UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSG 265 – DESIGN AND ANALYSIS OF EXPERIMENTS
[REKABENTUK DAN ANALISIS UJIKAJI]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FOUR** (4) questions.

*[**Arahan**: Jawab **semua empat** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. An experiment is conducted to study the effect of coating machine, operators and material type on the density of a coated fabric. Each of the four operators (Bob, Mary, Sam and Ben) ran each of the two machines in the plant (machines 15 and 21) with both types of material (A and B). Two repeated tests were obtained for each of the process conditions. Data from the study are shown in the table below.

Machine 15			Machine 21		
Operator	Material	Density	Operator	Material	Density
Bob	A	190.1	Bob	A	209.4
Bob	B	182.0	Bob	B	194.2
Mary	A	200.8	Mary	A	224.6
Mary	B	196.0	Mary	B	189.1
Sam	A	205.6	Sam	A	202.1
Sam	B	191.8	Sam	B	183.1
Ben	A	183.5	Ben	A	191.8
Ben	B	170.5	Ben	B	186.7
Bob	A	179.4	Bob	A	216.2
Bob	B	177.8	Bob	B	188.3
Mary	A	191.0	Mary	A	210.1
Mary	B	180.7	Mary	B	186.1
Sam	A	198.3	Sam	A	207.2
Sam	B	206.9	Sam	B	198.0
Ben	A	182.1	Ben	A	184.4
Ben	B	187.5	Ben	B	187.5

The sums of squares are given below:

$SS_{Total} = 4633.475$ $SS_{Machine*Operator} = 530.965$

$SS_{Machine} = 567.845$ $SS_{Machine*Material} = 283.220$

$SS_{Operator} = 1064.545$ $SS_{Operator*Material} = 287.710$

$SS_{Material} = 907.380$ $SS_{Machine*Operator*Material} = 172.690$

(a) State the linear statistical model for this experiment.

(b) Analyze the data and state the conclusion. Use $\alpha = 0.05$.

(c) Analyze the residuals for the model obtained in part (b). (Refer to the Appendix)

(d) Obtain the point estimates of the mean density for each machine and material types.

(e) Assume that the replicates are used as blocking variables. Present the ANOVA table for the full model of the three-factor factorial design. Compare the results obtained in part (e) with part (b).

[100 marks]

1. *Satu eksperimen dijalankan bagi mengkaji kesan mesin saduran, operator dan jenis bahan terhadap kepadatan fabrik yang disadur. Setiap operator (Bob, Mary, Sam dan Ben) melarikan dua mesin (mesin 15 dan 21) dengan dua jenis bahan (A dan B). Uji kaji diulang sebanyak dua kali pada setiap keadaan proses. Data daripada kajian tersebut diberikan dalam jadual di bawah:*

Mesin 15			*Mesin 21*		
Operator	*Bahan*	*Kepadatan*	*Operator*	*Bahan*	*Kepadatan*
Bob	*A*	*190.1*	*Bob*	*A*	*209.4*
Bob	*B*	*182.0*	*Bob*	*B*	*194.2*
Mary	*A*	*200.8*	*Mary*	*A*	*224.6*
Mary	*B*	*196.0*	*Mary*	*B*	*189.1*
Sam	*A*	*205.6*	*Sam*	*A*	*202.1*
Sam	*B*	*191.8*	*Sam*	*B*	*183.1*
Ben	*A*	*183.5*	*Ben*	*A*	*191.8*
Ben	*B*	*170.5*	*Ben*	*B*	*186.7*
Bob	*A*	*179.4*	*Bob*	*A*	*216.2*
Bob	*B*	*177.8*	*Bob*	*B*	*188.3*
Mary	*A*	*191.0*	*Mary*	*A*	*210.1*
Mary	*B*	*180.7*	*Mary*	*B*	*186.1*
Sam	*A*	*198.3*	*Sam*	*A*	*207.2*
Sam	*B*	*206.9*	*Sam*	*B*	*198.0*
Ben	*A*	*182.1*	*Ben*	*A*	*184.4*
Ben	*B*	*187.5*	*Ben*	*B*	*187.5*

