
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MAT 101 - Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **SIX** (6) questions.

Arahan: Jawab **ENAM** (6) soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Suppose $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ a+1 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \frac{\sin 2x}{x} & , \quad x < 0 \end{cases}$.

(i) Find $\lim_{x \rightarrow 2} f(x)$.

(ii) What should be the value of a if f is continuous at 2?

(iii) Find $\lim_{x \rightarrow 0^-} f(x)$.

[15 marks]

(b) Find $\lim_{x \rightarrow 4} \frac{x-4}{|4-x|}$ if it exists.

[10 marks]

(c) Suppose $\lim_{x \rightarrow a} f(x) = 0$. Using the Squeeze Theorem, show that

$$\lim_{x \rightarrow a} \left([f(x)]^2 \sin \frac{1}{x} \right) = 0.$$

[10 marks]

1. (a) Andaikan $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ a+1 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \frac{\sin 2x}{x} & , \quad x < 0 \end{cases}$

(i) Cari $\lim_{x \rightarrow 2} f(x)$.

(ii) Apakah nilai a yang sepatutnya jika f adalah selanjar pada 2?

(iii) Cari $\lim_{x \rightarrow 0^-} f(x)$.

[15 markah]

(b) Cari $\lim_{x \rightarrow 4} \frac{x-4}{|4-x|}$ jika ia wujud.

[10 markah]

(c) Andaikan $\lim_{x \rightarrow a} f(x) = 0$. Dengan menggunakan Teorem Himpitan,

tunjukkan bahawa $\lim_{x \rightarrow a} \left([f(x)]^2 \sin \frac{1}{x} \right) = 0$.

[10 markah]

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2. (a) $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$ represents the derivative of some function f at some number a .

State such a function f and number a . Then deduce the value of

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h}.$$

[10 marks]

- (b) Find the derivative of the function. **Do not simplify your answer.**

(i) $y = x^e + e$

(ii) $y = \frac{\tan x}{\sqrt{x}}$

(iii) $y = e^{(x^2+1)^5}$

(iv) $y = xe^x \sin x$

(v) $y = \cos(x \ln x)$

[20 marks]

- (c) Suppose $h(x) = f(g(x^2))$, where $f'(1) = 3$, $g(4) = 1$ and $g'(4) = 2$. Find $h'(2)$.

[10 marks]

2. (a) $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$ mewakili terbitan bagi suatu fungsi f pada suatu nombor a .
Nyatakan fungsi f dan nombor a tersebut. Kemudian, deduksikan nilai
bagi $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$.

[10 markah]

- (b) Cari terbitan bagi fungsi yang berikut. **Jangan permudahkan jawapan anda.**

(i) $y = x^e + e$

(ii) $y = \frac{\tan x}{\sqrt{x}}$

(iii) $y = e^{(x^2+1)^5}$

(iv) $y = xe^x \sin x$

(v) $y = \cos(x \ln x)$

[20 markah]

- (c) Andaikan $h(x) = f(g(x^2))$, $f'(1) = 3$, $g(4) = 1$ dan $g'(4) = 2$. Cari
 $h'(2)$.

[10 markah]

3. (a) Suppose $\int_1^7 f(x)dx = 10$, $\int_4^7 f(x)dx = 1$ and $\int_4^1 g(x)dx = 4$. Compute $\int_1^4 f(x) + 3g(x)dx$.

[10 marks]

(b) Find $F'(x)$, where $F(x) = \int_{x^2}^x \sin(t^2) dt$.

[10 marks]

(c) Show that the right Riemann sum R_n for $f(x) = x^2$ from 0 to 1 is $\sum_{i=1}^n \frac{i^2}{n^3}$.

Hence, find $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2}{n^3}$.

[10 marks]

3. (a) Andaikan $\int_1^7 f(x)dx = 10$, $\int_4^7 f(x)dx = 1$ dan $\int_4^1 g(x)dx = 4$. Hitungkan $\int_1^4 f(x) + 3g(x)dx$.

[10 markah]

(b) Cari $F'(x)$. Diberi $F(x) = \int_{x^2}^x \sin(t^2) dt$.

[10 markah]

(c) Tunjukkan bahawa hasil tambah Riemann kanan R_n bagi $f(x) = x^2$ dari 0

ke 1 ialah $\sum_{i=1}^n \frac{i^2}{n^3}$. Justeru itu, cari $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2}{n^3}$.

[10 markah]

4. (a) The region bounded by the graph of $y = x^2$, the horizontal line $y = 4$ and the y -axis is rotated about the vertical line $x = 2$. Compute the volume of the solid obtained using the shell method.

[15 marks]

- (b) Find the following integral.

