UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

...2/-

1. (a) Let $A=\begin{pmatrix} 1 & -1 & -1 & -1 & \cdots & -1 \\ 2 & 2 & -2 & -2 & \cdots & -2 \\ 3 & 3 & 3 & -3 & \cdots & -3 \\ \vdots & \vdots & & \ddots & & \vdots \\ n-1 & n-1 & \cdots & \cdots & n-1 & -n+1 \\ n & n & \cdots & \cdots & \cdots & n \end{pmatrix}_{n\times n}.$

Show that $\det A = \prod_{i=2}^{n} 2i$.

[70 marks]

(b) Solve the following system of linear equations

$$\begin{aligned} 2x + 3y - z &= -2 \\ x - 3y + z &= -1 \\ -x + 4y + 3z &= 1 \end{aligned}$$

[50 marks]

2. (a) Given $T:\mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation over $\mathbb{R}$.
Let $T_{\alpha,\beta}=\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ where $\alpha=\{(1,1),(1,0)\}$ and $\beta=\{(1,-1),(0,1)\}$ are 2 ordered bases of $\mathbb{R}^2$. Find the definition of *T*.

[80 marks]

(b) Let $T:P_2(\mathbb{C})\to P_2(\mathbb{C})$ be a linear transformation over $\mathbb{C}$ such that $(a_0+a_1x+a_2x^2)T=(a_0-a_1)+(a_1-a_2)x+a_2x^2$. Find $T_{\alpha,\beta}$ where $\alpha=\{1,ix,x^2\}$ and $\beta=\{i,x,ix^2\}$ are 2 ordered bases of $P_2(\mathbb{C})$.

[40 marks]

3. Let $V=\mathbb{R}^n$

(a) Show that *V* cannot be a vector space over $\mathbb{C}$.

[40 marks]

(b) Show that $V_{\mathbb{C}}=\mathbb{C}^n$.

[60 marks]

.../3-

1. (a) *Biar* $A = \begin{pmatrix} 1 & -1 & -1 & -1 & \cdots & -1 \\ 2 & 2 & -2 & -2 & \cdots & -2 \\ 3 & 3 & 3 & -3 & \cdots & -3 \\ \vdots & \vdots & & \ddots & & \vdots \\ n-1 & n-1 & \cdots & \cdots & n-1 & -n+1 \\ n & n & \cdots & \cdots & \cdots & n \end{pmatrix}_{n\times n.}$

Tunjukkan $\det A = \prod_{i=2}^{n} 2i$

[70 markah]

(b) *Selesaikan sistem persamaan linear yang berikut*

$$\begin{aligned} 2x + 3y - z &= -2 \\ x - 3y + z &= -1 \\ -x + 4y + 3z &= 1 \end{aligned}$$

[50 markah]

2. (a) *Diberi* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *ialah suatu transformasi linear atas* $\mathbb{R}$.
Biar $T_{\alpha,\beta} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, *dengan* $\alpha = \{(1,1),(1,0)\}$ dan $\beta = \{(1,-1),(0,1)\}$ *adalah 2 asas bertertib bagi* $\mathbb{R}^2$. *Cari takrifan bagi T.*

[80 markah]

(b) *Biar* $T : P_2(\mathbb{C}) \to P_2(\mathbb{C})$ *suatu transformasi linear atas* $\mathbb{C}$ *sedemikian hingga* $(a_0 + a_1x + a_2x^2)T = (a_0 - a_1) + (a_1 - a_2)x + a_2x^2$. *Cari* $T_{\alpha,\beta}$ *dengan* $\alpha = \{1, ix, x^2\}$ dan $\beta = \{i, x, ix^2\}$ *merupakan 2 asas bertertib bagi* $P_2(\mathbb{C})$.

[40 markah]

3. *Biar* $V = \mathbb{R}^n$

(a) *Tunjukkan V bukan suatu ruang vektor atas* $\mathbb{C}$.

[40 markah]

(b) *Tunjukkan* $V_{\mathbb{C}} = \mathbb{C}^n$.

[60 markah]

.../4-

4. Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

(i) Explain whether A can be digonalised over $\mathbb{R}$ are not.

[100 marks]

(ii) Find A^{1000}

[60 marks]

5. Let $x = (a_1, a_2, \ldots, a_n)$, $y = (b_1, b_2, \ldots, b_n) \in \mathbb{R}^n$. Define $< x, y >= \sum_{i=1}^{n} a_i b_i$.

(a) Show that $<,>$ is an inner product of $\mathbb{R}^n$.

[50 marks]

(b) Let $A = (a_{ij})$, $B = (b_{ij}) \in M_{2\times 2}(\mathbb{R})$. The inner product of $M_{2\times 2}(\mathbb{R})$ is defined as follows:

$$\langle A, B \rangle = \sum_{j=1}^{2} \sum_{i=1}^{2} a_{ij} b_{ij}$$

(i) Let $\alpha = \left\{ \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \right\}$. Show that α is a basis of $M_{2\times 2}(\mathbb{R})$.

[50 marks]

(ii) Using Gram-Schmidt process, find an orthornormal basis of $M_{2\times 2}(\mathbb{R})$ from α.

[100 marks]

.../5-

4. *Biar* $A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

(i) *Jelaskan sama ada A boleh diperpenjurukan atas* $\mathbb{R}$.

[100 markah]

(ii) *Cari* A^{1000}

[60 markah]

5. *Biar* $x = (a_1, a_2, \ldots, a_n)$, $y = (b_1, b_2, \ldots, b_n) \in \mathbb{R}^n$. *Takrifkan* $< x, y > = \sum_{i=1}^{n} a_i b_i$.

(a) *Tunjukkan* $<,>$ *ialah suatu hasil darab terkedalam bagi* $\mathbb{R}^n$.

[50 markah]

(b) *Biar* $A = (a_{ij})$, $B = (b_{ij}) \in M_{2\times2}(\mathbb{R})$. *Hasil darab terdalam bagi* $M_{2\times2}(\mathbb{R})$ *adalah ditakrif seperti berikut:*

$$\langle A, B \rangle = \sum_{j=1}^{2} \sum_{i=1}^{2} a_{ij} b_{ij}$$

(i) *Biar* $\alpha = \left\{ \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \right\}$. *Tunjukkan* α *adalah suatu asas bagi* $M_{2\times2}(\mathbb{R})$.

[50 markah]

(ii) *Dengan menggunakan proses Gram-Schmidt, cari suatu asas ortonormal bagi* $M_{2\times2}(\mathbb{R})$ *daripada* α.

[100 markah]

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