

(b) Diberi satu lengkung Bézier kubik $\boldsymbol{P}(t)$ *dengan titik-titik kawalan* $(1,\ 0)$, $(1,\ 1)$, $(2,\ 1)$ *dan* $(2,\ 0)$. *Lengkung itu dibahagi kepada tiga segmen lengkung kubik pada* $t = 1/3$ *dan* $t = 2/3$, *cari titik-titik kawalan baru segmen-segmen lengkung.*

(c) Katakan $\{(-8,\ 4),\ \boldsymbol{V},\ \boldsymbol{W}\}$ *dan* $\{\boldsymbol{V},\ \boldsymbol{W},\ (8,\ 4)\}$ *ialah dua set titik kawalan bagi lengkung Bézier kuadratik* $\boldsymbol{P}(t)$ *dan* $\boldsymbol{Q}(t)$ *masing-masing, di mana* $\boldsymbol{V}$, $\boldsymbol{W} \in \mathbb{R}^2$ *dan* $t \in [0,\ 1]$. *Andaikan* $\boldsymbol{P}(0.5) = (-3,\ 7)$ *dan* $\boldsymbol{Q}(0.5) = (3,\ 7)$, *nilaikan lengkung Bézier kubik dengan titik-titik kawalan* $(-8,\ 4)$, $\boldsymbol{V}$, $\boldsymbol{W}$ *dan* $(8,\ 4)$ *pada* $t = 0.5$.

[100 markah]

2. Let the Bernstein polynomials of degree n be denoted by

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i \left(1-t\right)^{n-i}, \qquad 0 \le t \le 1, \qquad \text{for } i = 0,\ 1,\ \ldots,\ n.$$

(a) Given a rational cubic Bézier function

$$r(t) = \frac{c_0 B_0^3(t) + \omega_1 c_1 B_1^3(t) + \omega_2 c_2 B_2^3(t) + c_3 B_3^3(t)}{B_0^3(t) + \omega_1 B_1^3(t) + \omega_2 B_2^3(t) + B_3^3(t)}, \qquad t \in [0,\ 1]$$

where $\omega_1 > 0$, $\omega_2 > 0$ and $c_i \in \mathbb{R}$, for $i = 0,\ 1,\ 2,\ 3$. Determine all the ω_i and c_i, such that $r(t) = 2t^2 - 4t + 1$.

(b) Find the minimum value of function

$$r(t) = \frac{B_0^3(t) - 2B_1^3(t) - 2B_2^3(t) + B_3^3(t)}{B_0^3(t) + 2B_1^3(t) + 2B_2^3(t) + B_3^3(t)}, \quad t \in [0, 1].$$

(c) Given a rational quadratic Bézier curve

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t) + w\boldsymbol{C}_1 B_1^2(t) + \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \quad t \in [0, 1]$$

where $w > 0$, $\boldsymbol{C}_0 = (1, 1)$, $\boldsymbol{C}_1 = (4, 5)$ and $\boldsymbol{C}_2 = (4, 0)$. Evaluate the w such that $\boldsymbol{R}$ is a circular arc.

[100 marks]

2. *Katakan polinomial Bernstein berdarjah n ditandakan sebagai*

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}, \qquad 0 \le t \le 1, \qquad \textit{bagi} \;\; i = 0, 1, \ldots, n.$$

(a) Diberi suatu fungsi Bézier kubik nisbah

$$r(t) = \frac{c_0 B_0^3(t) + \omega_1 c_1 B_1^3(t) + \omega_2 c_2 B_2^3(t) + c_3 B_3^3(t)}{B_0^3(t) + \omega_1 B_1^3(t) + \omega_2 B_2^3(t) + B_3^3(t)}, \quad t \in [0, 1]$$

di mana $\omega_1 > 0$, $\omega_2 > 0$ dan $c_i \in \square$, bagi $i = 0, 1, 2, 3$. Tentukan semua ω_i dan c_i, supaya $r(t) = 2t^2 - 4t + 1$.

