UNIVERSITI SAINS MALAYSIA

First Semester Examination
2013/2014 Academic Session

December 2013 / January 2014

MAT 101 - Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all six** [6] questions.

*[**Arahan:** Jawab **semua enam** [6] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Suppose $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ 2 + x & , \quad 0 \le x \le 2 \\ \dfrac{|x|}{x} + a & , \quad x < 0 \end{cases}$.

(i) Find $\lim_{x \to 2} f(x)$.

(ii) Why is f continuous at 2?

(iii) What should be the value of a if f is continuous at 0?

[9 marks]

(b) Find the following limit if it exists.

(i) $\lim_{t \to 0} \dfrac{\sin t^2}{\sin 2t}$

(ii) $\lim_{x \to \infty} (\sqrt{x^2 + x} - x)$

[16 marks]

1. (a) Andaikan $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ 2 + x & , \quad 0 \le x \le 2 \\ \dfrac{|x|}{x} + a & , \quad x < 0 \end{cases}$.

(i) Cari $\lim_{x \to 2} f(x)$.

(ii) Kenapa f *selanjar pada* 2 *?*

(iii) Apakah nilai a *yang sepatutnya jika* f *adalah selanjar pada* 0 *?*

[9 markah]

(b) Cari had yang berikut jika ia wujud.

(i) $\lim_{t \to 0} \dfrac{\sin t^2}{\sin 2t}$

(ii) $\lim_{x \to \infty} (\sqrt{x^2 + x} - x)$

[16 markah]

2. (a) (i) Show that $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$.

(ii) Suppose $f(x) = \begin{cases} x^3 \sin\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$. Using the definition of derivative, determine whether $f'(0)$ exists.

[15 marks]

(b) Suppose f is continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=0$ and $f'(x)>0$ for all $x\in(a,b)$, prove that $f(x)>0$ for all $x\in(a,b]$.

[10 marks]

2. (a) (i) Tunjukkan bahawa $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$.

(ii) Andaikan bahawa $f(x) = \begin{cases} x^3 \sin\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$. *Dengan menggunakan takrif terbitan, tentukan sama ada* $f'(0)$ *wujud.*

[15 markah]

(b) Andaikan f *adalah selanjar pada* $[a,b]$ *dan terbezakan pada* (a,b). *Jika* $f(a)=0$ *dan* $f'(x)>0$ *bagi semua* $x\in(a,b)$ *, buktikan bahawa* $f(x)>0$ *bagi semua* $x\in(a,b]$.

[10 markah]

3. (a) Prove $\lim_{x\to 1}(3x+1)=4$ using the ε - δ -definition.

[9 marks]

(b) Find the derivative of the function. **Do not simplify your answer.**

(i) $y=(\sqrt{x^2+1})\sin x$

(ii) $y=\tan^5(\ln(e^x+1))$

[6 marks]

(c) Write down the correct unique answer.

(i) Suppose $f:\mathbf{R}\to\mathbf{R}$. If $f(-2)\neq f(2)$, then f is not an odd function. True or false?
Ans: True / False

(ii) Suppose neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists. Can $\lim_{x\to a}[f(x)+g(x)]$ exist?
Ans: Can / Cannot

(iii) If $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ exists, then $\lim_{x\to a} f(x)$ exists. True or false?
Ans: True / False

(iv) The Fermat's Theorem says that if f has a local maximum or local minimum at c, then $f'(c)=0$. True or false?
Ans: True / False

(v) Are the following two statements equivalent?

- For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\delta < \varepsilon$.
- There exists $\delta > 0$ such that for every $\varepsilon > 0$, we have $\delta < \varepsilon$.

Ans: Yes / No

[10 marks]

3. (a) *Buktikan bahawa* $\lim_{x \to 1}(3x+1) = 4$ *dengan menggunakan takrif* ε - δ.

[9 markah]

(b) *Cari terbitan bagi fungsi yang berikut.* ***Jangan permudahkan jawapan anda.***

(i) $y = \sqrt{x^2+1}\sin x$

(ii) $y = \tan^5(\ln(e^x+1))$

[6 markah]

(c) *Tulis jawapan unik yang betul.*

(i) *Andaikan* $f : \mathbf{R} \to \mathbf{R}$. *Jika* $f(-2) \neq f(2)$, *maka* f *bukan fungsi ganjil. Benar atau palsu?*

Jawapan: Benar / Palsu

(ii) *Andaikan kedua-dua* $\underset{x \to a}{had}\, f(x)$ *dan* $\underset{x \to a}{had}\, g(x)$ *tidak wujud. Bolehkah* $\underset{x \to a}{had}[f(x)+g(x)]$ *wujud?*