Hasil tambah kuasa dua diberikan di bawah:

$SS_{Jumlah} = 4633.475$ $SS_{Mesin*Operator} = 530.965$

$SS_{Mesin} = 567.845$ $SS_{Mesin*Bahan} = 283.220$

$SS_{Operator} = 1064.545$ $SS_{Operator*Bahan} = 287.710$

$SS_{Bahan} = 907.380$ $SS_{Mesin*Operator*Bahan} = 172.690$

(a) Nyatakan model linear statistik bagi uji kaji ini.

(b) Analisis data tersebut dan nyatakan kesimpulan anda. Guna $\alpha = 0.05$.

(c) Analisa reja bagi model yang diperoleh dalam bahagian (b). (Rujuk lampiran)

(d) Dapatkan anggaran titik bagi min kepadatan untuk setiap mesin dan jenis bahan.

(e) Andaikan replika digunakan sebagai pemboleh ubah blok. Berikan jadual ANOVA bagi model penuh reka bentuk faktorial tiga faktor. Bandingkan keputusan yang diperoleh dalam bahagian (e) dengan bahagian (b).

[100 markah]

…4/-

2. (a) (i) List down and explain three basic principles of experimental design.

(ii) Give two important properties of replication.

[40 marks]

(b) An agricultural experiment, to test the effectiveness of certain crop stimulants on certain crops is carried out as follows. Three 100 hectare plots of land are selected and randomly labeled as 1, 2 and 3. In plot 1, wheat is grown. In plot 2, rye is grown and in plot 3, canola is grown. Then each plot is divided into four 25 hectare fields and the four possible stimulants are applied, one to a field, in random order. Each replicate of a factorial experiment requires 12 observations, and the experimenter has decided to run four replicates. However, due to time constraint, the experimenter is only capable of making 12 runs per day. So, the experimenter decides to run one replicate on each of the four days and to consider the days or replicates as blocks.

(i) What is the name of the design being used here?

(ii) Write down the linear model which you would use to analyze these data. Define all the parameters involved in this model.

(iii) Give the general analysis of variance for the data described in this experiment. Your ANOVA table should consist of source of variation, degrees of freedom, expected mean squares and test statistics, F_o.

[60 marks]

2. (a) *(i) Nyatakan dan huraikan tiga prinsip asas bagi analisis ujikaji.*

(ii) Berikan dua ciri penting pereplikaan.

[40 markah]

(b) *Suatu eksperimen pertanian bagi menguji keberkesanan perangsang tanaman tertentu ke atas beberapa tanaman dijalankan seperti berikut. Tiga plot tanah 100 hektar dipilih dan dilabelkan 1,2 dan 3 secara rawak. Dalam plot 1, gandum ditanam. Manakala dalam plot 2, 'rye' ditanam dan plot 3, kanola ditanam. Seterusnya, setiap plot dibahagikan kepada empat padang seluas 25 hektar dan empat perangsang diberikan secara rawak di setiap padang. Setiap replika bagi eksperimen faktorial memerlukan 12 cerapan, dan pengujikaji menetapkan untuk menjalankan empat replika. Walau bagaimanapun, disebabkan oleh kekangan masa, pengujikaji hanya boleh menjalankan 12 larian per hari. Maka, pengujikaji memutuskan untuk menjalankan satu replika setiap hari dan menganggap hari atau replika sebagai blok.*

(i) Apakah nama rekabentuk yang digunakan?

(ii) Tuliskan model linear yang akan anda gunakan bagi menganalisis data tersebut. Takrifkan semua parameter yang terlibat dalam model ini.

(iii) Berikan jadual analisis varians umum bagi data yang dijalankan dalam eksperimen ini. Jadual ANOVA anda perlu mengandungi sumber variasi, darjah kebebasan, jangkaan min kuasa dua dan ujian statistik, F_o.

