UNIVERSITI SAINS MALAYSIA

First Semester Examination

2013/2014 Academic Session

December 2013

MAT 263 – Probability Theory
[Teori Kebarangkalian]

Duration : 3 hours

[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all six** [6] questions.

*[**Arahan:** Jawab **semua enam** [6] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Given the function of, N, has

$$P(N=n)=\begin{cases}\dfrac{2}{3^{n+1}} & n=0,1,2,\ldots\\ 0 & \text{otherwise.}\end{cases}$$

   a. Show that N is a random variable.

[4 marks]

   b. Find mean of N

[5 marks]

   c. Let N_1 and N_2 be independent and identical distribution as above, and $X=N_1+N_2$, find $f_X(3)$.

[6 marks]

1. Diberi suatu fungsi, N, mempunyai

$$Kb(N=n)=\begin{cases}\dfrac{2}{3^{n+1}} & n=0,1,2,\ldots\\ 0 & \textit{selainnya.}\end{cases}$$

   a. Tunjukkan bahawa N adalah suatu pembolehubah rawak.

[4 markah]

   b. Cari min bagi N

[5 markah]

   c. Biarkan N_1 dan N_2 adalah taburan tidak bersandar dan secaman seperti di atas, dan $X=N_1+N_2$, cari $f_X(3)$.

[6 markah]

2. Let A, B and C be the independent random variables which have the respective moment generating functions as follows:

$$M_A(t)=\exp\left[2\left(e^t-1\right)\right]$$
$$M_B(t)=\exp\left[3\left(e^t-1\right)\right]$$
$$M_C(t)=\exp\left[4\left(e^t-1\right)\right]$$

Let $X=A+B+C$, and skewness coefficient, γ, is defined as $\gamma=\frac{E[(X-\mu)^3]}{\sigma^3}$, where μ is the mean and σ is the standard deviation of X.

   a. Find moment generating function of X, $M_X(t)$.

[3 marks]

   b. Calculate $E\left(X^k\right)$ for $k=1,2,3$

[5 marks]

   c. Find σ and finally,

[3 marks]

   d. Find γ.

[7 marks]

2. *Biarkan A, B dan C adalah pemboleh-pemboleh ubah tidak bersandar dan masing-masing mempunyai fungsi penjana momen seperti berikut:*

$$M_A(t) = \text{eks}\left[2\left(e^t - 1\right)\right]$$
$$M_B(t) = \text{eks}\left[3\left(e^t - 1\right)\right]$$
$$M_C(t) = \text{eks}\left[4\left(e^t - 1\right)\right]$$

Biarkan $X = A + B + C$, dan pekali kepencongan, γ ditakrifkan sebagai $\gamma = \frac{E[(X-\mu)^3]}{\sigma^3}$, yang mana μ adalah min dan σ adalah sisihan piawai bagi X.

a. *Cari fungsi penjana momen bagi X, $M_X(t)$.*

[3 markah]

b. *Kira $E\left(X^k\right)$ bagi $k = 1,2,3$.*

[5 markah]

c. *Cari σ dan akhirnya,*

[3 markah]

d. *Cari γ.*

[7 markah]

3. Let A, B, C and D be the events such that $B = A'$, $C \cap D = \varnothing$, and $\mathrm{P}(A) = \frac{1}{4}$, $\mathrm{P}(B) = \frac{3}{4}$, $\mathrm{P}(C \mid A) = \frac{1}{2}$, $\mathrm{P}(C \mid B) = \frac{3}{4}$, $\mathrm{P}(D \mid A) = \frac{1}{4}$ and $\mathrm{P}(D \mid B) = \frac{1}{8}$. Calculate

a. $\mathrm{P}(C \cup D)$.

[8 marks]

b. $\mathrm{P}(B \mid C \cup D)$.

[5 marks]

3. *Biarkan A, B, C dan D suatu peristiwa yang mana $B = A'$, $C \cap D = \varnothing$, dan $Kb(A) = \frac{1}{4}$, $Kb(B) = \frac{3}{4}$, $Kb(C \mid A) = \frac{1}{2}$, $Kb(C \mid B) = \frac{3}{4}$, $Kb(D \mid A) = \frac{1}{4}$ dan $Kb(D \mid B) = \frac{1}{8}$. Kira*

a. *$Kb(C \cup D)$.*

[8 markah]

b. *$Kb(B \mid C \cup D)$.*

[5 markah]

4. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 0.25xe^{-0.5x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

a. State the distribution of X, and thus, find its mean and variance.

