UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MGM 562 – Probability Theory
[Teori Kebarangkalian]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all seven** [7] questions.

*[**Arahan:*** *Jawab **semua tujuh** [7] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. X has the probability mass function with the domains of 0, 1, 2, 3 and 4. The moment generating function of this distribution is given as follows:

$$M_X(t) = \frac{1}{15} + \frac{2}{15}e^t + \frac{3}{15}e^{2t} + \frac{4}{15}e^{3t} + \frac{5}{15}e^{4t}$$

a) Derive the mean and the variance of this distribution

[10 marks]

b) Hence, find the probability mass function of this distribution

[3 marks]

1. X mempunyai fungsi jisim kebarangkalian dengan domain-domain 0, 1, 2, 3 dan 4. Fungsi penjana momen bagi taburan ini diberikan seperti berikut:

$$M_X(t) = \frac{1}{15} + \frac{2}{15}e^t + \frac{3}{15}e^{2t} + \frac{4}{15}e^{3t} + \frac{5}{15}e^{4t}$$

a) Dapatkan min dan varians bagi taburan ini

[10 markah]

b) Oleh itu, cari fungsi jisim kebarangkalian bagi taburan ini

[3 markah]

2. Anis is considering in participating two consecutive weekly competitions. Both competitions have prize worth RM 2,000 each. Losing each competition will gain nothing. She feels that she is 70 percent confident to win the first week competition. If she wins the first competition, her confidence of winning the second week increases to 80 percent. If she loses the first, her confidence decreases to 50 percent.

a) Construct a tree diagram viewing the possible total prizes of Anis in entering the two competitions.

[3 marks]

b) Find the first moment and second moment of the total prizes (in RM).

[4 marks]

c) Using the Markov's inequality, find the highest probability of her winning the total prizes of more than or equal to RM 3,500 from both competitions.

[3 marks]

2. *Anis sedang mempertimbangkan untuk menyertai dua pertandingan mingguan berturut-turut. Kedua-dua pertandingan itu mempunyai hadiah bernilai RM 2,000 setiap satu. Kalah dalam setiap pertandingan tidak akan menerima apa-apa hadiah. Beliau berasa bahawa beliau yakin 70 peratus memenangi pertandingan pada minggu pertama. Jika beliau menang pertandingan yang pertama, keyakinan beliau untuk memenangi pertandingan pada minggu kedua meningkat kepada 80 peratus. Jika beliau kalah pada pertandingan pertama, keyakinannya menurun kepada 50 peratus.*

   a) *Bina suatu gambarajah pokok memperihalkan jumlah hadiah yang mungkin bagi Anis menyertai dua pertandingan tersebut.*

   [3 markah]

   b) *Cari momen pertama dan momen kedua bagi hadiah (dalam RM).*

   [4 markah]

   c) *Dengan menggunakan ketaksamaan Markov, cari kebarangkalian tertinggi bagi beliau memenangi jumlah hadiah lebih daripada RM 3,500 bagi kedua-dua pertandingan tersebut.*

   [3 markah]

3. Let $f_X(x) = \dfrac{5^x e^{-5}}{x!}$, for $x = 0,1,2,\ldots$. Let $Y = 5X + 9$.

   a) Find the moment generating function of Y.

   [3 marks]

   b) Find $E\left(Y^3\right)$.

   [9 marks]

3. *Biarkan* $f_X(x) = \dfrac{5^x e^{-5}}{x!}$, *bagi* $x = 0,1,2,\ldots$. *Biarkan* $Y = 5X + 9$.

   a) *Cari fungsi penjana momen bagi* Y.

   [3 markah]

   b) *Cari* $E\left(Y^3\right)$.

   [9 markah]

4. Find $P(3 \le X < 6 \mid X \ge 2)$ when X has the moment generating function as follows:

a) $M_X(t) = \left[0.5\left(e^t + 1\right)\right]^8$

[6 marks]

b) $M_X(t) = e^{\left(4t + 8t^2\right)}$

[6 marks]

4. Cari $Kb(3 \le X < 6 \mid X \ge 2)$ apabila X mempunyai fungsi penjana momen seperti berikut:

a) $M_X(t) = \left[0.5\left(e^t + 1\right)\right]^8$

[6 markah]

b) $M_X(t) = e^{\left(4t + 8t^2\right)}$

[6 markah]

5. Given $X_1, \ldots, X_{20}$ are independent and identically distributed, having exponential distribution with mean 0.5. Also, let $S = X_1 + \ldots + X_{20}$.

a) Find the moment generating function of S.

[5 marks]

b) Calculate mean and variance of S.

[6 marks]

c) Using Chebychev's inequality, find the highest probability of $P\left[\left|S - E(S)\right| \ge 10\right]$.

[5 marks]

5. Diberi $X_1, \ldots, X_{20}$ adalah tertabur secara tidak bersandar dan secaman dan mempunyai taburan eksponen dengan min 0.5. Juga, biarkan $S = X_1 + \ldots + X_{20}$.

a) Cari fungsi penjana momen bagi S.

