UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all seven** [7] questions.

*[**Arahan:** Jawab **semua tujuh** [7] soalan.]*

1. (a) Let $A = \begin{pmatrix} 0 & 0 & 0 & \alpha_4 & \alpha_5 \\ 0 & 0 & 0 & \beta_4 & \beta_5 \\ 0 & 0 & 0 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \end{pmatrix} \in M_{5\times 5}(\mathbb{R})$. Show that $\det(A) = 0$

[60 marks]

(b) Solve the following system of linear equations using Camers rule

$$\begin{array}{rcrcrcr} 3x & + & y & + & z & = & -1 \\ x & - & y & + & z & = & 1 \\ -x & + & y & + & 2z & = & -1 \end{array}$$

[40 marks]

2. Let $V = \{p(x) \in P_3(\mathbb{C}) | p(0) = 0\}$

(i) Show that V is a vector space over $\mathbb{C}$ with $\dim_{\mathbb{C}} V = 3$.

[40 marks]

(ii) Show that V is a vector space over $\mathbb{R}$ with $\dim_{\mathbb{R}} V = 6$.

[60 marks]

3. Let $A = (a_{ij}) \in M_{2\times 2}(\mathbb{R})$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation over $\mathbb{R}$ such that $T_{\alpha,\beta} = A$, where $\alpha = \{v_1, v_2\}$ and $\beta = \{w_1, w_2\}$ are bases of $\mathbb{R}^2$. Find the definition of T.

[100 marks]

4. Let $T : V \to V$ be a linear transformation over F.

(a) Show that T^k has λ^k as an eigen value if λ is an eigen value of T.

[50 marks]

(b) Show that if T is a nilpotent transformation of index n then all eigen value of T must be zero.

[50 marks]

.../3-

1. (a) *Biar* $A = \begin{pmatrix} 0 & 0 & 0 & \alpha_4 & \alpha_5 \\ 0 & 0 & 0 & \beta_4 & \beta_5 \\ 0 & 0 & 0 & a_4 & a_5 \\ b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \end{pmatrix} \in M_{5\times 5}(\mathbb{R})$. *Tunjukkan* $\det(A) = 0$

[60 markah]

(b) *Selesaikan sistem persamaan linear yang berikut dengan menggunakan petua Cramers*

$$\begin{array}{rcrcrcr} 3x & + & y & + & z & = & -1 \\ x & - & y & + & z & = & 1 \\ -x & + & y & + & 2z & = & -1 \end{array}$$

[40 markah]

2. *Biar* $V = \{p(x) \in P_3(\mathbb{C}) | p(0) = 0\}$

(i) *Tunjukkan* V *ialah suatu ruang vector atas* $\mathbb{C}$ *dengan* $\dim_{\mathbb{C}} V = 3$.

[40 markah]

(ii) *Tunjukkan* V *ialah suatu ruang vector atas* $\mathbb{R}$ *dengan* $\dim_{\mathbb{R}} V = 6$.

[60 markah]

3. *Biar* $A = (a_{ij}) \in M_{2\times 2}(\mathbb{R})$. *Biar* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *suatu transformasi linear atas* $\mathbb{R}$ *sedemikian hingga* $T_{\alpha,\beta} = A$, *dengan* $\alpha = \{v_1, v_2\}$ *dan* $\beta = \{w_1, w_2\}$ *ialah asas bagi* $\mathbb{R}^2$. *Cari definasi bagi T.*

[100 markah]

4. *Biar* $T : V \to V$ *merupakan suatu transformasi linear atas F.*

(a) *Tunjukkan* T^k *mempunyai* λ^k *sebagai suatu nilai eigen jika* λ *ialah suatu nilai eigen bagi T.*

[50 markah]

(b) *Tunjukkan jika T ialah suatu transformasi nilpotent berdarjah n maka semua nilai eigen bagi T semestinya kosong.*

[50 markah]

...*/4-*

5. *Biar* $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$

(a) *Tunjukkan A tidak boleh diperpenjurukan*

[35 markah]

(b) *Cari* A^{1000}

(Petunjuk: Anda mungkin perlu cari suatu asas bagi $\mathbb{R}^3$ *untuk menurunkan A ke bentuk berkanun Jordan).*

[65 markah]

6. *Biar* $A = (a_{ij}), B = (b_{ij}) \in M_{n\times m}(\mathbb{R})$.

Takrifkan $A \cdot B = \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}b_{ij}$

(a) *Tunjukkan* '$\cdot$' *yang ditakrifkan di atas adalah suatu hasil darab terkedalaman bagi* $M_{n\times m}(\mathbb{R})$.

[50 markah]

(b) *Biar V ialah suatu ruang hasil darab terkedalaman. Suatu set* $\alpha = \{v_2, v_2, \cdots, v_n\} \subseteq V$ *di namakan sebagai set ortonomal jika dan hanya jika*

$$v_i \cdot v_j = \begin{cases} 0 & \text{jika} \quad i \neq j \\ 1 & \text{jika} \quad i = j \end{cases}.$$

Tunjukkan suatu set ortonomal semestinya set tak bersandar linear.

[50 markah]