UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MAT 122 – DIFFERENTIAL EQUATIONS I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>SIX</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>ENAM</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>: Answer **FIVE** (5) questions.

*[**<u>Arahan</u>:** Jawab **<u>LIMA</u>**(5) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. The rate of change of population for a certain species is given by the differential equation $\frac{dp}{dt} = -0.5(p-50)(100-p)$.

(a) Without solving the equation, find

(i) $\lim_{t\to\infty} p(t)$, with initial value $p(0) = 60$.

(ii) $\lim_{t\to\infty} p(t)$, with initial value $p(0) = 40$.

(iii) $p(100)$ if $p(0) = 100$.

(b) Solve the problems in part (a) by solving the differential equation.

[20 marks]

1. Kadar perubahan populasi suatu spesis tertentu diberikan oleh persamaan pembezaan $\frac{dp}{dt} = -0.5(p-50)(100-p)$.

(a) Tanpa menyelesaikan persamaan tersebut, cari

(i) $\underset{t\to\infty}{had}\ p(t)$, dengan nilai awal $p(0) = 60$.

(ii) $\underset{t\to\infty}{had}\ p(t)$, dengan nilai awal $p(0) = 40$.

(iii) $p(100)$ jika $p(0) = 100$.

(b) Selesaikan masalah dalam bahagian (a) dengan menyelesaikan persamaan pembezaan.

[20 markah]

2. (a) Consider the initial value problem $y' = x + y,\ y(0) = 1$. Using Euler's method with step size 0.5, approximate the value of y at $x = 2$.

(b) Given an initial value problem $ty' = \frac{1}{y+2},\ y(1) = 0,\ y$ is a function of t, find the solution and determine the interval where the solution is defined.

(c) The growth of two competing species are related by a system of differential equations

$$\frac{dx}{dt} = x - 2y,$$
$$\frac{dy}{dt} = -2x + y.$$

(i) Show that $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 0.$

(ii) Find the general solution of $x(t)$.

[20 marks]

2. (a) Pertimbangkan masalah nilai awal $y' = x + y,\ y(0) = 1$. Guna kaedah Euler dengan saiz langkah 0.5 untuk mendapatkan nilai hampir y pada $x = 2$.

(b) Diberi suatu masalah nilai awal $ty' = \frac{1}{y+2},\ y(1) = 0,\ y$ suatu fungsi dalam sebutan t, cari penyelesaiannya dan tentukan selang agar penyelesaian tertakrif.

(c) Pertambahan dua spesis yang bersaing dikait oleh suatu sistem persamaaan pembezaan

$$\frac{dx}{dt} = x - 2y,$$
$$\frac{dy}{dt} = -2x + y.$$

(i) Tunjukkan $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 0.$

(ii) Cari penyelesaian am $x(t)$.

[20 markah]

…4/-

3. (a) Show that the differential equation

$$\left(2y^2x-2y^3\right)dx+\left(4y^3-6y^2x+2yx^2\right)dy=0$$

is exact and then find the general solution.

(b) The body of a dead man was discovered at $11.00\,pm$. A doctor arrived at $11.30\,pm$ and he took the temperature of the body which read $94.6^0 F$. An hour later he took the temperature again, it was $93.4^0 F$. The doctor also noted that the room temperature is $70^0 F$. Using the Newton's law of cooling determine the time the man was dead assuming that at the time of death the temperature of the body is $96.4^0 F$.

[20 marks]

3. (a) *Tunjukkan persamaan pembezaan*

$$\left(2y^2x-2y^3\right)dx+\left(4y^3-6y^2x+2yx^2\right)dy=0$$

adalah tepat, dan seterusnya dapatkan penyelesaian amnya..

(b) *Mayat seoarng lelaki telah ditemui pada* $11.00\,\text{malam}$. *Seorang doktor tiba pada* $11.30\ \text{malam}$ *mengambil suhu mayat tersebut dengan bacaan* $94.6^0 F$. *Selepas satu jam dia mengambil suhunya semula, ia adalah* $93.4^0 F$. *Doktor juga mendapati suhu bilik adalah* $70^0 F$. *Menggunakan hukum penyejukkan Newton, tentukan waktu lelaki tersebut mati dengan mengandaikan suhu badan ketika lelaki itu mati adalah* $98.6^0 F$..

[20 markah]

…5/-

4. (a) Find the general solution of the differential equation $y''-2y'-3y=e^{2x}$.

(b) Solve the initial value problem $y'+2(x+1)y^2=0,\ y(0)=-\frac{1}{8}$ where y is a function of $x,$ and indicates the interval where the solution is valid.

(c) Find the general solution to the differential equation $x^2y''-5xy'+9y=x,\ x\neq 0,$ by substitution $z=\ln x.$

[20 marks]

4. (a) *Dapatkan penyelesaian am persamaan pembezaan* $y''-2y'-3y=e^{2x}$.

(b) Selesaikan masalah nilai awal $y'+2(x+1)y^2=0,\ y(0)=-\frac{1}{8},$ *y fungsi dalam sebutan* x *dan tentukan selang agar penyelesaian adalah sah.*

(c) Dapatkan penyelesaian am persamaan pembezaan $x^2y''-5xy'+9y=x,\ x\neq 0,$ *dengan penggantian* $z=\ln x.$

[20 markah]

…6/-

5. (a) Find the general solution to a system of differential equation $\boldsymbol{X}'=\begin{bmatrix}1 & -4\\ -2 & 3\end{bmatrix}\boldsymbol{X}$.

(b) Consider the differential equation $ty''-2(1+t)y'+(2+t)y=0.$

(i) Show that $y_1(t)=e^t$ is a solution

(ii) Find another solution $y_2(t)$ and then show that $y_1(t)$ and $y_2(t)$ are linearly independent.

(iii) Find the general solution.

[20 marks]

5. (a) *Cari penyelesaian am suatu sistem persamaan pembezaan* $\boldsymbol{X}'=\begin{bmatrix}1 & -4\\ -2 & 3\end{bmatrix}\boldsymbol{X}$.

(b) Pertmbangkan suatu persamaan pembezaan $ty''-2(1+t)y'+(2+t)y=0.$

(i) Tunjukkan $y_1(t)=e^t$ *ialah suatu penyelesaian.*

(ii) Cari penyelasiaan lain $y_2(t)$*, dan seterusnya tunjukkan bahawa* $y_1(t)$ dan $y_2(t)$ *adalah tak bersandar secara linar.*

(iii) Cari penyelesaian am.

[20 markah]

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