(b) Cari nilai minimum bagi fungsi

$$r(t) = \frac{B_0^3(t) - 2B_1^3(t) - 2B_2^3(t) + B_3^3(t)}{B_0^3(t) + 2B_1^3(t) + 2B_2^3(t) + B_3^3(t)}, \quad t \in [0, 1].$$

(c) Diberi lengkung Bézier kuadratik nisbah

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t) + w\boldsymbol{C}_1 B_1^2(t) + \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \quad t \in [0, 1]$$

di mana $w > 0$, $\boldsymbol{C}_0 = (1, 1)$, $\boldsymbol{C}_1 = (4, 5)$ dan $\boldsymbol{C}_2 = (4, 0)$. Nilaikan w supaya $\boldsymbol{R}$ ialah satu lengkok bulatan.

[100 markah]

3. The normalised basis spline of order k on a non-decreasing knot vector $\boldsymbol{u} = (u_0, u_1, \ldots, u_m)$, $m \geq k$, can be formulated recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \text{for} \quad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{otherwise}. \end{cases}$$

(a) Suppose $\boldsymbol{u} = (-2, -1, 0, 0, 1, 2, 3, 4)$, determine the number of basis splines of order 3 that can be formed and the order of continuity of the spline $N_1^3(u)$ at each knot.

(b) Suppose a cubic B-spline curve $\boldsymbol{P}(u)$ is defined with four de Boor points $\boldsymbol{D}_i$, $i = 0, 1, 2, 3$, on $\boldsymbol{u} = (-3, -2, -0.5, 0, 1, 3, 4, 5)$, find the de Boor points such that

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}(1-u)^3 + \begin{pmatrix} 6 \\ 3 \end{pmatrix}(1-u)^2 u + \begin{pmatrix} 9 \\ 3 \end{pmatrix}(1-u)u^2 + \begin{pmatrix} 4 \\ 2 \end{pmatrix}u^3.$$

(c) A biquadratic B-spline surface

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{2}\sum_{j=0}^{2} \boldsymbol{D}_{i,j}\, N_j^3(v)\, N_i^3(u), \qquad 0 \leq u \leq 1, \qquad 0 \leq v \leq 1,$$

is defined over the knot vectors $\boldsymbol{u} = (-1.5, -0.5, 0, 1, 1.75, 2.5)$ and $\boldsymbol{v} = (-2, -1, 0, 1, 2, 3)$ with the de Boor points

$\boldsymbol{D}_{0,0} = (0, 0, 1)$, $\boldsymbol{D}_{1,0} = (1, 0, 3)$, $\boldsymbol{D}_{2,0} = (2, 0, 1)$,

$\boldsymbol{D}_{0,1} = (0, 1, 2)$, $\boldsymbol{D}_{1,1} = (1, 1, 1)$, $\boldsymbol{D}_{2,1} = (2, 1, 2)$,

$\boldsymbol{D}_{0,2} = (0, 2, 1)$, $\boldsymbol{D}_{1,2} = (1, 2, 3)$, $\boldsymbol{D}_{2,2} = (2, 2, 1)$.

Use the de Boor algorithm to evaluate $\boldsymbol{S}$ at $(u, v) = (0.5, 0.5)$.

[100 marks]

3. Splin asas ternormal berperingkat k pada suatu vektor simpulan tak menyusut $\boldsymbol{u} = (u_0, u_1, \ldots, u_m)$, $m \geq k$, *boleh dirumuskan secara rekursi oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \textit{bagi} \quad k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \textit{sebaliknya}. \end{cases}$$

(a) *Andaikan* $\boldsymbol{u} = (-2\,,\ -1,\ 0\,,\ 0\,,\ 1,\ 2\,,\ 3\,,\ 4)$, *tentukan bilangan splin asas berperingkat* 3 *yang boleh dibentuk dan peringkat keselanjaran bagi splin* $N_1^3(u)$ *pada setiap simpulan.*