[5 markah]

b) Kira min dan varians bagi S.

[6 markah]

c) Dengan menggunakan ketaksamaan Chebychev, cari kebarangkalian tertinggi bagi $Kb\left[\left|S - E(S)\right| \ge 10\right]$.

[5 markah]

6. Let A, B and C be independent random variables and having the respective moment generating functions as follows:

$$M_A(t) = \exp\left(t + t^2\right)$$
$$M_B(t) = \exp\left(2t + 2t^2\right)$$
$$M_C(t) = \exp\left(3t + 3t^2\right)$$

Let $X = A + B + C$, and skewness coefficient for X, γ is defined as $\gamma = E\left[(X-\mu)^3\right]/\sigma^3$, where μ is the mean and σ is the standard deviation of X,

a) By using moment generating function, calculate $E\left(X^k\right)$ for $k = 1,2,3$.

[10 marks]

b) Show that, $\gamma = 0$.

[4 marks]

c) Determine the distribution of X.

[2 marks]

6. *Biarkan* A, B *dan* C *masing-masing merupakan pembolehubah- pembolehubah tidak bersandar dan mempunyai fungsi penjana momen seperti berikut:*

$$M_A(t) = \text{eks}\left(t + t^2\right)$$
$$M_B(t) = \text{eks}\left(2t + 2t^2\right)$$
$$M_C(t) = \text{eks}\left(3t + 3t^2\right)$$

Biarkan $X = A + B + C$, *dan pekali kepencongan bagi* X, γ *ditakrifkan sebagai* $\gamma = E\left[(X-\mu)^3\right]/\sigma^3$, *yang mana* μ *adalah min dan* σ *adalah sisihan piawai bagi* X.

a) *Dengan menggunakan fungsi penjana momen, kira* $E\left(X^k\right)$ *bagi* $k = 1,2,3$.

[10 markah]

b) *Tunjukkan bahawa,* $\gamma = 0$.

[4 markah]

c) *Tentukan taburan bagi* X.

[2 markah]

7. Let the variable of X and Y has a function of

$$f(x,y)=\begin{cases} kxy & x=1,2,3 \text{ and } 1\le y\le x \\ 0 & \text{otherwise.} \end{cases}$$

a) Find k so that the above function be a jointly random variable.

[3 marks]

b) Find the marginal distribution of X and Y, respectively.

[9 marks]

c) What is $Cov(X,Y)$?

[9 marks]

7. Biarkan pembolehubah X dan Y mempunyai fungsi

$$f(x,y)=\begin{cases} kxy & x=1,2,3 \text{ dan } 1\le y\le x \\ 0 & \text{selainnya.} \end{cases}$$

a) Cari k supaya fungsi di atas menjadi suatu taburan rawak tercantum .

[3 markah]

b) Cari taburan sut masing-masing bagi X dan Y .

[9 markah]

c) Apakah $Kov(X,Y)$?

[9 markah]

Appendix

Random Variable, X	Probability distribution function, $f_X(x)$	Mean, $E(X)$	Variance, $Var(X)$	Moment Generating Function, $M_X(t) = E\left(e^{tX}\right)$
$bin(n,p)$	$\binom{n}{x} p^x (1-p)^{n-x}$, $x = 0,1,\ldots,n$	np	$np(1-p)$	$\left(pe^t + 1 - p\right)^n$
$Poisson(\lambda)$	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0,1,2,\ldots$	λ	λ	$\exp\left[\lambda\left(e^t - 1\right)\right]$
$NB(r,p)$	$\binom{x+r-1}{x} p^r (1-p)^x$, $x = 0,1,2,\ldots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$
$U(a,b)$	$\frac{1}{b-a}$, $a < x < b$	$\frac{a+b}{2}$	$\frac{(a-b)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$
$N\left(\mu,\sigma^2\right)$	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}(x-\mu)^2\right]$, $-\infty < x < \infty$	μ	σ^2	$\exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$
$\exp(\theta)$	$\frac{1}{\theta}e^{-x/\theta}$, $x > 0$	θ	θ^2	$(1-\theta t)^{-1}$
$Gamma(\alpha,\theta)$	$\frac{1}{\Gamma(\alpha)}\frac{x^{\alpha-1}}{\theta^\alpha}\exp(-x/\theta)$, $x > 0$	$\alpha\theta$	$\alpha\theta^2$	$(1-\theta t)^{-\alpha}$
$Beta(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, $0 \le x \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	

Binomial Expansion:-

$$(a+b)^n = \sum_{x=0}^{n} {}^nC_x a^x b^{n-x}$$

Taylor's Series:-

$$g(x) = \sum_{m=0}^{\infty} \frac{g^{(m)}(x_0) \times (x-x_0)^m}{m!}$$

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