UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSS 212– FURTHER LINEAR ALGEBRA
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FIVE** (5) questions.

*[**Arahan:** Jawab **LIMA** (5) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Answer all questions.

1. Suppose that $T:\mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by

$$T(x, y, z) = (x - y + z, x + y + z, x)$$

(i) Show that

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -2 & -2 & 0 \end{bmatrix}$$

is the matrix of T relative to the ordered basis $\{(1,1,1),(1,1,0),(1,0,0)\}$ of P^3.

(ii) Show that if A is the matrix of T relative to the standard basis of P^3, then A is equivalent to B given in (i) by finding invertible matrices P and Q^{-1} such that $A = PBQ^{-1}$.

[20 marks]

1. Andai $T:\mathbb{R}^3 \to \mathbb{R}^3$ *ialah suatu transformasi linear tertakrif dengan*

$$T(x, y, z) = (x - y + z, x + y + z, x)$$

(i) Tunjukkan bahawa

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -2 & -2 & 0 \end{bmatrix}$$

ialah matriks bagi T yang relatif kepada asas $\{(1,1,1),(1,1,0),(1,0,0)\}$ *bagi* P^3.

(ii) Tunjukkan bahawa jika A ialah matrix bagi T relatif kepada asas piawai bagi P^3, *maka A adalah setara dengan B yang diberi dalam (i) dengan mencari suatu matriks yang tersongsangkan P dan* Q^{-1} *sedemikian hingga* $A = PBQ^{-1}$.

[20 markah]

2. (a) Consider two subspaces U and W of a finite-dimensional vector space V.

(i) Is the intersection $U \cap W$ necessarily a subspace of *V*? Justify your answer.

(ii) Is the intersection $U \cup W$ necessarily a subspace of *V*? Justify your answer.

(b) Let $T : \mathbb{C} \to \mathbb{R}^2$ be defined by

$$T(a+bi) = (a,b)$$

Show that *T* is an isomorphism and $\left|T(z)\right|^2 = z\overline{z}$ for all $z = a+bi \in \mathbb{C}$.
Note: $\left|T(z)\right|$ denotes the length of $T(z)$ and $\overline{z}$ denotes the conjugate of *z*.

[20 marks]

2. (a) *Pertimbangkan dua subruang U dan W dari ruang vektor terhingga V.*

(i) Adakah persilangan $U \cap W$ semestinya suatu subruang dari V? Beri justifikasi terhadap jawapan anda.

(ii) Adakah gabungan $U \cup W$ semestinya suatu subruang dari V? Beri justifikasi terhadap jawapan anda.

(b) *Biar $T : \mathbb{C} \to \mathbb{R}^2$ tertakrif dengan*

$$T(a+bi) = (a,b)$$

Tunjukkan bahawa T ialah suatu isomorfisma dan $\left|T(z)\right|^2 = z\overline{z}$ bagi semua $z = a+bi \in \mathbb{C}$.
Catatan: $\left|T(z)\right|$ menandakan panjang $T(z)$ dan $\overline{z}$ menandakan konjugasi bagi z.

[20 markah]

3. (a) Prove that an orthogonal set of nonzero vectors *S* in an inner product space $\left(V, \langle\ ,\ \rangle\right)$ is linearly independent.

(b) Consider the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}$ defined by $T(x, y, z) = x + y + z$.

(i) Find a basis for the kernel of *T*.

(ii) Change the basis in (i) into an orthogonal basis for the kernel of *T*.

(iii) Extend the basis obtained in (ii) to an orthogonal basis for P^3.

[20 marks]

3. *(a) Buktikan bahawa set vektor berortogon bukan-sifar S dalam suatu ruang hasildarab terkedalam* $\left(V, \langle\ ,\ \rangle\right)$ *adalah tak bersandar linear.*

(b) Pertimbangkan transformasi linear $T:\mathbb{R}^3 \to \mathbb{R}$ *tertakrif dengan* $T(x, y, z) = x + y + z$.

(i) Cari asas bagi inti T.

(ii) Tukar asas dalam (i) kepada asas berortogon bagi inti T.

(iii) Lanjutkan asas yang diperoleh dalam (ii) kepada asas berortogon bagi P^3.

[20 markah]

4. (a) Determine whether $T:\mathbb{R}^2 \to \mathbb{R}^4$ defined by

$$T(x,y) = \frac{1}{\sqrt{2}}(x,x,y,y)$$

is an isometric embedding of P^2 in P^4.

(b) Consider the matrix

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 4 & -6 & 4 \end{bmatrix}$$

with characteristic polynomial $\det(M - \lambda I) = (\lambda - 1)^4$.

(i) Show that *M* is not diagonalizable.

(ii) Determine the Jordan Canonical Form of *M*.

[20 marks]

4. *(a) Tentukan sama ada* $T : \mathbb{R}^2 \to \mathbb{R}^4$ *yang tertakrif dengan*

$$T(x,y) = \frac{1}{\sqrt{2}}(x,x,y,y)$$

adalah suatu pembenaman isometrik bagi P^2 *dalam* P^4*.*

(b) Pertimbangkan matriks

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 4 & -6 & 4 \end{bmatrix}$$

dengan polinomial cirian $\det(M - \lambda I) = (\lambda - 1)^4$.

(i) Tunjukkan bahawa M tak terpepenjurukan.

(ii) Tentukan Bentuk Berkanun Jordan bagi M.

[20 markah]

5. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by

$$T(x,y,z) = (3x - z, 2y, -x + 3z).$$

(i) Show that T is self-adjoint.

(ii) Hence, using (i), explain why T is diagonalizable.

(iii) Find an orthonormal basis that will diagonalize T.

[20 marks]

5. *Andai* $T:\mathbb{R}^3 \to \mathbb{R}^3$ *ialah suatu transformasi linear linear tertakrif dengan*

$$T(x,y,z)=(3x-z,2y,-x+3z).$$

(i) Tunjukkan bahawa T ialah swa-dampingan.

(ii) Dengan itu, menggunakan (i), terangkan mengapa T adalah terpepenjurukan.

(iii) Cari suatu asas ortonormal yang mempepenjurukan T.

[20 markah]

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