[60 markah]

3. A wool shrinkage experiment was run to identify the effects of seven factors each at two levels on percentange of weight loss (%). The factors are:

A = NaOH in grams/litre (1 , 3)
B = Liquid Dilution Ratio (1:20,1:30)
C = Time in minutes (20 , 40)
D = GA in grams/litre (0 , 1)
E = DD in grams/litre (0 , 3)
F = H_2O_2 (0 , 20 ml/L)
G = Enzyme in percent (0 , 2)

(a) How many treatment combinations are involved in this experiment?

(b) If the experimenter can only afford to run 8 treatment combinations, what is the appropriate design that he would choose?

(c) Given that the generating relations are $I = ABD$, $I = ACE$, $I = BCF$ and $I = ABCG$. Identify the complete defining relation and the alias relationships of the main effects.

(d) Data for the wool shrinkage experiment are given in the table below:

Factors							Weight loss (%)
A	B	C	D	E	F	G	
-1	-1	-1	1	1	1	-1	1.18
1	-1	-1	-1	-1	1	1	22.30
-1	1	-1	-1	1	-1	1	23.10
1	1	-1	1	-1	-1	-1	1.73
-1	-1	1	1	-1	-1	1	27.00
1	-1	1	-1	1	-1	-1	1.72
-1	1	1	-1	-1	1	-1	0.56
1	1	1	1	1	1	1	39.00

List down the treatment combinations that will be run in this experiment and estimate the factor effects.

[100 marks]

3. *Ujikaji pengecutan kapas dijalankan bagi mengenalpasti kesan tujuh faktor masing-masing pada dua aras terhadap peratusan kehilangan berat (%). Faktor-faktor tersebut adalah:*

A = NaOH dalam gram/liter (1 , 3)
B = Nisbah pencairan cecair (1:20,1:30)
C = Masa dalam minit (20 , 40)
D = GA dalam gram/liter (0 , 1)
E = DD dalam gram/liter (0 , 3)
F = H_2O_2 (0 , 20 ml/L)
G = Enzim dalam peratusan (0 , 2)

(a) Berapa kombinasi rawatan terlibat dalam ujikaji ini?

(b) Sekiranya pengujikaji hanya mampu melarikan 8 kombinasi rawatan, apakah rekabentuk yang sesuai untuk dipilih?

(c) Diberikan bahawa hubungan penjana adalah $I = ABD$, $I = ACE$, $I = BCF$ dan $I = ABCG$. Kenalpasti hubungan pentakrif yang lengkap dan perhubungan alias bagi kesan utama.

(d) Data bagi ujikaji pengecutan kapas diberikan dalam jadual di bawah:

Faktor							*Pengurangan berat (%)*
A	*B*	*C*	*D*	*E*	*F*	*G*	
-1	*-1*	*-1*	*1*	*1*	*1*	*-1*	*1.18*
1	*-1*	*-1*	*-1*	*-1*	*1*	*1*	*22.30*
-1	*1*	*-1*	*-1*	*1*	*-1*	*1*	*23.10*
1	*1*	*-1*	*1*	*-1*	*-1*	*-1*	*1.73*
-1	*-1*	*1*	*1*	*-1*	*-1*	*1*	*27.00*
1	*-1*	*1*	*-1*	*1*	*-1*	*-1*	*1.72*
-1	*1*	*1*	*-1*	*-1*	*1*	*-1*	*0.56*
1	*1*	*1*	*1*	*1*	*1*	*1*	*39.00*

Senaraikan kombinasi rawatan yang akan dijalankan dalam ujikaji ini dan anggar faktor kesan utama.

[100 markah]

4. (a) Consider the three-factor model

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk}$$

Assuming that all the factors are random:

(i) Complete the analysis of variance table below.

(ii) Propose appropriate test statistics for all effects.

