UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

# MAA 111 – Algebra for Science Students
## *[Aljabar untuk Pelajar Sains]*

Duration : 3 hours
*[Masa : 3 jam]*

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

**Instructions**: Answer **NINE** (9) questions.

*[**Arahan**: Jawab **SEMBILAN** (9) soalan.]*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. If $A$ is a non-singular matrix and $\left(5A^{T}\right)^{-1}=\begin{bmatrix}-3 & -1\\ 5 & 2\end{bmatrix}$, find $A$.

[ 5 marks ]

*1. Jika $A$ merupakan matriks bukan singular dan $\left(5A^{T}\right)^{-1}=\begin{bmatrix}-3 & -1\\ 5 & 2\end{bmatrix}$, dapatkan $A$.*

*[ 5 markah ]*

2. Write $B=\begin{bmatrix}0 & 0 & 1\\ 0 & 2 & 4\\ 1 & 2 & 3\end{bmatrix}$ as a product of elementary matrices.

[ 7 marks ]

*2. Tuliskan $B=\begin{bmatrix}0 & 0 & 1\\ 0 & 2 & 4\\ 1 & 2 & 3\end{bmatrix}$ sebagai hasildarab matriks permulaan.*

*[ 7 markah ]*

3. Find all $2\times 2$ diagonal matrices $A$ that satisfy the equation $A^{2}-3A+2I=\mathbf{0}$.

[ 9 marks ]

*3. Dapatkan semua $2\times 2$ matriks pepenjuru $A$ yang memenuhi persamaan $A^{2}-3A+2I=\mathbf{0}$.*

*[ 9 markah ]*

4. Use Cramer's rule to solve

$$2z+3 = y+3x$$
$$x-z = 2y+1$$
$$3y+z = 2-2x$$

[ 10 marks ]

*4. Gunakan petua Cramer untuk menyelesaikan*

$$2z+3 = y+3x$$
$$x-z = 2y+1$$
$$3y+z = 2-2x$$

*[ 10 markah ]*

5. Consider the linear system

$$x+2y+2z=1$$
$$x+ay+3z=3$$
$$x+11y+az=b$$

Find,

(a) all values of $a$ so that the system has a unique solution.

(b) the pairs of values $(a,b)$ so that the system has more than one solution.

[ 15 marks ]

*5. Pertimbangkan sistem linear*

$$x+2y+2z=1$$
$$x+ay+3z=3$$
$$x+11y+az=b$$

*Dapatkan,*

*(a) semua nilai a supaya sistem mempunyai penyelesaian unik.*

*(b) pasangan nilai* $(a,b)$ *supaya sistem mempunyai lebih daripada satu penyelesaian.*

*[ 15 markah ]*

6. Show that the following vectors set is linearly dependent.

(a) $S = \{ \ (1,-3,2),(1,6,-16),(1,0,-4) \ \}.$

(b) $S = \{ \ 1+x-2x^2, 2+5x-x^2, x+x^2 \ \}.$

[ 6 marks ]

*6. Tunjukkan bahawa set vektor berikut adalah bersandar secara linear.*

*(a)* $S = \{ \ (1,-3,2),(1,6,-16),(1,0,-4) \ \}.$

*(b)* $S = \{ \ 1+x-2x^2, 2+5x-x^2, x+x^2 \ \}.$

*[ 6 markah ]*

7. (a) Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 3 & 5 \\ 1 & 1 & 3 & 4 & 3 \end{bmatrix}$$

Find a basis for the subspace $W \subset R^5$ if $W = \{ \mathbf{v} : A\mathbf{v} = 0, \ \mathbf{v} \in \mathbb{R}^5 \ \}.$

(b) Given

$$S = \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right\}$$

(i) Express $\begin{bmatrix} 1 & -3 \\ 1 & 5 \end{bmatrix}$ as a linear combination vectors in *S*.

(ii) Prove or disprove that *S* is a basis of $M_{2\times2}$ matrix.

(iii) What is the dimension of $M_{2\times2}$ matrix?