(b) *Andaikan suatu lengkung splin-B kubik* $\boldsymbol{P}(u)$ *ditakrif dengan empat titik de Boor* $\boldsymbol{D}_i$, $i = 0\,,\ 1,\ 2\,,\ 3$, *pada* $\boldsymbol{u} = (-3\,,\ -2\,,\ -0.5\,,\ 0\,,\ 1,\ 3\,,\ 4\,,\ 5)$, *cari titik-titik de Boor supaya*

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\2\end{pmatrix}(1-u)^3 + \begin{pmatrix}6\\3\end{pmatrix}(1-u)^2 u + \begin{pmatrix}9\\3\end{pmatrix}(1-u)u^2 + \begin{pmatrix}4\\2\end{pmatrix}u^3 .$$

(c) *Suatu permukaan splin-B dwikuadratik*

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{2}\sum_{j=0}^{2} \boldsymbol{D}_{i,\,j}\, N_j^3(v)\, N_i^3(u)\,, \qquad 0 \le u \le 1\,, \qquad 0 \le v \le 1\,,$$

ditakrif ke atas vektor-vektor simpulan $\boldsymbol{u} = (-1.5\,,\ -0.5\,,\ 0\,,\ 1,\ 1.75\,,\ 2.5)$ *dan* $\boldsymbol{v} = (-2\,,\ -1,\ 0\,,\ 1,\ 2\,,\ 3)$ *dengan titik-titik de Boor*

$$\boldsymbol{D}_{0,0} = (0,\ 0,\ 1)\,, \qquad \boldsymbol{D}_{1,0} = (1,\ 0,\ 3)\,, \qquad \boldsymbol{D}_{2,0} = (2,\ 0,\ 1)\,,$$

$$\boldsymbol{D}_{0,1} = (0,\ 1,\ 2)\,, \qquad \boldsymbol{D}_{1,1} = (1,\ 1,\ 1)\,, \qquad \boldsymbol{D}_{2,1} = (2,\ 1,\ 2)\,,$$

$$\boldsymbol{D}_{0,2} = (0,\ 2,\ 1)\,, \qquad \boldsymbol{D}_{1,2} = (1,\ 2,\ 3)\,, \qquad \boldsymbol{D}_{2,2} = (2,\ 2,\ 1)\,.$$

Gunakan algoritma de Boor untuk menilai $\boldsymbol{S}$ *pada* $(u,v) = (0.5,\ 0.5)$.

[100 markah]

4. Let the generalised Bernstein polynomials of degree n be defined as

$$B_{i,\,j,k}^n(u,v,w) = \frac{n!}{i!\,j!\,k!}u^i v^j w^k\,, \qquad 0 \le u\,,\ v,\ w \le 1, \qquad u+v+w=1\,,$$

where i, j, k are non-negative integers with $i+j+k=n$ and (u,v,w) are barycentric coordinates of a point relative to a domain triangle T.

(a) Given a cubic patch on T as

$$S(u,v,w) = u^2(u-v-w) + 2v^2(v-u-w) + 3v^2(w-u-v) + 3uvw\,,$$

evaluate S if $2u = 3v = 6w$.

(b) Find a cubic Bézier patch $S(u,v,w)$ in terms of $B_{i,\,j,k}^3(u,v,w)$ such that

$$S(u,v,w) = u^2 + 2v^2 + 3w^2 - uv - vw - wu\,.$$

(c) Given two Bézier patches by

$$S_1(u, v, w) = \sum_{i+j+k\geq 2} b_{i,j,k} B^2_{i,j,k}(u, v, w),$$

$$S_2(r, s, t) = \sum_{i+j+k\geq 2} c_{i,j,k} B^2_{i,j,k}(r, s, t),$$

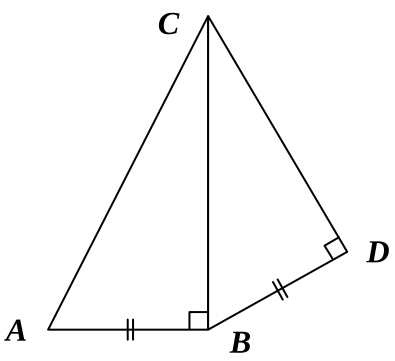
where (u, v, w) and (r, s, t) are barycentric coordinates with respect to two adjoining domain triangles $\boldsymbol{ABC}$ and $\boldsymbol{DBC}$, see figure below. Suppose the patches join C^1 parametric continuously along the common edge of the triangles by