Source	DF	E(MS)
A		$\sigma^2 + c\sigma^2_{\tau\beta} + bc\sigma^2_{\tau}$
	b-1	$\sigma^2 + c\sigma^2_{\tau\beta} + a\sigma^2_{\beta\gamma} + ac\sigma^2_{\beta}$
C		$\sigma^2 + a\sigma^2_{\beta\gamma} + ab\sigma^2_{\gamma}$
	(a-1)(b-1)	$\sigma^2 + c\sigma^2_{\tau\beta}$
BC		$\sigma^2 + a\sigma^2_{\beta\gamma}$
	b(a-1)(c-1)	σ^2
Total		

[40 marks]

(b) Consider a 2^4 factorial design to identify the effect of pH, concentration, time and pressure on the yield in an industrial process. The experiment were replicated twice. The data are given below:

Treatment	pH	Concentration	Time	Pressure	Yield (Rep I)	Yield (Rep II)
1	3	10	0.2	0	28.64	31.06
2	3	10	0.07	0	27.93	28.71
3	10	10	0.2	0	8.86	10.844
4	10	10	0.07	0	17.99	19.05
5	3	100	0.2	0	3.86	5.02
6	3	100	0.07	0	24.91	23.67
7	10	100	0.2	0	11.31	10.17
8	10	100	0.07	0	12.32	12.3
9	3	10	0.07	3000	36.69	36.03
10	3	10	0.2	3000	23.96	21.76
11	10	10	0.07	3000	23.38	24
12	10	10	0.2	3000	13.71	12.31
13	3	100	0.2	3000	8.68	10.9
14	3	100	0.07	3000	20.09	22.33
15	10	100	0.07	3000	17.45	17.29
16	10	100	0.2	3000	10.43	10.95

Given that:

$contrast_A = -121.876$ $contrast_{AB} = 87.396$

$contrast_B = -143.244$ $contrast_{AC} = 31.284$

$contrast_C = -141.676$ $contrast_{AD} = 20.036$

$contrast_D = 33.316$ $contrast_{BC} = -16.404$

$contrast_{BD} = -4.196$ $contrast_{CD} = -27.444$

$contrast_{ABC} = 60.796$ $contrast_{ABD} = -9.076$

$contrast_{ACD} = -1.044$ $contrast_{BCD} = 40.724$

$contrast_{ABCD} = -53.116$

Due to some restrictions, it is impossible to perform a complete replicate of a factorial design in one block. To overcome this problem, the experimenter decided to run the experiment in two blocks for each replicate.

(i) Give the outline of the design. What is the name of the design?

(ii) Analyze the data and draw conclusion.

[60 marks]

4. *(a) Pertimbangkan model tiga-faktor*

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk}$$

Andaikan semua faktor adalah rawak:

(i) Lengkapkan jadual analisis varians di bawah.

(ii) Cadangkan ujian statistik yang bersesuaian bagi semua kesan.

Sumber	*Darjah kebebasan*	*E(MS)*
A		$\sigma^2 + c\sigma^2_{\tau\beta} + bc\sigma^2_{\tau}$
	b-1	$\sigma^2 + c\sigma^2_{\tau\beta} + a\sigma^2_{\beta\gamma} + ac\sigma^2_{\beta}$
C		$\sigma^2 + a\sigma^2_{\beta\gamma} + ab\sigma^2_{\gamma}$
	(a-1)(b-1)	$\sigma^2 + c\sigma^2_{\tau\beta}$
BC		$\sigma^2 + a\sigma^2_{\beta\gamma}$
	b(a-1)(c-1)	σ^2
Jumlah		

[40 markah]

(b) Pertimbangkan reka bentuk faktoran 2^4 bagi mengenalpasti kesan pH, kepekatan, masa dan tekanan ke atas hasil dalam suatu proses industri. Eksperimen diulang sebanyak dua kali. Data bagi eksperimen diberikan di bawah:

