UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MAT 102 – ADVANCED CALCULUS
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **NINE** (9) questions.

*[**Arahan:** Jawab **semua sembilan** (9) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. Determine whether the sequence $\left\{ 2 - \dfrac{2}{n} - \dfrac{1}{2^n} \right\}$ is monotonic and bounded.

[7 marks]

1. *Tentukan sama ada jujukan* $\left\{ 2 - \dfrac{2}{n} - \dfrac{1}{2^n} \right\}$ *berekanada dan terbatas.*

[7 markah]

2. Test the convergence of the following series:

    (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-3)^{n-1}}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1+3^n}{1+4^n}$

[12 marks]

2. *Uji penumpuan siri berikut:*

    (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-3)^{n-1}}{\sqrt{n}}$ *(b)* $\displaystyle\sum_{n=1}^{\infty} \dfrac{1+3^n}{1+4^n}$

[12 markah]

3. Find the radius and interval of convergence of power series $\displaystyle\sum_{n=0}^{\infty} \left(\dfrac{\sqrt{x}}{2} - 1 \right)^n$.

[10 marks]

3. *Dapatkan jejari dan selang penumpuan siri kuasa* $\displaystyle\sum_{n=0}^{\infty} \left(\dfrac{\sqrt{x}}{2} - 1 \right)^n$.

[10 markah]

4. Using the power series representation for the function $f(x) = \dfrac{1}{1-2x}$ for $|x| < \dfrac{1}{2}$, find a power series for $h(x) = \dfrac{x^2}{(1-2x)^2}$.

[6 marks]

4. *Dengan menggunakan perwakilan siri kuasa bagi fungsi $f(x) = \dfrac{1}{1-2x}$ untuk $|x| < \dfrac{1}{2}$, dapatkan siri kuasa bagi $h(x) = \dfrac{x^2}{(1-2x)^2}$.*

[6 markah]

5. Evaluate the following integral:

(a) $\displaystyle\int_1^{\infty} \dfrac{|\sin x|}{x^2}\,dx$

(b) $\displaystyle\int_{-1}^{1} \dfrac{dx}{(2x+1)^{\frac{1}{3}}}$

[12 marks]

5. *Nilaikan kamiran berikut:*

(a) $\displaystyle\int_1^{\infty} \dfrac{|\sin x|}{x^2}\,dx$

(b) $\displaystyle\int_{-1}^{1} \dfrac{dx}{(2x+1)^{\frac{1}{3}}}$

[12 markah]

6. (a) If $f(x,y) = \dfrac{xy - y - 2x + 2}{x-1}$, does $\displaystyle\lim_{(x,y)\to(1,1)} f(x,y)$ exist? Justify your answer.

(b) Find the value of $\dfrac{\partial z}{\partial y}$ at point $P(1,1,1)$, if $z^3 - xy + yz + y^3 - 2 = 0$.

[10 marks]

6. (a) *Jika $f(x,y) = \dfrac{xy - y - 2x + 2}{x-1}$, adakah $\displaystyle\lim_{(x,y)\to(1,1)} f(x,y)$ wujud? Jelaskan jawapan anda.*

(b) *Dapatkan nilai $\dfrac{\partial z}{\partial y}$ pada titik $P(1,1,1)$, jika $z^3 - xy + yz + y^3 - 2 = 0$.*

[10 markah]

7. (a) Find the maximum rate of change of $f(x,y,z) = \dfrac{x+y}{z}$ at the point $(1,1,-1)$ and the direction in which it occurs.

(b) Suppose f is a differentiable function of x and y, and $g(u,v) = f(e^u + \sin v, e^u + \cos v)$. Use the table of values below to calculate $g_u(0,0)$ and $g_v(0,0)$.

	f	g	f_x	f_y
$(0,0)$	3	6	4	8
$(1,2)$	6	3	2	5

[13 marks]

7. (a) *Dapatkan kadar perubahan maksimun bagi* $f(x,y,z) = \dfrac{x+y}{z}$ *pada titik* $(1,1,-1)$ *dan arah ia berlaku.*

(b) *Katakan f fungsi terbezakan terhadap x dan y dan* $g(u,v) = f(e^u + \sin v, e^u + \cos v).$ *Guna jadual nilai dibawah untuk menghitung* $g_u(0,0)$ *dan* $g_v(0,0).$

	f	g	f_x	f_y
$(0,0)$	*3*	*6*	*4*	*8*
$(1,2)$	*6*	*3*	*2*	*5*

[13 markah]

8. A closed rectangular box has a volume $27\,cm^3$. Find the smallest surface area of the box.

[15 marks]

8. *Satu kotak segi empat tepat tertutup mempunyai isipadu* $27\,cm^3$. *Dapatkan luas permukaan terkecil kotak.*

[15 markah]

9. Evaluate the following integral.

(a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \cos\left(x^2 + y^2\right) dydx$

(b) $\displaystyle\iiint_E z\, dV$, where E is bounded by the cylinder $y^2 + z^2 = 9$ and the planes

$x = 0,\ y = 3x$ and $z = 0$ in the first octant.

[15 marks]

9. *Nilaikan kamiran berikut.*

(a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \cos\left(x^2 + y^2\right) dydx$

(b) $\displaystyle\iiint_E z\, dV$, *yang mana E dibatasi oleh silinder* $y^2 + z^2 = 9$ *dan satah-satah*

$x = 0,\ y = 3x$ *dan* $z = 0$ *dalam sukuan pertama.*

[15 markah]

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