UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **SIX** (6) questions.

*[**Arahan**: Jawab **ENAM** (6) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Consider the following LP and its optimal tableau:

$$\begin{aligned} \text{maximise} \quad & z = x_1 - x_2 + 2x_3 \\ \text{subject to} \quad & \\ & 2x_1 - 2x_2 + 3x_3 \le 5 \\ & x_1 + x_2 - x_3 \le 3 \\ & x_1 - x_2 + x_3 \le 2 \\ & x_1, \quad x_2, \quad x_3 \ge 0. \end{aligned}$$

Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	2	A	B	1	1	C	D
a	5	1	E	1	3	F	G
b	2	H	J	0	1	1	K
c	4	L	N	1	2	P	Q

Find the values of A, B, C, D, E, F, G, H, J, K, L, N, P, Q and the basic variables a, b, c **without** solving the LP from the beginning.

(b) Consider the following LP:

$$\begin{aligned} \text{maximise} \quad & z = 2x_1 - 2x_2 + 3x_3 \\ \text{subject to} \quad & \\ & -x_1 + x_2 + x_3 \le 4 \\ & 2x_1 - x_2 + x_3 \le 2 \\ & x_1 + x_2 + 3x_3 \le 12 \\ & x_1, \quad x_2, \quad x_3 \ge 0. \end{aligned}$$

Solve the LP using the Revised Simplex Method.

[20 marks]

1. *(a)* *Pertimbangkan PL berikut serta tablo optimumnya:*

$$\begin{aligned} &\textit{maksimumkan} && z = x_1 - x_2 + 2x_3 \\ &\textit{terhadap} \\ & && 2x_1 - 2x_2 + 3x_3 \le 5 \\ & && x_1 + x_2 - x_3 \le 3 \\ & && x_1 - x_2 + x_3 \le 2 \\ & && x_1,\quad x_2,\quad x_3 \ge 0. \end{aligned}$$

Asas	x_1	x_2	x_3	s_1	s_2	s_3	*Penyelesaian*
z	*2*	*A*	*B*	*1*	*1*	*C*	*D*
a	*5*	*1*	*E*	*1*	*3*	*F*	*G*
b	*2*	*H*	*J*	*0*	*1*	*1*	*K*
c	*4*	*L*	*N*	*1*	*2*	*P*	*Q*

Berikan nilai-nilai A, B, C, D, E, F, G, H, J, K, L, N, P, Q dan pembolehubah-pembolehubah asas a, b, c ***tanpa*** *menyelesaikan PL dari mula.*

(b) *Pertimbangkan PL berikut:*

$$\begin{aligned} &\textit{memaksimumkan} && z = 2x_1 - 2x_2 + 3x_3 \\ &\textit{terhadap} \\ & && -x_1 + x_2 + x_3 \le 4 \\ & && 2x_1 - x_2 + x_3 \le 2 \\ & && x_1 + x_2 + 3x_3 \le 12 \\ & && x_1,\quad x_2,\quad x_3 \ge 0. \end{aligned}$$

Selesaikan PL dengan menggunakan Kaedah Simpleks Tertilik Semula.

[20 markah]

2. (a) Consider the following LP

$$\begin{aligned} &\text{maximise} && z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ &\text{subject to} \\ & && x_1 + x_2 + \ldots + x_n = 1 \\ & && x_1, \quad x_2, \quad \ldots, \quad x_n \geq 0 \\ &\text{where} \\ & && c_1, c_2, \ldots, c_n \in \mathbb{R}. \end{aligned}$$

(i) Find the dual of this LP.

(ii) Suppose that $c_4 > c_i \quad \forall i$ such that $i \neq 4$. Use this information to solve the dual.

(iii) Use the solution in (ii) to solve the primal LP. State any and all theorems used.

(b) Consider the following LP and its optimal tableau

$$\begin{aligned} &\text{maximise} && z = -5x_1 + 5x_2 + 13x_3 \\ &\text{subject to} \\ & && -x_1 + x_2 + 3x_3 \leq 20 \\ & && 12x_1 + 4x_2 + 10x_3 \leq 90 \\ & && x_1, \quad x_2, \quad x_3 \geq 0 \end{aligned}$$

Basic	x_1	x_2	x_3	s_1	s_2	Solution
z	0	0	2	5	0	100
x_2	-1	1	3	1	0	20
s_2	16	0	-2	-4	1	10

Investigate the following (separate) changes to the original LP. Determine whether the current basis remains optimal. If the current basis is no longer optimal, determine the new optimal solution.

(i) Change b_1 to 30.

(ii) Add the constraint $2x_1 + 3x_2 + 5x_3 \leq 50$ to the LP.