Rawatan	*pH*	*Kepekatan*	*Masa*	*Tekanan*	*Hasil (Rep I)*	*Hasil (Rep II)*
1	*3*	*10*	*0.2*	*0*	*28.64*	*31.06*
2	*3*	*10*	*0.07*	*0*	*27.93*	*28.71*
3	*10*	*10*	*0.2*	*0*	*8.86*	*10.844*
4	*10*	*10*	*0.07*	*0*	*17.99*	*19.05*
5	*3*	*100*	*0.2*	*0*	*3.86*	*5.02*
6	*3*	*100*	*0.07*	*0*	*24.91*	*23.67*
7	*10*	*100*	*0.2*	*0*	*11.31*	*10.17*
8	*10*	*100*	*0.07*	*0*	*12.32*	*12.3*
9	*3*	*10*	*0.07*	*3000*	*36.69*	*36.03*
10	*3*	*10*	*0.2*	*3000*	*23.96*	*21.76*
11	*10*	*10*	*0.07*	*3000*	*23.38*	*24*
12	*10*	*10*	*0.2*	*3000*	*13.71*	*12.31*
13	*3*	*100*	*0.2*	*3000*	*8.68*	*10.9*
14	*3*	*100*	*0.07*	*3000*	*20.09*	*22.33*
15	*10*	*100*	*0.07*	*3000*	*17.45*	*17.29*
16	*10*	*100*	*0.2*	*3000*	*10.43*	*10.95*

Diberikan:

$kontrast_A = -121.876$ $kontrast_{AB} = 87.396$

$kontrast_B = -143.244$ $kontrast_{AC} = 31.284$

$kontrast_C = -141.676$ $kontrast_{AD} = 20.036$

$kontrast_D = 33.316$ $kontrast_{BC} = -16.404$

$kontrast_{BD} = -4.196$ $kontrast_{CD} = -27.444$

$kontrast_{ABC} = 60.796$ $kontrast_{ABD} = -9.076$

$kontrast_{ACD} = -1.044$ $kontrast_{BCD} = 40.724$

$kontrast_{ABCD} = -53.116$

Disebabkan beberapa kekangan, adalah mustahil bagi menjalankan reka bentuk ujikaji faktoran dengan replika penuh dalam satu blok. Bagi mengatasi masalah tersebut, pengujikaji memutuskan untuk menjalankan ujikaji dalam dua blok bagi setiap replika.

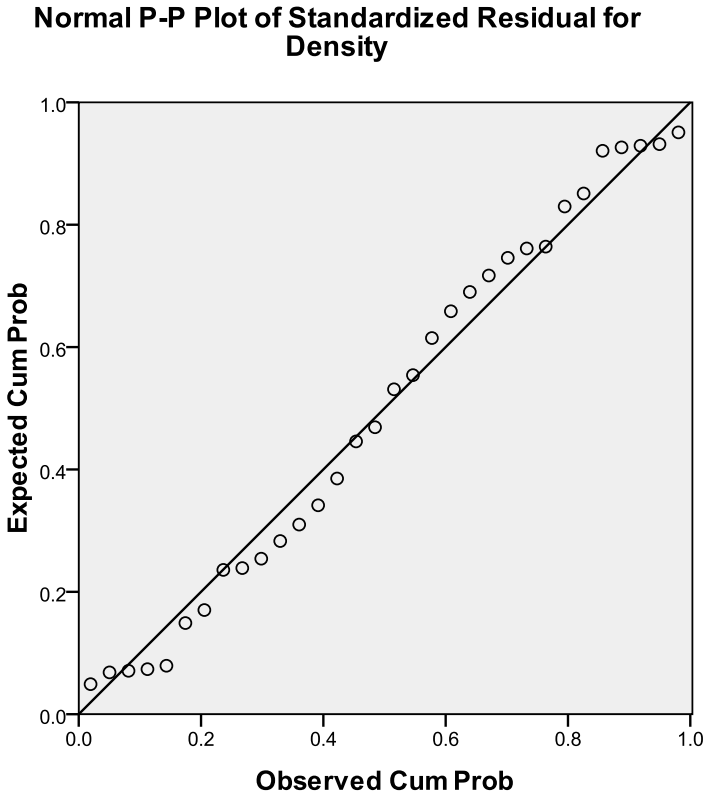
(i) *Beri gambaran bagi reka bentuk ujikaji ini. Apakah nama reka bentuk ini?*

