UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSG 284 – Introduction to Geometric Modelling
[Pengenalan kepada Pemodelan Geometri]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) State a difference between parametric continuity C^1 and geometric continuity G^1.

   (b) Find a quadratic polynomial that interpolates three points $(1,1)$, $(2,3)$ and $(3,4)$.

   (c) Given a circular arc $x^2 + y^2 - 2(x + y + 1) = 0$, where $x \geq 0$ and $y \geq 0$. Sketch the curve and find its appropriate parametric equation.

[100 marks]

1. (a) Nyatakan satu perbezaan antara keselanjaran berparameter C^1 dan keselanjaran geometri G^1.

(b) Cari polinomial kuadratik yang menginterpolasi tiga titik $(1,1)$, $(2,3)$ dan $(3,4)$.

(c) Diberi lengkok bulatan $x^2 + y^2 - 2(x + y + 1) = 0$, di mana $x \geq 0$ dan $y \geq 0$. Lakarkan lengkung dan cari persamaan berparameter yang berkenaan.

[100 markah]

2. Let the Bernstein polynomials of degree n be defined by

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i \left(1-t\right)^{n-i}, \qquad 0 \leq t \leq 1, \qquad \text{for } i = 0, 1, \ldots, n.$$

   (a) Show that $B_i^n(t) = (1-t)\,B_i^{n-1}(t) + t\,B_{i-1}^{n-1}(t)$.

   (b) Find the minimum value of a cubic Bézier polynomial

$$P(t) = B_0^3(t) - B_1^3(t) + 2B_2^3(t) + B_3^3(t), \qquad 0 \leq t \leq 1.$$

   (c) Given a spline function

$$s(u) = \begin{cases} u\,(au^2 + bu + c), & 0 \leq u \leq 1, \\ p(u), & 1 \leq u \leq 3, \end{cases}$$

   where a, b, $c \in \mathbb{R}$. The function p can be represented locally as

$$p(t) = B_0^2(t) + B_1^2(t) + 3B_2^2(t), \qquad 0 \leq t \leq 1.$$

   Evaluate a, b and c such that $s(u)$ is C^2 parametric continuous.

[100 marks]

2. *Katakan polinomial Bernstein berdarjah n ditakrif sebagai*

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i \left(1-t\right)^{n-i}, \qquad 0 \le t \le 1, \qquad \textit{bagi} \;\; i = 0, 1, \ldots, n.$$

(a) *Tunjukkan bahawa* $B_i^n(t) = (1-t)\, B_i^{n-1}(t) \; + \; t\, B_{i-1}^{n-1}(t)$.

(b) *Cari nilai minimum bagi polinomial Bézier kubik*

$$P(t) = B_0^3(t) - B_1^3(t) + 2B_2^3(t) + B_3^3(t), \qquad 0 \le t \le 1.$$

(c) *Diberi fungsi splin*

$$s(u) = \begin{cases} u\,(au^2 + bu + c), & 0 \le u \le 1, \\ p(u), & 1 \le u \le 3, \end{cases}$$

di mana a, b, $c \in \mathbb{R}$. *Fungsi* p *boleh diwakili secara setempat sebagai*

$$p(t) = B_0^2(t) + B_1^2(t) + 3B_2^2(t), \qquad 0 \le t \le 1.$$

Nilaikan a, b *dan* c *supaya* $s(u)$ *adalah selanjar berparameter* C^2.

[100 markah]

3. Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a non-decreasing knot vector where n and k are positive integers with $n \ge k-1$. The normalized B-spline basis functions of order k are defined recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \text{for} \quad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

where $i = 0, 1, \ldots, n$.

(a) Give a knot vector $\boldsymbol{u}$ such that $N_i^4(u) = B_i^3(u)$, $u \in [0, 1]$, for $i = 0, 1, 2, 3$, where $B_i^3(u)$ are the Bernstein polynomials of degree 3.

(b) Let $\boldsymbol{u} = (-2, -1, 0, 1, 2, 3)$, show that $\sum_{i=0}^{2} N_i^3(u) = 1$, for $u \in [0, 1]$.

(c) Given a uniform B-spline curve of order 3

$$\boldsymbol{P}(u) = \begin{pmatrix} x(u) \\ y(u) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ d \end{pmatrix} N_1^3(u) + \begin{pmatrix} 3 \\ 1 \end{pmatrix} N_2^3(u),$$

$u \in [u_2, u_3]$, evaluate d such that the curve $\boldsymbol{P}$ touches the x-axis.

[100 marks]

3. *Katakan* $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ *ialah suatu vektor simpulan tak menyusut di mana* n *dan* k *adalah integer positif dengan* $n \geq k-1$. *Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursi oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad bagi \quad k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & sebaliknya \end{cases}$$

di mana $i = 0, 1, \ldots, n$.

(a) *Berikan suatu vektor simpulan* $\boldsymbol{u}$ *supaya* $N_i^4(u) = B_i^3(u)$, $u \in [0, 1]$, *bagi* $i = 0, 1, 2, 3$, *di mana* $B_i^3(u)$ *adalah polinomial Bernstein berdarjah* 3.

(b) *Katakan* $\boldsymbol{u} = (-2, -1, 0, 1, 2, 3)$, *tunjukkan bahawa* $\sum_{i=0}^{2} N_i^3(u) = 1$, *bagi* $u \in [0, 1]$.