[20 marks]

2. (a) *Pertimbangkan PL berikut*

memaksimumkan $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

terhadap

$$x_1 + x_2 + \ldots + x_n = 1$$
$$x_1, \quad x_2, \quad \ldots, \quad x_n \geq 0$$

dan

$$c_1, c_2, \ldots, c_n \in \mathbb{R}.$$

(i) *Berikan rumus dual bagi PL di atas.*

(ii) *Katakan* $c_4 > c_i \quad \forall i$ *dengan* $i \neq 4$. *Gunakan maklumat tersebut untuk menyelesaikan masalah dual.*

(iii) *Gunakan penyelesaian (ii) untuk menyelesaikan PL primal. Nyatakan segala teorem yang anda gunakan.*

(b) *Pertimbangkan PL berikut dan tablo optimumnya*

memaksimumkan $z = -5x_1 + 5x_2 + 13x_3$

terhadap

$$-x_1 + x_2 + 3x_3 \leq 20$$
$$12x_1 + 4x_2 + 10x_3 \leq 90$$
$$x_1, \quad x_2, \quad x_3 \geq 0$$

Asas	x_1	x_2	x_3	s_1	s_2	*Penyelesaian*
z	*0*	*0*	*2*	*5*	*0*	*100*
x_2	*-1*	*1*	*3*	*1*	*0*	*20*
s_2	*16*	*0*	*-2*	*-4*	*1*	*10*

Pertimbangkan perubahan-perubahan berikut yang dikenakan (secara berasingan) pada PL di atas. Tentiukan sama ada asas semasa masih optimum. Jika tidak, tentukan penyelesaian optimum terkini.

(i) *Ubah* b_1 *kepada 30.*

(ii) *Tambahkan kekangan* $2x_1 + 3x_2 + 5x_3 \leq 50$ *pada PL di atas.*

[20 markah]

3. Solve the following IP using the branch-and-bound method:

$$\begin{aligned} \text{minimise} \quad & z = 5x_1 + 4x_2 \\ \text{subject to} \quad & \\ & 3x_1 + 2x_2 \geq 5 \\ & 2x_1 + 3x_2 \geq 7 \\ & x_1, \quad x_2 \geq 0 \text{ and integer.} \end{aligned}$$

[15 marks]

3. Selesaikan PI berikut menggunakan kaedah cabang dan batas:

$$\begin{aligned} \textit{meminimumkan} \quad & z = 5x_1 + 4x_2 \\ \textit{terhadap} \quad & \\ & 3x_1 + 2x_2 \geq 5 \\ & 2x_1 + 3x_2 \geq 7 \\ & x_1, \quad x_2 \geq 0 \textit{ dan integer}. \end{aligned}$$

[15 markah]

4. Consider the following 0-1 IP:

$$\begin{aligned} \text{maximise} \quad & z = 5x_1 + 6x_2 + 2x_3 + 3x_4 + 3x_5 \\ \text{subject to} \quad & \\ & x_1 + x_2 \leq 1 \\ & x_1 + x_4 + 2x_5 \leq 2 \\ & 4x_1 + 6x_2 + 2x_3 + 3x_4 + 2x_5 \leq 12 \\ & x_1 + x_2 + x_3 + x_4 \leq 2 \\ & x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5 = 0 \text{ or } 1. \end{aligned}$$

Solve the 0-1 IP.

[15 marks]

4. *Pertimbangkan PI 0-1 berikut:*

$$\begin{aligned} \textit{memaksimumkan} \quad & z = 5x_1 + 6x_2 + 2x_3 + 3x_4 + 3x_5 \\ \textit{terhadap} \quad & \\ & x_1 + x_2 \leq 1 \\ & x_1 + x_4 + 2x_5 \leq 2 \\ & 4x_1 + 6x_2 + 2x_3 + 3x_4 + 2x_5 \leq 12 \\ & x_1 + x_2 + x_3 + x_4 \leq 2 \\ & x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5 = 0 \textit{ atau } 1. \end{aligned}$$

Selesaikan PI 0-1 di atas.

[15 markah]

5. Consider the following IP:

$$\begin{aligned} &\text{minimise} && z = -5x_1 - 8x_2 \\ &\text{subject to} \\ & && x_1 + x_2 \le 6 \\ & && 5x_1 + 9x_2 \le 45 \\ & && x_1, \quad x_2 \ge 0 \text{ and integer.} \end{aligned}$$

(a) Given that x_1 and x_2 are basic variables in the optimal solution (of the linear relaxation), show that

$$B^{-1} = \begin{bmatrix} 9/4 & -1/4 \\ -5/4 & 1/4 \end{bmatrix}.$$

(b) Using B^{-1}, and without performing any simplex/revised simplex iterations, show that the optimal tableau (of the linear relaxation) is as follows:

Basic	x_1	x_2	s_1	s_2	Solution
z	0	0	$-5/4$	$-3/4$	$-165/4$
x_1	1	0	$9/4$	$-1/4$	$9/4$
x_2	0	1	$-5/4$	$1/4$	$15/4$

(c) Get the integer solution using the cutting plane method.