Jawapan: Boleh / Tidak Boleh

(iii) *Jika* $\underset{h \to 0}{had}\dfrac{f(a+h)-f(a)}{h}$ *wujud, maka* $\underset{x \to a}{had}\, f(x)$ *wujud. Benar atau palsu?*

Jawapan: Benar / Palsu

(iv) *Teorem Fermat menyatakan bahawa jika* f *mempunyai maksimum setempat atau minimum setempat pada* c, *maka* $f'(c) = 0$. *Benar atau palsu?*

Jawapan: Benar / Palsu

(v) *Adakah dua kenyataan berikut bersamaan?*

- *Bagi setiap* $\varepsilon > 0$, *wujud* $\delta > 0$ *di mana* $\delta < \varepsilon$.
- *Wujud* $\delta > 0$ *di mana bagi setipa* $\varepsilon > 0$, *kita ada* $\delta < \varepsilon$.

Jawapan: Ya / Tidak

[10 markah]

4. (a) (i) If $\int_0^7 f(x)dx = 10,\quad \int_4^7 f(x)dx = 1,$ and $\int_0^1 f(x)dx = 4.$ Compute $\int_1^4 2f(x)dx.$

(ii) If $F(x) = (\cos x)\int_1^{2x} \frac{\cos 2t}{t}dt,$ find $F'(\pi).$

[13 marks]

(b) (i) Integrate $\int \frac{x}{e^{3x}}dx$.

(ii) Let f be a continuous function on the interval $[0,5]$, and it satisfies $\int_1^5 f(x)dx = 6.$ With this information evaluate $\int_0^2 t\, f(t^2+1)dt.$

[12 marks]

4. (a) (i) Jika $\int_0^7 f(x)dx = 10,\quad \int_4^7 f(x)dx = 1,$ *dan* $\int_0^1 f(x)dx = 4.$ *Hitung* $\int_1^4 2f(x)dx.$

(ii) Jika $F(x) = (\cos x)\int_1^{2x} \frac{\cos 2t}{t}dt,$ *cari* $F'(\pi).$

[13 markah]

(b) (i) Kamirkan $\int \frac{x}{e^{3x}}dx$.

(ii) Biar f *suatu fungsi selanjar pada selang* $[0,5]$, *dan ia memenuhi* $\int_1^5 f(x)dx = 6.$ *Dengan maklumat ini nilaikan* $\int_0^2 t\, f(t^2+1)dt.$

[12 markah]

5. (a) The region bounded by $y = 1 - x^4$ and $y = 0$ is rotated through 360^0 about the vertical axis $x = 2.$ Compute the volume of rotation by using the shell method.

[13 marks]

(b) Find the arc length of the curve $y = 1 - x^{\frac{3}{2}}$ which lies above the $x-$axis.

[12 marks]

5. (a) Rantau yang dibendung oleh $y = 1 - x^4$ *dan* $y = 0$ *dikisar melalui* 360^0 *pada paksi tegak* $x = 2.$ *Hitung isipadu kisaran denagn menguna kaedah silinder.*

[13 markah]

(b) Cari panjang lengkuk graf $y = 1 - x^{\frac{3}{2}}$ *yang terletak sebelah atas paksi* $-x.$

[12 markah]

6. (a) Evaluate the following integrals

(i) $\int_0^{\frac{\pi}{4}} (1+\tan^2 x+\sin x)dx.$

(ii) $\int_{-1}^{1} \frac{x^3}{x^2-4}dx.$

[12 marks]

(b) (i) For $x>0$ and $f(x)=\sqrt{1+x}$, using Mean Value Theorem show that there is $c\in(0,x)$ such that $\frac{\sqrt{1+x}-1}{x}=\frac{1}{2\sqrt{1+c}}$.

(ii) From part b(i) deduce that $\sqrt{1+x}\le 1+\frac{1}{2}x$ for $x>0$.

(iii) Using part b(ii), show that $\int_0^1 \sqrt{1+x^2}\ dx \le \frac{7}{6}.$

[13 marks]

6. *(a) Nilaikan kamiran berikut.*

(i) $\int_0^{\frac{\pi}{4}} (1+\tan^2 x+\sin x)dx.$

(ii) $\int_{-1}^{1} \frac{x^3}{x^2-4}dx.$

[12 markah]

(b) (i) Untuk $x>0$ *dan* $f(x)=\sqrt{1+x}$, *dengan Teorem Nilai Min tunjukkan terdapat* $c\in(0,x)$ *supaya* $\frac{\sqrt{1+x}-1}{x}=\frac{1}{2\sqrt{1+c}}$.

(ii) Dari bahagian b(i) simpulkan $\sqrt{1+x}\le 1+\frac{1}{2}x$ *untuk* $x>0$.

(iii) Dari bahagian b(ii) tunjukkan $\int_0^1 \sqrt{1+x^2}\ dx \le \frac{7}{6}.$

[13 markah]

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