(c) *Diberi lengkung splin-B seragam berperingkat* 3

$$\boldsymbol{P}(u) = \begin{pmatrix} x(u) \\ y(u) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ d \end{pmatrix} N_1^3(u) + \begin{pmatrix} 3 \\ 1 \end{pmatrix} N_2^3(u),$$

$u \in [u_2, u_3]$, *nilaikan* d *supaya lengkung* $\boldsymbol{P}$ *menyentuh paksi-* x.

[100 markah]

4. (a) Consider a biquadratic Bézier surface

$$\begin{aligned} \boldsymbol{S}(u, v) &= \big(x(u,v), \ y(u,v), \ z(u,v) \big) \\ &= \sum_{i=0}^{2} \sum_{j=0}^{2} \boldsymbol{C}_{i,j} \, B_j^2(v) B_i^2(u), \qquad 0 \leq u, \ v \leq 1, \end{aligned}$$

where $B_s^2(t)$, $0 \leq t \leq 1$, $s = 0, 1, 2$, indicate the Bernstein polynomials of degree 2 and $\boldsymbol{C}_{i,j}$ are the control points

$\boldsymbol{C}_{0,0} = (1, 0, 1)$, $\boldsymbol{C}_{0,1} = (1, 2, 2)$, $\boldsymbol{C}_{0,2} = (1, 4, 1)$,

$\boldsymbol{C}_{1,0} = (2, 0, 2)$, $\boldsymbol{C}_{1,1} = (2, 2, 4)$, $\boldsymbol{C}_{1,2} = (2, 4, 2)$,

$\boldsymbol{C}_{2,0} = (3, 0, 1)$, $\boldsymbol{C}_{2,1} = (3, 2, 2)$, $\boldsymbol{C}_{2,2} = (3, 4, 1)$.

(i) Prove that the surface $\boldsymbol{S}$ lies within the convex hull of the control net.

(ii) Determine the unit normal vector to the surface $\boldsymbol{S}$ at $(x, y) = (1, 1)$.

(b) Define a bilinearly blended Coons patch $\boldsymbol{S}(u,\ v)$, $0 \le u$, $v \le 1$, with four boundaries given below

$$\boldsymbol{S}(0,\ v) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(1-v) + \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}v, \qquad \boldsymbol{S}(u,\ 0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(1-u) + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}u,$$

$$\boldsymbol{S}(1,\ v) = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}(1-v)^2 + \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}2v(1-v) + \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}v^2,$$

$$\boldsymbol{S}(u,\ 1) = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}(1-u)^2 + \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}2u(1-u) + \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}u^2.$$

Then evaluate the patch $\boldsymbol{S}$ at $(u,\ v) = (0.5,\ 0.5)$.

[100 marks]

4. (a) *Pertimbangkan permukaan Bézier dwikuadratik*

$$\boldsymbol{S}(u,v) = \left(x(u,v),\ y(u,v),\ z(u,v) \right)$$

$$= \sum_{i=0}^{2}\sum_{j=0}^{2} \boldsymbol{C}_{i,j}\, B_j^2(v)\, B_i^2(u), \qquad 0 \le u,\ v \le 1,$$

di mana $B_s^2(t)$, $0 \le t \le 1$, $s = 0,\ 1,\ 2$, *menandakan polinomial Bernstein berdarjah* 2 *dan* $\boldsymbol{C}_{i,j}$ *ialah titik-titik kawalan*

$\boldsymbol{C}_{0,0} = (1,\ 0,\ 1)$, $\quad \boldsymbol{C}_{0,1} = (1,\ 2,\ 2)$, $\quad \boldsymbol{C}_{0,2} = (1,\ 4,\ 1)$,

$\boldsymbol{C}_{1,0} = (2,\ 0,\ 2)$, $\quad \boldsymbol{C}_{1,1} = (2,\ 2,\ 4)$, $\quad \boldsymbol{C}_{1,2} = (2,\ 4,\ 2)$,

$\boldsymbol{C}_{2,0} = (3,\ 0,\ 1)$, $\quad \boldsymbol{C}_{2,1} = (3,\ 2,\ 2)$, $\quad \boldsymbol{C}_{2,2} = (3,\ 4,\ 1)$.

(i) *Buktikan bahawa permukaan* $\boldsymbol{S}$ *terletak di dalam hul cembung jaring kawalan.*

(ii) *Tentukan vektor unit normal kepada permukaan* $\boldsymbol{S}$ *pada* $(x,\ y) = (1,\ 1)$.

(b) *Takrifkan satu tampalan Coons teraduan dwilinear* $\boldsymbol{S}(u,\ v)$, $0 \le u\ ,\ v \le 1$, *dengan empat sempadan diberikan di bawah*

$$\boldsymbol{S}(0,\ v) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(1-v) + \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} v, \qquad \boldsymbol{S}(u,\ 0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(1-u) + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} u,$$

$$\boldsymbol{S}(1,\ v) = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}(1-v)^2 + \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} 2v(1-v) + \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} v^2,$$

$$\boldsymbol{S}(u,\ 1) = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}(1-u)^2 + \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} 2u(1-u) + \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} u^2.$$

Kemudian nilaikan tampalan $\boldsymbol{S}$ *pada* $(u,\ v) = (0.5,\ 0.5)$.

[100 markah]

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