$$S_1(0, v, w) = S_2(0, s, t),$$

$$c_{1,0,1} = \alpha b_{1,0,1} + \beta b_{0,0,2} + \gamma b_{0,1,1},$$

$$c_{1,1,0} = \alpha b_{1,1,0} + \beta b_{0,1,1} + \gamma b_{0,2,0},$$

with $v = s$, evaluate α, β and γ.



A pair of right-angled triangles in a plane with the length $\|\boldsymbol{BC}\| = 2\|\boldsymbol{BA}\| = 2\|\boldsymbol{BD}\|$.

[100 marks]

4. *Katakan polinomial Bernstein teritlak berdarjah n ditakrif sebagai*

$$B^n_{i,j,k}(u, v, w) = \frac{n!}{i!\,j!\,k!} u^i v^j w^k, \qquad 0 \leq u,\ v,\ w \leq 1, \qquad u + v + w = 1,$$

di mana i, j, k adalah integer tak negatif dengan $i + j + k = n$ dan (u, v, w) ialah koordinat baripusat bagi suatu titik relatif kepada segi tiga domain T.

(a) *Diberi satu tampalan kubik pada T sebagai*

$$S(u, v, w) = u^2(u - v - w) + 2v^2(v - u - w) + 3v^2(w - u - v) + 3uvw,$$

nilaikan S jika $2u = 3v = 6w$.

(b) *Cari satu tampalan Bézier kubik* $S(u,v,w)$ *dalam sebutan* $B^3_{i,j,k}(u,v,w)$ *supaya*

$$S(u,v,w) = u^2 + 2v^2 + 3w^2 - uv - vw - wu\,.$$

(c) *Diberi dua tampalan Bézier*

$$S_1(u,v,w) = \sum_{i+j+k\geq 2} b_{i,j,k} B^2_{i,j,k}(u,v,w)\,,$$

$$S_2(r,s,t) = \sum_{i+j+k\geq 2} c_{i,j,k} B^2_{i,j,k}(r,s,t)\,,$$

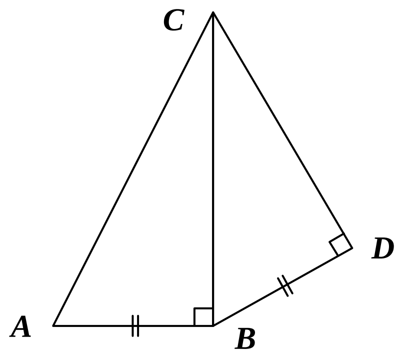
di mana (u,v,w) *dan* (r,s,t) *adalah koordinat baripusat terhadap dua segi tiga domain yang bersebelahan* ***ABC*** *dan* ***DBC*** *, lihat rajah di bawah. Andaikan dua tampalan bertemu secara selanjar* C^1 *di sepanjang sisi umum segi tiga oleh*

$$S_1(0,v,w) = S_2(0,s,t)\,,$$

$$c_{1,0,1} = \alpha b_{1,0,1} + \beta b_{0,0,2} + \gamma b_{0,1,1}\,,$$

$$c_{1,1,0} = \alpha b_{1,1,0} + \beta b_{0,1,1} + \gamma b_{0,2,0}\,,$$

dengan $v = s$, *nilaikan* α, β *dan* γ.



Sepasang segi tiga bersudut tegak dalam satah dengan panjang $\|\mathbf{BC}\| = 2\|\mathbf{BA}\| = 2\|\mathbf{BD}\|$.

[100 markah]

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