[5 marks]

b. Use Markov's Theorem to show that the probability that X is greater than 5 is less than 0. 8.

[3 marks]

c. Find the probability that X is greater than 5.

[7 marks]

4. *Biarkan X menjadi pemboleh ubah rawak dengan fungsi ketumpatan kebarangkalian*

$$f(x) = \begin{cases} 0.25xe^{-0.5x} & x \geq 0 \\ 0 & selainnya. \end{cases}$$

a. Nyatakan taburan bagi X , dan seterusnya, cari min dan variansnya.

[5 markah]

b. Gunakan Teorem Markov untuk menuunjukkan bahawa kebarangkalian X lebih besar daripada 5 adalah kurang daripada 0.8.

[3 markah]

c. Cari kebarangkalian X lebih besar daripada 5.

[7 markah]

5. Let X be the Geometric distribution with parameter p.

a. Derive the moment generating function of X, $M_X(t)$.

[4 marks]

b. Now, by using the above moment generating function, derive the mean and the second moment of X.

[7 marks]

c. Let X has a Geometric distribution with $p = 0.25$, and let $Y = 2X + 3$. Find the variance of Y, $Var(Y)$.

[8 marks]

5. *Biarkan X menjadi taburan Geometrik dengan parameter p.*

a. Dapatkan fungsi penjana momen bagi X, $M_X(t)$.

[4 markah]

b. Sekarang, dengan menggunakan fungsi penjana momen di atas, dapatkan min dan momen kedua bagi X.

[7 markah]

c. Biarkan X mempunyai taburan Geometrik dengan $p = 0.25$, dan biarkan $Y = 2X + 3$.Cari varians bagi Y, $Var(Y)$.

[8 markah]

6. Let the jointly variable of X and Y has the function of

$$f(x,y)=\begin{cases} c\left[2-(x+y)\right] & x>0, y>0 \text{ and } x+y<2 \\ 0 & \text{otherwise.} \end{cases}$$

a. Find c so that the function above be a jointly random variable.

[5 marks]

b. Find the marginal distribution of X.

[3 marks]

c. Find $\mathrm{P}(X<1)$.

[3 marks]

d. What is $Cov(X,Y)$?

[9 marks]

6. Biarkan pembolehubah tercantum X dan Y mempunyai fungsi

$$f(x,y)=\begin{cases} c\left[2-(x+y)\right] & x>0, y>0 \ dan \ x+y<2 \\ 0 & selainnya. \end{cases}$$

a. Cari c supaya fungsi di atas menjadi suatu pemboleh ubah rawak tercantum.

[5 markah]

b. Cari taburan sut bagi X.

[3 markah]

c. Cari $Kb(X<1)$.

[3 markah]

d. Apakah $Kov(X,Y)$?

[9 markah]

Appendix

Random Variable, X	**Probability distribution function, $f_X(x)$**	**Mean, $E(X)$**	**Variance, $Var(X)$**	**Moment Generating Function, $M_X(t) = E(e^{tX})$**
$Uniform(a,b)$	$\frac{1}{b-a}, \quad a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
$exp(\theta)$	$\frac{1}{\theta}e^{-x/\theta}, \quad x > 0$	θ	θ^2	$(1-\theta t)^{-1}$
$bin(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}, \quad x = 0,1,\dots,n$	np	$np(1-p)$	$(pe^t+1-p)^n$
$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma}exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty$	μ	σ^2	$exp\left(\mu t+\frac{t^2\sigma^2}{2}\right)$
$Gamma(\alpha,\theta)$	$\frac{1}{\Gamma(\alpha)}x^{\alpha-1}\left(\frac{1}{\theta}\right)^{\alpha}exp\left(-\frac{x}{\theta}\right), \quad x > 0$	$\alpha\theta$	$\alpha\theta^2$	$(1-\theta t)^{-\alpha}$
$Po(\lambda)$	$\frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\dots$	λ	λ	$exp[\lambda(e^t-1)]$
$NB(r,p)$	$\binom{x+r-1}{x}(1-p)^x p^r, x = 0,1,2,\dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$

Arithmetic series

$$S(x) = a + (a+d) + (a+2d) + \cdots + [a+(x-1)d] = ax + \frac{x(x-1)d}{2}$$

Geometric series

$$S(x) = a + aq + aq^2 + \cdots + aq^{x-1} = \frac{a(1-q^x)}{1-q}$$

Binomial series

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$$

Taylor Series

$$f(x) = \sum_{k=0}^{\infty}\frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$$