[15 marks]

5. *Pertimbangkan PI berikut:*

$$\begin{aligned} &\textit{meminimumkan} && z = -5x_1 - 8x_2 \\ &\textit{terhadap} \\ & && x_1 + x_2 \le 6 \\ & && 5x_1 + 9x_2 \le 45 \\ & && x_1, \quad x_2 \ge 0 \textit{ dan integer}. \end{aligned}$$

(a) Diberi x_1 and x_2 adalah asas dalam penyelesaian optimum (PL tak tegang), tunjukkan bahawa

$$B^{-1} = \begin{bmatrix} 9/4 & -1/4 \\ -5/4 & 1/4 \end{bmatrix}.$$

(b) Menggunakan B^{-1}, dan tanpa melakukan lelaran simpleks/simpleks tertilik semula, tunjukkan bahawa tablo optimum (PL tak tegang) adalah seperti berikut:

Asas	x_1	x_2	s_1	s_2	*Penyelesaian*
z	*0*	*0*	$-{}^{5}/_{4}$	$^{-3}/_{4}$	$-{}^{165}/_{4}$
x_1	*1*	*0*	$^{9}/_{4}$	$-{}^{1}/_{4}$	$^{9}/_{4}$
x_2	*0*	*1*	$-{}^{5}/_{4}$	$^{1}/_{4}$	$^{15}/_{4}$

(c) Dapatkan penyelesaian integer menggunakan kaedah potongan satah.

[15 markah]

6. The board of directors of Superposh Theatre is worried that lately, the residents of Affluent City (hipsters, art buffs and architects) are becoming less interested in the performing arts. Thus, to generate interest, the chairman of the board has decided that Superposh will hold two types of events: open days (where visitors are allowed to tour Superposh's premises), and free performances (given by Affluent City's Drama Club). Superposh has a budget of RM75,000 and will have to spend RM15,000 to hold each open day and RM7500 to hold each free performance. The board of directors estimate the attendance of these two event types as follows:

Event	Number of People Attending per Event Held		
	Hipsters	Art Buffs	Architects
Free Performance	150	300	0
Open Day	600	0	375

The chairman has set the following goals (in order of importance):

Goal 1: At least 1200 architects attend the events.
Goal 2: At least 1800 hipsters attend the events.
Goal 3: At least 1500 art buffs attend the events.

Formulate and solve a preemptive goal programming model for Superposh Theatre's board of directors.

[15 marks]

6. *Lembaga pengarah Superposh Theatre bimbang bahawa sejak kebelakangan ini, penduduk di Affluent City ("hipster", peminat seni dan arkitek) menjadi kurang berminat dalam seni persembahan. Oleh itu, untuk menjana minat, pengerusi lembaga telah memutuskan bahawa Superposh akan mengadakan dua jenis acara: hari terbuka (orang ramai dibenarkan melawat premis Superposh), dan persembahan percuma (diberikan oleh Kelab Drama Affluent City). Superposh mempunyai belanjawan sebanyak RM75,000 dan perlu menggunakan RM15,000 untuk mengadakan setiap hari terbuka dan RM7500 untuk mengadakan setiap persembahan percuma. Lembaga pengarah menganggarkan kehadiran bagi kedua-dua jenis acara seperti berikut:*

Acara	*Bilangan yang Hadir Setiap Kali Acara Diadakan*		
	"Hipster"	*Peminat Seni*	*Arkitek*
Persembahan Percuma	*150*	*300*	*0*
Hari Terbuka	*600*	*0*	*375*

Pengerusi lembaga telah menetapkan gol-gol berikut (mengikut kepentingan):

***Gol 1**: Sekurang-kurangnya 1200 orang arkitek hadir acara-acara diadakan.*
***Gol 2**: Sekurang-kurangnya 1800 orang "hipster" hadir acara-acara diadakan.*
***Gol 3**: Sekurang-kurangnya 1500 orang peminat seni hadir acara-acara diadakan.*

Rumuskan suatu model pengaturcaraan gol "preemptive" untuk lembaga pengarah Superposh Theatre dan selesaikannya.

[15 markah]

- ooo0ooo -