[ 14 marks ]

7. *(a)* *Biarkan*

$$A=\begin{bmatrix}1&1&1&1&2\\1&2&2&3&5\\1&1&3&4&3\end{bmatrix}$$

*Dapatkan suatu asas bagi subruang* $W \subset R^5$ *if* $W=\{\mathbf{v} : A\mathbf{v}=0,\ \mathbf{v}\in \mathbb{R}^5\}$.

*(b)* *Diberi*

$$S=\left\{\begin{bmatrix}0&1\\1&0\end{bmatrix}, \begin{bmatrix}-1&0\\0&1\end{bmatrix}, \begin{bmatrix}-1&1\\1&-1\end{bmatrix}, \begin{bmatrix}1&-1\\0&1\end{bmatrix}\right\}$$

*(i)* *Nyatakan* $\begin{bmatrix}1&-3\\1&5\end{bmatrix}$ *sebagai kombinasi linear vektor-vektor dalam S.*

*(ii)* *Buktikan atau sangkalkan bahawa S ialah asas bagi matrik* $M_{2\times 2}$.

*(iii)* *Apakah dimensi* $M_{2\times 2}$ *?*

*[ 14 markah ]*

8. (a) Given $\mathbf{u}=(2,\frac{-1}{2},1)$, and $\mathbf{v}=(\frac{3}{2},2,-1)$. Find the inner product represented by $\langle \mathbf{u},\mathbf{v}\rangle = u_1v_1+2u_2v_2+3u_3v_3$ and hence find the distance between **u** and **v**.

(b) Show that the vectors

$$\mathbf{u}_1=(1,-1,0),\ \mathbf{u}_2=(3,3,0)\ \text{ and }\ \mathbf{u}_3=(0,0,2)$$

form an orthogonal basis for $\mathbb{R}^3$ with using the Euclidean inner product and find the orthonormal basis.

(c) Let $T$ be the linear transformation represented by $T(x,y,z)=(z,x-2y,3x+z)$. Use the standard matrix for $T$ to find $T(\mathbf{x})$ when $\mathbf{x}=(2,1,-3)$.

[ 14 marks ]

8. (a) *Diberi* $\mathbf{u} = (2, \frac{-1}{2}, 1)$, and $\mathbf{v} = (\frac{3}{2}, 2, -1)$. *Dapatkan hasildarab terkedalam yang diwakili oleh by* $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$ *dan seterusnya cari jarak di antara* ***u*** *dan* ***v****.*

(b) *Tunjukkan bahawa vektor-vektor*

$$\mathbf{u}_1 = (1, -1, 0), \quad \mathbf{u}_2 = (3, 3, 0) \text{ and } \mathbf{u}_3 = (0, 0, 2)$$

*membentuk suatu asas berortogon untuk* $\mathbb{R}^3$ *dengan menggunakan hasildarab terkedalam Euclidean dan dapatkan asas berortonormal.*

(c) *Biar* $T$ *suatu transformasi linear yang diwakili oleh* $T(x, y, z) = (z, x - 2y, 3x + z)$. *Gunakan matrik piawai untuk* $T$ *bagi mendapatkan* $T(\mathbf{x})$ *apabila* $\mathbf{x} = (2, 1, -3)$.

*[ 14 markah ]*

9. (a) Let

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

(i) Show that eigenvalues of $A$ are $\lambda = 1, 3$ and $4$.

(ii) Find a basis for the eigenspace corresponding to each eigenvalue.

(iii) Is $A$ diagonalizable? Justify your answer.

(b) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Find the least squares solution of the linear system $A\mathbf{x} = \mathbf{b}$.

[ 20 marks ]

9. (a) *Biar*

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

(i) *Tunjukkan nilai-nilai eigen bagi A ialah* $\lambda = 1, 3$ dan $4$.

(ii) *Dapatkan suatu asas ruang eigen bersesuaian dengan setiap nilai eigen.*

(iv) *Adakah A terpepenjurukan? Jelaskan jawapan anda.*

(b) *Biar*

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} \quad \textit{dan} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

*Dapatkan penyelesaian kuasa dua terkecil bagi sistem linear* $A\mathbf{x} = \mathbf{b}$.

*[ 20 markah ]*

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