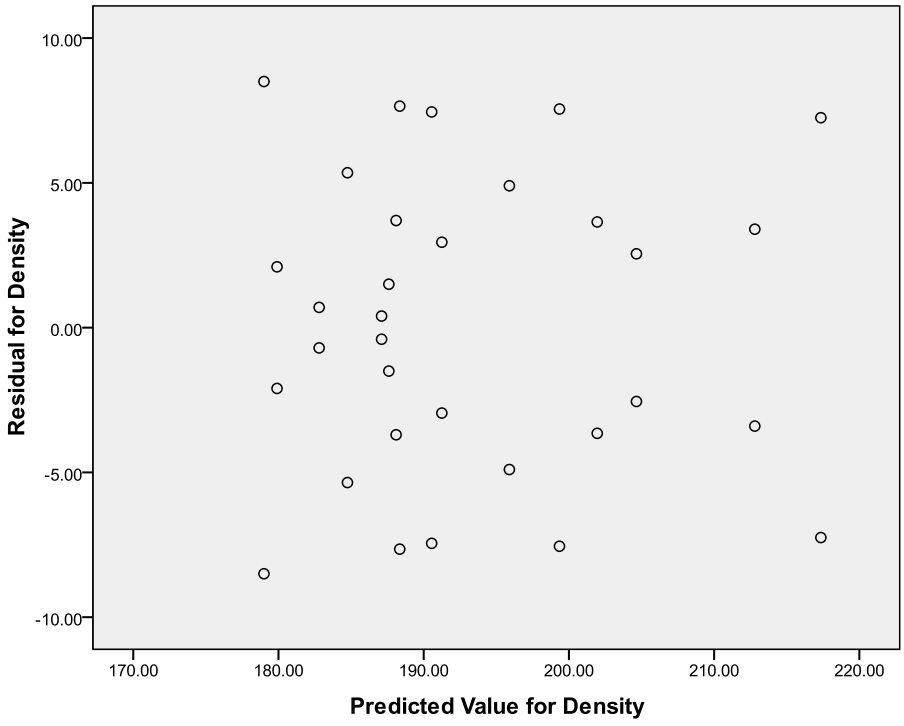
(ii) *Analisa data dan berikan kesimpulan.*

[60 markah]

Appendix

Residual Plots





…13/-

Formula

1. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SS_B = \sum_{j=1}^{b}\frac{y_{.j..}^2}{acn} - \frac{y_{....}^2}{abcn}$$

$$SS_C = \sum_{k=1}^{c}\frac{y_{..k.}^2}{abn} - \frac{y_{....}^2}{abcn}$$

$$SS_{Subtotal(AB)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij..}^2}{cn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$

$$SS_{Subtotal(AC)} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$SS_E = SS_T - SS_{Subtotal}$ atau $SS_E = SS_T - SS_A - SS_B - SS_{AB}$

2. Expected Mean Square : Split-Plot Design

$$\tau_i : \sigma^2 + ab\sigma_\tau^2$$

$$\beta_j : \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb\sum\beta_j^2}{a-1}$$

$$\gamma_k : \sigma^2 + \frac{ra\sum\gamma_k^2}{ab-1}$$

$$(\tau\beta)_{ij} : \sigma^2 + b\sigma_{\tau\beta}^2$$

$$(\beta\gamma)_{jk} : \sigma^2 + \frac{r\sum\sum(\beta\gamma)_{jk}^2}{(a-1)(b-1)}$$

$$\varepsilon_{ijk} : \sigma^2$$

- ooo O ooo -