(i)
$$\int \frac{\sec^2 3x}{(1 + \tan 3x)^2} dx$$

(ii)
$$\int x^2 \ln x dx$$

[20 marks]

4. (a) *Rantau yang dibendung oleh graf bagi $y = x^2$, garis lintang $y = 4$ dan paksi y dikisarkan sekitar garis tegak $x = 2$. Hitungkan isipadu kisanan dengan menggunakan kaedah silinder.*

[15 markah]

- (b) *Cari kamiran yang berikut.*

(i)
$$\int \frac{\sec^2 3x}{(1 + \tan 3x)^2} dx$$

(ii)
$$\int x^2 \ln x dx$$

[20 markah]

5. (a) Prove $\lim_{x \rightarrow 1} (2x - 1) = 1$ using the ε - δ -definition.
[10 marks]
- (b) Show by contradiction that the equation $x^3 + x = 1$ cannot have more than one real roots.
[10 marks]
- (c) Suppose f and g are continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . If $f(a) + g(b) = f(b) + g(a)$, prove that there exists a number $c \in (a, b)$ such that $f'(c) = g'(c)$.
[10 marks]

5. (a) *Buktikan bahawa $\lim_{x \rightarrow 1} (2x - 1) = 1$ dengan menggunakan takrif ε - δ .*
[10 markah]
- (b) *Tunjuk dengan cara percanggahan bahawa persamaan $x^3 + x = 1$ tidak boleh mempunyai lebih daripada satu punca nyata.*
[10 markah]
- (c) *Andaikan f dan g adalah selanjar pada selang tertutup $[a, b]$ dan terbezakan pada selang terbuka (a, b) . Jika $f(a) + g(b) = f(b) + g(a)$, buktikan bahawa wujud suatu nombor $c \in (a, b)$ supaya $f'(c) = g'(c)$.*
[10 markah]

6. (a) Suppose $y = x^{\cos x}$. Find $\frac{dy}{dx}$ using logarithmic differentiation.

[10 marks]

- (b) Is the following statement true or false? Just write down the correct answer, that is, either “TRUE” or “FALSE”.

- (i) Suppose f is defined everywhere. If $f(-2) \neq f(2)$, then f is not an even function.
- (ii) $\lim_{x \rightarrow \pi} \sin\left(\frac{1}{x}\right)$ does not exist.
- (iii) If $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist, then it cannot be the case that both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist.
- (iv) If a function f is differentiable at a , then $\lim_{x \rightarrow a^+} f(x)$ exists.
- (v) Fermat's Theorem says that if f has a local maximum or local minimum at c , then $f'(c) = 0$.
- (vi) For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\varepsilon = \delta$.
- (vii) If $f(a)$ is undefined, then $\lim_{x \rightarrow a} f(x)$ does not exist.
- (viii) A function f is one-to-one if and only if $f(a) = f(b)$ whenever $a = b$.
- (ix) Suppose f is continuous on an open interval (a, b) . The Extreme Value Theorem tells us that the absolute maximum of f exists.
- (x) If $\lim_{x \rightarrow \infty} f(x) = 5$, then the line $y = 5$ is the horizontal asymptote of the graph of f .

[20 marks]

6. (a) Andaikan $y = x^{\cos x}$. Cari $\frac{dy}{dx}$ dengan menggunakan cara pembezaan logaritma.

[10 markah]

- (b) Adakah kenyataan berikut benar atau palsu? Cuma tuliskan jawapan yang betul, iaitu sama ada “BENAR” atau “PALSU”.

(i) Andaikan f tertakrif di semua tempat. Jika $f(-2) \neq f(2)$, maka f bukan suatu fungsi genap.

(ii) $\lim_{x \rightarrow \pi} \sin\left(\frac{1}{x}\right)$ tidak wujud.

(iii) Jika $\lim_{x \rightarrow a} [f(x) + g(x)]$ tidak wujud, maka tidak mungkin berlakunya bahawa kedua-dua $\lim_{x \rightarrow a} f(x)$ dan $\lim_{x \rightarrow a} g(x)$ wujud.

(iv) Jika suatu fungsi f terbezakan pada a , maka $\lim_{x \rightarrow a^+} f(x)$ wujud.

(v) Teorem Fermat mengatakan bahawa jika f mempunyai maksimum tempatan atau minimum tempatan pada c , maka $f'(c) = 0$.

(vi) Bagi setiap $\varepsilon > 0$, wujud $\delta > 0$ supaya $\varepsilon = \delta$.

(vii) Jika $f(a)$ tidak tertakrif, maka $\lim_{x \rightarrow a} f(x)$ tidak wujud.

(viii) Suatu fungsi f adalah satu-satu jika dan hanya jika $f(a) = f(b)$ bila-bila $a = b$.

(ix) Andaikan f adalah selanjar pada selang terbuka (a, b) . Teorem Nilai Lampau memberitahu kita bahawa maksimum mutlak bagi f wujud.

(x) Jika $\lim_{x \rightarrow \infty} f(x) = 5$, maka garis $y = 5$ adalah asimptot melintang graf bagi f .

[20 markah]